Strategic Gaming Analysis for Electric Power Systems: An MPEC Approach

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***Abstract*—Transmission constraints and market concentration may prevent power markets from being fully competitive, allowing firms to exercise market power and raise prices above marginal cost. We present a strategic gaming model for analyzing such markets; it represents an oligopolistic market economy consisting of several dominant firms in an electric power network. Each generating firm submits bids to an ISO, choosing its bids to maximize profits subject to anticipated reactions by rival firms. The single-firm model is formulated as a Mathematical Program with Equilibrium Constraints (MPEC) with a parameter-dependent spatial price equilibrium problem as the inner problem. Power flows and pricing strategies are constrained by the ISO's linearized DC optimal power flow (OPF) model. A penalty interior point algorithm is used to compute a local optimal solution of the MPEC. Numerical examples based on a 30 bus network are presented, including multi-firm Nash equilibria in which each player solves an MPEC of the single-firm type.**

***Index Terms*—Game theory, operating economics, deregulation, algorithms, market models.**

I. Introduction

IN THE next few years, most electric power markets in the United States will be opened to competition. Competition will then displace government regulation as the major factor in determining prices [17]. But if competition is weak, it may fail to force prices down to marginal cost. For instance, restructuring in England and Wales allowed two large power companies to control 79% of the market in 1990. Several analyzes have presented evidence that the two companies have been able to use their market power to maintain prices above marginal cost [23], [33], [34].

Examples of important questions being asked about the price effects of restructuring include the following (e.g. [23]). What will happen to prices? How will they be affected by market power? Do the peculiarities of electric networks offer opportunities to exercise market power that are absent in other commodity markets? How are these answers affected by particular market characteristics and what degree of market power results in significant market inefficiencies and inequities that should be mitigated? This paper presents a numerical approach for projecting the short-run price implications of market power. Long run effects (firm entry and exit) are not addressed, nor is the question of what threshold is appropriate for determining whether market power requires mitigation.

The industrial organization literature is rich with theoretical and empirical examples that show that peculiarities of markets can make a large difference in market power [26]. In power markets, relevant considerations include the system's physical characteristics (e.g., transmission bottlenecks and their location relative to generation capacity and demand), auction design (such as multipart bids as in the UK versus single price auctions as in the California PX-Wilson auction), transmission pricing, the ability to bypass auctions via bilateral transactions, whether firms are vertically integrated, and market power mitigation (such as "must-run" provisions for California generators that are needed for voltage support).

There exist at least four distinct approaches to answering such questions. First are *ex post* analyzes of existing markets, such as attempts to empirically determine whether UK prices have diverged from marginal cost (e.g., [34]). The other approaches, market concentration analyzes, laboratory experiments, and modeling, are used in *ex ante* studies of proposed market structures. Concentration analyzes, such as the Herfindahl Index used by the U.S. Department of Justice for merger analyzes, do not address prices directly, but instead consider whether one or a few firms have a dominant share of the market. Lab experiments [4], [30], [32] can investigate subtle interactions of market structure and participant behavior, especially in dynamic (repeated auction) settings; however, they often involve naive (student) subjects, and their expense makes replication, sensitivity analysis, and generalization to other situations difficult. Using artificial adaptation agents [20], [28] instead of live subjects in experiments could address these objections. The final approach, modeling, calculates (usually static) price equilibria, and is more easily generalized and analyzed for sensitivity.

Many modeling studies of market power in electricity markets have already been undertaken. The literature can be classified in terms of the market mechanisms that are simulated, how the electric network is modeled, and what types of interactions occur between rival power producers. Relative to market structures, most modeling studies implicitly or explicitly assume that a bidding process supervised by an ISO results in a set of market clearing prices (e.g., [24]), although bilateral trading has also been simulated [16]. In terms of network modeling, many studies disregard transmission constraints entirely [3], [13], [31] or use a transshipment network that ignores Kirchhoff's voltage law [5], [9], [16]; however, this means that

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unique opportunities that electric networks provide for market manipulation are disregarded. To correct this shortcoming, some studies have used AC [12] or linearized DC [24], [7] load flow models. Such studies have found, for example, that small noneconomic generators or phase shifters can be used by companies to manipulate flows, congesting selected lines so that competition in local markets is weakened [6], [11], [36].

A crucial difference among models is the type of interaction that is assumed among rival generators. The interactions range from intense competition to collusion. Collusion has been modeled, for example, as a cooperative Nash bargaining game [5] and as limit-pricing, where existing firms collude to prevent new firms from entering [16]. Another uncompetitive situation is a Stackelberg game, in which a leader with market power manipulates prices, generation capacity, or transmission capacity in order to maximize profits, subject to the anticipated reactions of naive followers who believe they cannot affect prices [15].

In more competitive models, the type of interaction can often be summarized in terms of a firm's "conjectural variations": what does each firm assume about its rivals' responses to its actions? The most intense competition results from Bertrand competition [16], [36], in which each firm believes that its rivals will not alter their prices. Less intense, and more commonly assumed, is Cournot competition, where rivals are presumed to hold their output fixed [3], [10], [11], [37]. As an example of the results of such an analysis, Oren [24] shows that under one particular set of Cournot assumptions, generators will set outputs in order to destroy the value of transmission congestion charges. However, the Cournot and Bertrand conjectures seem naive for ISO-type auctions, in which firms bid a (generally) upward sloping supply curve. A more reasonable conjecture for a firm would be that if it changes its bid curve, other firms would not alter theirs. This is termed supply function competition [19], and is the basis of several power market models [7], [12], [14], [25], [31], [36]. The resulting price equilibria generally lie between the Bertrand and Cournot extremes; however, these supply function models are either designed for very simple systems (e.g., 1 or 3 nodes) or use a grid search approach to find the optimal pricing strategy for each firm, which drastically limits the number of bidding strategies that can be considered.

The models presented below make two contributions. First, unlike previous approaches, they calculate an oligopolistic price equilibria for general linearized DC networks using the supply function conjectural variation, while considering a continuous range of bidding strategies for each supplier. The resulting optimization problem for each generating firm is a two level program [1], in which the upper (leader) level chooses the parameters of the firm's bid curve and the lower (follower) level simulates the market-clearing algorithm of the ISO. Such bilevel problems are inherently nonconvex and more difficult to solve than normal mathematical programs, as pointed out in [11]. The second contribution is that an advanced interior point algorithm is used to solve the bilevel problem. Although no algorithm (short of complete enumeration) can guarantee optimal solutions to nonconvex problems, computational experience indicates that the approach is effective and efficient in finding good solutions.

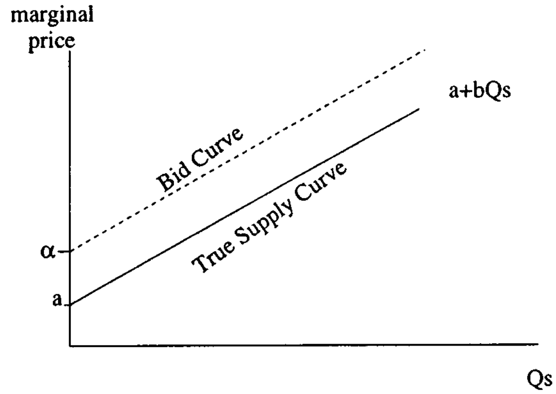

Fig. 1. Marginal cost (supply) function and bid curve.

II. Market Assumptions

In our models, there are a number of generating firms, each owning some number of units. Each unit submits an hourly bid to provide power to an independent system operator (ISO). Each bid is in the form of a linear nondecreasing marginal price function (\$/MWh). Demand is represented as a downward sloping curve, so demand bidding can also be included. The ISO then decides how much power to buy from which units, how much power to deliver to consumers, and what prices to charge, based upon an optimal power flow (OPF) calculation a la Schweppe et al. [27]. The bids, rather than the true cost functions, are what the ISO uses in its OPF. The units determine their own bids. By deviating their bids from marginal cost (unbeknownst to the outside world), a firm may be able to increase its profit (Fig. 1); in general, optimal bids will differ from marginal cost. A Nash supply function equilibrium will occur when no company has any incentive to unilaterally change its bids. The extent to which the equilibrium bids exceed marginal cost is a measure of market power (the so-called Lerner index).

In this model, we assume that firms only manipulate the intercept α of the bid functions, and not its slope b. There are several reasons for this assumption. First, slopes of marginal cost functions for individual generators are usually very shallow, so the very steep slopes that would result from manipulating just b would not be credible. Second, the steepness of an aggregate bid curve for an entire firm can be manipulated by having different markups $\alpha - a$ for different units. Finally, if both a and b can be chosen, we have found that unique solutions rarely exist; in general, the literature recognizes that if bid functions can assume any form, unique equilibria exist only under very restrictive conditions [14], [25].

We have developed two supply function-based models that represent the interactions of the firms, analogous to the Cournot model of Cardell et al. [11]. The first is a bilevel model that represents the individual firm's problem: it determines the bids that are profit maximizing for that firm, while the other firms' bids are held constant (see also [8]). The second model coordinates the results of individual firm models in an attempt to find an equilibrium for all generators' bids.

The single-firm model can be phrased as follows. There are a number of dominant firms in the market, each making bids to the ISO. The model tries to determine the optimal bids for one firm. This firm can be thought of as a leader of a Stackelberg game, and the leader calculates its bids based on what it anticipates the other firms would do. The other firms' assumed reactions are based on their bids, and are considered by solving one quadratic program representing the ISO's (linearized) OPF problem.

The model presented in [11] is similar to ours, except for three differences. The first difference is that in [11], the quantities supplied by the rival generating units are fixed (Cournot assumption). We instead consider their reactions based on their bid curves (a possibility suggested in [11]), which we have argued is a more realistic conjectural variation. A second difference is that Cardell et al. allow generators to also demand power and own transmission congestion contracts; this is a generalization that is easily accommodated in our framework.

Third, the methods used to solve the problem differ. Cardell et al. [11] solved their model using GAMS, an optimization software package, solving the problem a number of times using different penalty parameters in each run until the equilibrium conditions are approximately satisfied. We instead use an interior point algorithm to find an equilibrium point. Note that the algorithm presented below could also be used to efficiently solve single firm models based on other conjectural variations, such as the Cournot assumption in [11].

III. Mathematical Formulation

Mathematically, the electric power market can be formulated as an oligopolistic market equilibrium model on a network consisting of the node set $\mathcal{N}$ and arc set $\mathcal{A}$. There are several dominant firms in the market, each controlling a certain number of units with quadratic supply curves. In this section, we give the precise formulation of the single-firm problem. The following section describes an algorithm to solve the single-firm problem along with numerical results.

A. The Single-Firm Problem

In essence, the single-firm problem is a two-level constrained optimization problem [1] in which the dominant firm in question takes as inputs its perceived market conditions (including any competitor firms' bids and supply and demand functions) and maximizes profit under a set of spatial price equilibrium constraints. In the terminology of a bilevel optimization problem, the first-level variables consist of the firm's bids and the second-level problem is the ISO's single commodity spatial price equilibrium (SPE) problem including additional constraints due to Kirchhoff's voltage law [35]. The SPE/Kirchhoff second-level problem (actually, an OPF problem) is parametrized by the firm's bids which are restricted by given bounds; such bounds constitute the first-level constraints. The first-level objective is the firm's profit, equal to its revenues less its costs.

There are two equivalent ways of stating the second-level parametric problem: either in the form of a (parametric) convex quadratic program or as a (parametric) linear complimentarily problem (LCP). In the latter form, the resulting two-level optimization problem is an instance of a mathematical program with equilibrium constraints (MPEC); this is a class of constrained optimization problems that is the subject of a recent comprehensive study [21]. In what follows, we will give both formulations of the second-level problem and identify the overall MPEC formulation of the single-firm decision problem.

The single-firm problem focuses on a dominant firm denoted f. The supply and demand functions are assumed to be separable and affine. The following is the notation used in the formulation of this problem.

Indices:

i	nodes (busses) in the network
ij	arc from i to j
m	Kirchhoff voltage loops in the network.

Sets in the problem:

$\mathcal{N}$	set of all nodes
$\mathcal{A}$	set of all arcs
$\mathcal{S}_f$	set of nodes with generators under control of firm f
$\mathcal{P}$	set of all nodes with generators
$\mathcal{D}$	set of all demand nodes
$\mathcal{L}$	set of Kirchhoff's voltage loops m
$\mathcal{L}_m$	ordered set of arcs (clockwise) associated with loop m

In practice, the sets $\mathcal{P}$ and $\mathcal{D}$ are not necessarily disjoint and their union could be a proper subset of $\mathcal{N}$. If there are multiple units at a node, artificial nodes could be defined so that there is only one unit at each node, so this is the case that we will consider. In the linearized DC models [27], [35], the Kirchhoff loops ensure the uniqueness of the net flow on each arc in the solution to the second-level problem, for fixed but arbitrary first-level inputs. Thus if m is the number of undirected arcs and n is the number of nodes in the network, then the number of (independent) loops needed is $m-n+1$.

Parameters:

a_i, b_i	intercept and slope of supply (marginal cost) function $Q_{Si} \mapsto a_i + b_i Q_{Si}$ for the generator at node $i \in \mathcal{P}$
c_i, d_i	intercept and slope of demand function $Q_{Di} \mapsto c_i - d_i Q_{Di}$ for consumers at node $i \in \mathcal{D}$
$\overline{\alpha}_i$	upper bound of the bid for the unit at node $i \in \mathcal{S}_f$
$\overline{Q}_{Si}$	upper bound of production capacity for the unit at node $i \in \mathcal{P}$
$\overline{T}_{ij}$	maximum transmission capacity on arc $ij \in \mathcal{A}$
r_{ij}	reactance on arc $ij \in \mathcal{A}$. In the DC model, resistance is assumed negligible relative to reactance, and is ignored.
s_{ijm}	± 1 corresponding to the orientation of arc ij in loop $m \in \mathcal{L}$.

By dropping the subscripts i and j, we will use the same letters to denote the vectors of the above parameters; for instance $\overline{Q}_S$ denotes the vector of $\overline{Q}_{Si}$ for $i \in \mathcal{P}$. This convention applies also to the variables introduced below.

First-level decision variables of firm f:

α_i	bid for the unit at $node i \in \mathcal{S}_f$

Primal variables in 2nd-level SPE/OPF:

Q_{Si}	quantity of power generated by the unit at node i
Q_{Di}	quantity of power demanded at node i
T_{ij}	MW transmitted from i to j

Throughout, it is understood that $Q_{Si} = 0$ for all $i \notin \mathcal{P}$ and $Q_{Dj=0}$ for all $j \notin \mathcal{D}$; thus these supply and demand variables can be taken to be defined at all nodes in the network.

Dual variables in 2nd-level SPE/OPF:

λ_i	marginal cost at node i
μ_i	marginal value of generation capacity for unit at node i
θ_{ij}	marginal value of transmission capacity, arc ij
γ_m	shadow price for Kirchhoff voltage law, loop m

Let α_i for all nodes $i \in \mathcal{P}\backslash\mathcal{S}_f$ be fixed at the levels previously bid by the rival firms, so that α_i is a variable only for $i \in \mathcal{S}_f$. The second-level SPE/OPF problem is formally stated as the following convex quadratic program in variables Q_S, Q_D, and T, parametrized by bids α_i for $i \in \mathcal{S}_f$.

- Objective function: maximization of social welfare (consumer surplus minus apparent costs)

$$\max \sum_{i\in\mathcal{D}} \left(c_i Q_{Di} - \tfrac{1}{2}\, d_i Q_{Di}^2\right) - \sum_{i\in\mathcal{P}} \left(\alpha_i Q_{Si} - \tfrac{1}{2} b_i Q_{Si}^2\right) \quad (1)$$

- Nonnegative demand and transmission variables: for all nodes $i \in \mathcal{D}$ and arcs $ij \in \mathcal{A}$,

$$Q_{Di} \geq 0; \qquad T_{ij} \geq 0 \quad (2)$$

- Lower bounds for supply variables: for all $i \in \mathcal{P}$,

$$Q_{Si} \geq 0 \quad (3)$$

- Capacity constraints for transmission and supply variables: for all nodes $i \in \mathcal{P}$ and arcs $ij \in \mathcal{A}$,

$$Q_{Si} \leq \overline{Q}_{Si} \quad (4)$$

$$T_{ij} \leq \overline{T}_{ij} \quad (5)$$

- Conservation constraints: for all $i \in \mathcal{N}$,

$$Q_{Di} - Q_{Si} + \sum_{j:\, ij\in\mathcal{A}} T_{ij} - \sum_{j:\, ji\in\mathcal{A}} T_{ji} = 0 \quad (6)$$

- Kirchhoff's voltage law: for all $m \in \mathcal{L}$,

$$\sum_{ij\in\mathcal{L}_m} s_{ijm} r_{ij} = 0. \quad (7)$$

We introduce some matrices in order to write the above formulation in vector-matrix notation. Let Δ denote the (node, arc) incidence matrix of the electric network; i.e.,

$$\Delta_{i\ell} \equiv \begin{cases} 1 & \text{if } \ell = ij \in \mathcal{A} \text{ for some } j \in \mathcal{N} \\ -1 & \text{if } \ell = ji \in \mathcal{A} \text{ for some } j \in \mathcal{N} \\ 0 & \text{otherwise.} \end{cases}$$

Let R denote the (arc, loop) incidence matrix of signed reactance coefficients; i.e.,

$$R_{ij,\,m} \equiv \begin{cases} s_{ijm} r_{ij} & \text{if } ij \in \mathcal{L}_m \\ 0 & \text{otherwise.} \end{cases}$$

Forming the Karush-Kuhn-Tucker optimality conditions for the above primal problem and using the dual variables, μ, θ, λ, and γ, we obtain the following mixed linear complimentarily formulation of the second-level problem. (The notation $u \perp v$ means that two vectors u and v are perpendicular, and diag(w) is a diagonal matrix whose diagonal entries are the components of the vector w.)

$$\begin{array}{lll} 0 \leq \overline{Q}_S - Q_S & \perp & \mu \geq 0 \\ 0 \leq Q_S - \underline{Q}_S & \perp & -\lambda + \mu + \alpha + \text{diag}(b) Q_S \geq 0 \\ 0 \leq Q_D & \perp & \lambda - c + \text{diag}(d) Q_D \geq 0 \\ 0 \leq \theta & \perp & \overline{T} - T \geq 0 \\ 0 \leq T & \perp & \Delta^T \lambda + \theta + R^T \gamma \geq 0 \\ \lambda \text{ free} & & Q_D - Q_S + \Delta T = 0 \\ \gamma \text{ free} & & RT = 0. \end{array} \quad (8)$$

The loops in $\mathcal{L}$ are determined so that if T'_a denotes the net flow on the undirected arc a in the network, then for all fixed but arbitrary vectors α, the quadratic program (1)–(7) has a unique solution in the supply and demand quantities Q_{S_i} and Q_{D_j} and net flows T'_a. A useful property of this solution is presented in the proposition below which follows easily from some well-known sensitivity results for convex quadratic programs; see [21] for instance.

Proposition.: For each vector α *or, there exists a unique globally optimal solution of the quadratic program (1)–(7), denoted* $(Q_D(\alpha), Q_S(\alpha), T'(\alpha))$, *with* $T'(\alpha)$ *denoting the vector of net flows on the arcs; furthermore, this solution is a piecewise linear function in* α.

With the second-level problem defined, we may now complete the first-level problem that firm f solves in order to determine its bids and other decision variables. Specifically, taking α_i for all $i \in \mathcal{S}_f$ as given, firm f computes a vector of bids $\alpha^f \equiv (\alpha_i\colon i \in \mathcal{S}_f)$, a vector of supplies Q_S, a vector of demands Q_D, and a vector of flows T in order to maximize its profit:

$$\begin{array}{ll} \text{maximize} & \pi \equiv \displaystyle\sum_{i\in\mathcal{S}_f} \left(\lambda_i Q_{Si} - a_i Q_{Si} - \frac{b_i}{2} Q_{Si}^2\right) \\ \text{subject to} & 0 \leq \alpha_i \leq \overline{\alpha}_i \quad \forall\, i \in \mathcal{S}_f \\ \text{and} & \text{the mixed LCP (8).} \end{array} \quad (9)$$

The above constrained optimization problem is an instance of an MPEC, more specifically, a mathematical program with linear complimentarily constraints [21]. In the above form, the objective function is neither convex nor concave in its arguments, because of the bilinear term $\lambda_i Q_{Si}$. For various reasons, it would be useful to reformulate the objective function as a concave function. This reformulation is indeed possible by exploiting the constraints in the system (8). This equivalent resulting objective function, which we will refer to as $P_f(Q_D, Q_S, T, \theta, \lambda, \mu)$, is used in the computational procedure for solving firm f's profit maximization problem (9):

$$\begin{aligned} \pi_f(\lambda, Q_S) &\equiv P_f(Q_D, Q_S, T, \theta, \lambda, \mu) \\ &= \sum_{i\in\mathcal{D}} (c_i Q_{Di} - d_i Q_{Di}^2) \\ &\quad - \sum_{i\in\mathcal{S}_f} \left(a_i Q_{Si} + \frac{b_i}{2} Q_{Si}^2\right) - \sum_{ij\in\mathcal{A}} \theta_{i,j} \overline{T}_{i,j} \\ &\quad - \sum_{i\in\mathcal{P}\backslash\mathcal{S}_f} \left[\mu_i \left(\overline{Q}_{Si} - \underline{Q}_{Si}\right) + a_i \left(Q_{Si} - \underline{Q}_{Si}\right)\right. \end{aligned}$$

$$+b_i Q_{Si}^2 + \lambda_i \underline{Q}_{Si} - b_i \underline{Q}_{Si} Q_{Si}\Big]; \tag{10}$$

IV. The Single-Firm Algorithm

A. PIPA

To solve the single-firm problem, we use a penalty interior point algorithm (PIPA). In this section, we will give a brief description of PIPA. See [21] for more details.

PIPA solves the following general problem:

$$\begin{array}{lcl} \text{minimize} & & f(w, x, y, z) \\ \text{subject to} & & x \in \boldsymbol{X} \\ 0 \le y & \perp & w = q + Nx + My + Lz \ge 0 \\ 0 & = & r + Gx + Hy + Kz \end{array} \tag{11}$$

To relate this to our model, $x \equiv \alpha^f$, $y \equiv (Q_D, Q_S, T, \mu, \theta)$, $z \equiv (\lambda, \gamma)$, and $f(x, y, z) \equiv P_f(Q_D, Q_S, T, \theta, \lambda, \mu)$. M, L, N, G, H and K are all matrices of appropriate dimensions containing the corresponding coefficients for x, y, and z, and q and r are constant vectors. Note that M is symmetric positive semidefinite, L is the negative transpose of H, and K is the zero matrix. The result is that this is equivalent to a convex quadratic program if x is fixed.

There are four basic steps to this algorithm.

0. Initialization. Choose an initial point, (x^0, y^0, w^0, z^0) such that y^0, w^0 are strictly positive. There are also PIPA parameters that must be chosen, but the details are omitted.
1. Direction Generation. Find a direction $(dx^\nu, dy^\nu, dw^\nu, dz^\nu)$ by solving the following convex quadratic program.

$$\text{minimize} \sum_{s=w,x,y,z} (df_s)^T\, ds + \frac{1}{2}(dx)^T Q\, dx$$

subject to

$$x + dx \in \boldsymbol{X}$$
$$\|dx\|_\infty \le c(\|r + Gx + Hy + Kz\|_2^2 + (w)^T y)$$
$$diag(w)dy + diag(y)dw = -diag(yy)w + \sigma\mu e$$
$$\begin{bmatrix} N \\ G \end{bmatrix} dx + \begin{bmatrix} M \\ H \end{bmatrix} dy + \begin{bmatrix} I \\ 0 \end{bmatrix} dw + \begin{bmatrix} L \\ K \end{bmatrix} dz = \begin{bmatrix} w - q + Nx + My + Lz \\ -r - Gx - Hy - Kz \end{bmatrix} \tag{12}$$

where for $s = w, x, y, z$, $df_s \equiv \nabla_s f(x, y, w, z)$, μ is a barrier parameter equal to $y^T w / \dim(y)$, σ and c are chosen parameters, and Q is a chosen positive semidefinite matrix. (In the implementation, Q is chosen to be the identity matrix.) The objective function, f, is concave and thus, we can use the Hessian matrix of f to find a "good" direction.
2. Step Size. A step size is determined by finding the root of a quadratic polynomial. If the root lies outside the interval (0, 1), then the value 0.99 is used. A merit function with a penalized term of the form

$$Pen_\beta \equiv f(x, y, w, z) + \beta\phi(x, y, w, z)$$

where $\beta > 0$ is the penalty parameter and $\phi(x, y, w, z) \equiv (r + Gx + Hy + Kz)^T(r + Gx + Hy + Kz) + w^T y$, is used to guide the progress of the algorithm.
3. Termination. If the norm of the direction is smaller than some given tolerance, stop. That is, $\|(dx^\nu, dy^\nu, dw^\nu, dz^\nu)\| < tol$, where tol is the chosen tolerance. Otherwise, let $\nu = \nu + 1$ and go to step 1.

B. Overall Algorithm

While PIPA is the heart of the main algorithm, we have developed an algorithm that uses PIPA as a subroutine to solve the single-firm problem. In this case, we also have a subroutine, *fixalpha*, that solves the quadratic program, SPE/OPF stated in Section III, equations (1)–(7).

0. Choose a random seed. [0 pt]
1. Generate three random sets of bids for Firm A. Solve the SPE/OPF, using *fixalpha*, for each set of bids.[0 pt]
2. For each solution found in Step 1, determine the value of the function, π (in (9)), at that point. Let (x, y, w, z) be the solution point giving the largest value of π [0 pt]
3. For $c = 0.2, 0.3, 0.4, 0.5$, use $(x, y + c * \mathbf{1}, w + c * \mathbf{1}, z)$ as the initial point and run PIPA, where $\mathbf{1}$ is a vector of all ones. Other positive perturbations could be employed, but these worked well.[0 pt]

In this case, there will be four final solutions found by PIPA. After running numerous examples, it seems that the best of these four solutions will be close to the largest value we can find. This can be seen in the following section.

C. Single-Firm Results

For the first problem, the generation and transmission data was based upon [2]. It includes a network with 30 busses(nodes), 41 lines, 12 loops, 6 supply buses, and 21 demand busses. Also given were generator cost functions, reactances, upper bounds on the supply quantities, and upper bounds on the transmission flows. The transmissions limits $\overline{T}$ are set to 60% of the values assumed in [2] so that at least some limits would be binding in the solutions. We have split the six supply nodes into two sets, so that Firm A owns three units and Firm B owns three units. Then, we solved the single firm problem for both firms, assuming the other firm bids its marginal costs for all its units ($\alpha = a$). The generator assumptions are:

	Firm A			Firm B		
Unit	1	2	3	1	2	3
Bus	8	11	13	1	2	5
a[$/MWh]	32.5	30	30	20	17.5	10
Capacity[MW]	35	30	40	200	80	50

We assume the following demand curve at each of the 21 demand busses: $P_i = 40 - d_i Q_{Di}$ where d_i is chosen so that $P_i = \$30/MWh$ when Q_{Di} equals the value assumed in [2]. This relatively price responsive demand might be reflective of a market in which demand bidding is active and self generation is an economic option for many customers.

To solve the single-firm problem for Firm A, we follow the procedure of Section IV-B. We first start with three random sets

TABLE I
RESULTS OF MULTI-FIRM ALGORITHM RUN ON A 30 NODE NETWORK

Results for Multi-firm Algorithm						
Loop	Firm A bid	π_A	PIPA iterations	Firm B bid	π_B	PIPA iterations
1	33.78, 31.20, 31.34	25.0	17	28.95, 26.65, 18.55	2107.8	39
2	33.20, 30.70, 30.71	38.6	172	28.07, 24.50, 16.63	2124.2	31
3	33.31, 30.75, 30.74	25.3	18	28.17, 24.19, 16.09	2122.1	25
4	33.05, 30.76, 30.74	25.5	21	28.24, 24.17, 16.38	2121.4	26
5	33.08, 30.76, 30.75	26.1	9	27.93, 23.40, 15.77	2130.0	22
6	33.09, 30.86, 30.83	22.7	22	28.04, 24.43, 16.15	2132.7	50
7	33.14, 30.79, 30.77	24.7	22	28.03, 24.29, 16.55	2129.8	48
8	33.16, 30.79, 30.78	24.6	21	27.99, 24.15, 16.42	2130.8	49
9	33.16, 30.81, 30.78	23.9	21	28.00, 24.19, 16.56	2131.2	50

of bids for Firm A, and solved the SPE/OPF for each set. Next, we chose the solution, (x, y, w, z) that gives the largest value when plugged into the profit function, π. Note that the vector x is equal to the vector of bids. As an initial point of PIPA, we use $(x, y + c * \mathbf{1}, w + c * \mathbf{1}, z)$, where c is a positive perturbation factor and $\mathbf{1}$ is a vector of ones. This ensures the positivity of the y and w vectors required by PIPA.

Random Bids and SPE results			
Bid 1	Bid 2	Bid 3	π
4.45	32.29	4.81	-968.1
5.01	6.05	5.03	-1137.9
21.03	26.65	10.51	-777.0

In this case, the third run gave us the best objective value of -777.0. We use the solution from this run to start PIPA with different perturbations.

PIPA results				
c	Bid 1	Bid 2	Bid 3	π
0.2	33.04	31.10	22.96	-92.04
0.3	33.58	31.55	31.33	24.42
0.4	33.80	31.54	31.34	24.47
0.5	33.67	31.54	31.34	24.45

The optimum profit found was approximately 24.47. We have done many more runs and have always achieved a similar value. Obviously, the more runs we do, the more confident we become that this is a global solution. Also, as a check, we solved the SPE/OPF for 50 random sets of bids (one for each of A's units); none of these solutions yielded a profit over 20. It should be noted, however, that there could be more than one local optimum because the MPEC is nonconvex due the complementarily constraints.

For Firm B, we ran this procedure and the best PIPA results are seen below. (In this case, we omit the randomly generated bids.)

PIPA Results				
c	Bid 1	Bid 2	Bid 3	π
0.3	27.80	23.89	16.29	2094

The best profit found is 2094, which is two orders of magnitude above A's profits. This is because of B's relatively low marginal costs. These single-firm solutions can be viewed as Stackelberg equilibria in which one firm (the leader) manipulates its bids in order to maximize profit while the other firm(s) are a "competitive fringe" who always set their bids equal to their units' marginal cost. In the next section, we consider the case in which two or more firms behave strategically.

These strategic results can be contrasted with the outcome when each firm bids honestly ($\alpha = a$). In that case, A earns $\pi_A = 21.19$ and B earns $\pi_B = 979.45$ compared to the strategic profits of $\pi_A = 24.47$ and $\pi_B = 2094$.

V. The Multi-Firm Problem

In a game theoretic context, the multi-firm problem can be phrased as a Nash game with multiple players, each being a dominant firm with respect to the ISO, able to predict how the ISO will process the bids of all players. The main feature of this Nash game is that each player is solving a MPEC, rather than a standard optimization problem. In this multiple-firm case, each leader is trying to maximize its profits based on what the market does as well as what the other dominant firms do. The goal in the multi-firm problem is to find an "equilibrium," such that if any firm changes its actions unilaterally, its profit will decrease. An equilibrium exists when there is no incentive for any firm to change its behavior unilaterally. The equilibrium conditions can be stated as follows:

$$(\overline{\alpha}^1, \overline{\alpha}^2, \ldots, \overline{\alpha}^q) \text{ is a multi-firm equilibrium if}$$
$$\overline{\alpha}^f \in SOL(\overline{\alpha}^i, i \in \mathcal{P}/\mathcal{S}_f) \;\; \forall f = 1 \ldots q,$$

where $SOL(\overline{\alpha}^i, i \in \mathcal{P}/\mathcal{S}_f)$ is the set of bids that solve the single-firm problem for firm f.

It is important to note that, in general, a Nash supply function equilibrium in pure strategies does not necessarily exist, nor is it necessarily unique [14], [19]. Indeed, simple examples can be defined in which an infinite up and down cycling of prices occurs [7]. This results from an Edgeworth-like process in which rival firms undercut each other's bids until one realizes that it is better off conceding defeat. The latter firm then jacks up its price for the small portion of the market that rivals cannot serve due to capacity constraints. Borenstein et al. [10] find a similar dynamic for a Cournot market. Nash's theorem guarantees that an equilibrium in mixed strategies (in which players choose bids randomly) still exists. However, we have found that for a wide range of capacity, demand, and cost conditions on a simple network, a pure strategy equilibrium does exist and can be computed [7]. Therefore, although the models of this paper cannot guarantee that an equilibrium will be found, we anticipate that in most cases they will succeed.

A. Multi-Firm Algorithm

We use a diagonalization algorithm, analogous to that used in [11], to solve the multi-firm problem. It performs the following steps for q firms:

Initialization:

0. Set all firms' bids to their supply curve intercept, a_i, for $i = 1 \dots q$. Set $i = 1$.
1. For Firm i, generate three random sets of bids, then solve the SPE/OPF for each set, where the bids from [7] Firm $j(\neq i)$ are fixed.
2. Solve the single-firm problem for Firm i using PIPA with perturbation of 0.4 on the best of the three results from step 1. Call this solution (x^i, y^i, w^i, z^i). Update firm i's bids as x^i.
3. If $i \neq q$, set $i = i + 1$ and go to step 1.

General Step:

4. Use $(x^i, y^i + c*\mathbf{1}, w^i + c*\mathbf{1}, z^i)$, for a chosen parameter c, as an initial point and run PIPA to solve the single-firm problem for Firm i, to get a new solution, (x^i, y^i, w^i, z^i). Update Firm i's bids as x^i. Repeat for $i = 1 \dots q$.
5. Repeat step 4 until the maximum number of iterations is reached or a satisfactory solution is found.

B. Multi-Firm Numerical Results

We have applied the algorithm defined in Section IV-C, but allowing both A and B to behave strategically. We set $c = 0.4$. See Table I.

Table I shows nine iterations. The solution has converged to an equilibrium in which A's prices and profits are lower than in the Stackelberg (single-firm) case (Section IV-C). B's profits, on the other hand, are greater than in its Stackelberg solution. This points out that asymmetrics in networks and costs can mean that alternative assumptions about strategic interactions can affect different firms in different ways.

VI. Conclusion

A practical and efficient MPEC-based procedure for calculating oligopolistic price equilibria for an electric power market has been developed and illustrated. The equilibrium is a "supply function" equilibrium, in which rival generators optimize their bid curves under the assumption that other firms will not change theirs. Future work will investigate the properties of the algorithm, incorporate resistance losses, and explore generator strategies under alternate cost, transmission, and demand conditions.

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References

[1] "Hierarchical Optimization," *Annals Oper. Res.*, vol. 34, 1992.

[2] O. Alsac and B. Stott, "Optimal Load Flow with Steady-State Security," *IEEE Trans. Power Appar. Sys.*, vol. 93, no. 3, pp. 745–751, 1973.

[3] B. Andersson and L. Bergman, "Market Structure and the Price of Electricity: An Ex Ante Analysis of Deregulated Swedish Markets," *Energy J.*, vol. 16, no. 2, pp. 97–109, 1995.

[4] S.R. Backerman, S.J. Rassenti, and V.L. Smith, "Efficiency and Income Shares in High Demand Networks: Who Gets the Congestion Rents When Lines are Congested?," Dept. of Economics, Univ. of Ariz., Jan. 1997.

[5] X. Bai, S.M. Shahidehpour, V.C. Ramesh, and E. Yu, "Transmission Analysis by Nash Game Method," *IEEE Trans. Power Systems*, vol. 12, no. 3, pp. 1046–1052, 1997.

[6] R. Baldick and E. Kahn, "Contract Paths, Phase-Shifters, and Efficient Electricity Trade," *IEEE Trans. Power Systems*, vol. 12, no. 2, pp. 749–755, 1997.

[7] C. Berry, B.F. Hobbs, W. Meroney, R. O'Neill, and W. Stewart, "Analyzing Strategic Bidding Behavior in Transmission Networks," IEEE Tutorial on Game Theory Applications in Power Systems, to be published.

[8] B. Biewald, D.E. White, and W. Steinhurst, "Horizontal Industrial Market Structure and Market Power in New England Electricity Markets: Simulation Results and a Review of NEPOOL's Analysis," Vermont Dept. of Public Service, Tech. Rep. no. 39, 1997.

[9] S. Borenstein and J. Bushnell, "An Empirical Analysis of Bohn, Spot Pricing of the Potential for Market Power in California's Electricity Industry," Univ. of Cal. Energy Inst., 1997.

[10] S. Borenstein, J. Bushnell, and S. Stoft, "The Competitive Effects of Transmission Capacity in a Deregulated Electricity Industry," Univ. Calif. Energy Inst., Berkeley, CA, PWP-040, 1996.

[11] J. Cardell, C.C. Hitt, and W.W. Hogan, "Market Power and Strategic Interaction in Electricity Networks," *Res. and Energy Econ.*, vol. 19, no. 1-2, pp. 109–137, 1997.

[12] R.W. Ferrero, S.M. Shahidehpour, and V.C. Ramesh, "Transaction Analysis in Deregulated Power Systems Using Game Theory," *IEEE Trans. Power Systems*, vol. 12, no. 3, pp. 1340–1347, 1997.

[13] R. Green, "Increasing Competition in the British Electricity Spot Market," *J. Ind. Econ.*, vol. 44, pp. 205–216, 1996.

[14] R. Green and D. Newbery, "Competition in the British Electric Spot Market," *J. Poli. Econ.*, vol. 100, pp. 929–953, 1992.

[15] B.F. Hobbs and K.A. Kelly, "Using Game Theory to Analyze Electric Transmission Pricing Policies in the U.S.," *Euro. J. Oper. Res.*, vol. 56, no. 2, pp. 154–171, 1992.

[16] B.F. Hobbs and R.E. Schuler, "Assessment of the Deregulation of Electric Power Generation Using Network Models of Imperfect Spatial Competition," *Papers Reg. Sci. Asso.*, vol. 57, pp. 75–89, 1985.

[17] S. Hunt and G. Shuttleworth, *Competition and Choice in Electricity*, N.Y.: J. Wiley, 1996.

[18] Y. Ji and B.F. Hobbs, "Including a DC Network Approximation in a Multiarea Probablistic Production Costing Model," *IEEE Trans. Power Systems*, vol. 13, no. 3, pp. 1121–1127, Aug. 1998.

[19] P.D. Klemperer and M.A. Meyer, "Supply Function Equilibria," *Econometrica*, vol. 57, pp. 1243–1277, 1989.

[20] V. Krishna and V.C. Ramesh, "Intelligent Agents in Negotiations in Market Games, Part 2: Application," *IEEE trans. Power Systems*, vol. 13, no. 3, pp. 1109–1114, Aug. 1998.

[21] Z.Q. Luo, J.S. Pang, and D. Ralph, *Mathematical Programs with Equilibrium Constraints*, N.Y.: Cambridge Univ. Press, 1996.

[22] T.C. Miller, T.L. Friesz, and R.L. Tobin, *Equilibrium Facility Location on Networks*, N.Y.: Springer, 1996.

[23] D.M. Newbery, "Power Markets and Market Power," *Energy J.*, vol. 16, pp. 39–66, 1995.

[24] S.S. Oren, "Economic Inefficiency of Passive Transmission Rights in Congested Electricity Systems with Competitive Generation," *Energy J.*, vol. 18, no. 1, pp. 63–83, 1997.

[25] A. Rudkevich and M. Duckworth, "Strategic Bidding in a Deregulated Generation Market: Implications for Electricity Process, Asset Valuation and Regulatory Response," *The Electricity Journal*, Jan. 1998.
[26] F.M. Scherer and D. Ross, *Economic Performance*, 3d ed. Boston: Houghton-Mifflin, 1990.
[27] F.C. Schweppe, M.C. Caramanis, R.E. Tabors, and R.E. Bohn, *Spot Pricing of Electricity*. Norwell, MA: Kluwer, 1998.
[28] G. Sheble, "EPRI Auction Market Simulator: Genetic Algorithm Solution of Electric Power Markets Using Available Transfer Capability,", www.ee.istate.edu/ sheble.
[29] T. Takayama and G. Judge, *Spatial and Temporal Price and Allocation Models*. London: North-Holland, 1971.
[30] R.J. Thomas, R.D. Zimmerman, and R. Ethier, "Power Web User's Manual," School Elect. Comp. Engin., Cornell University, Ithaca, NY, PSerc 97-10, 1997.
[31] N.M. von der Fehr and D. Harbord, "Spot Market Competition in the UK Electricity Industry," *Econ. J.*, vol. 103, pp. 531–546, 1993.
[32] J. Weiss, "Market Power Issues in the Restructuring of the Electricity Industry: An Experimental Investigation," Harvard Business School, Boston, MA, Nov. 1997.
[33] F.A. Wolak and R.H. Patrick, "The Impact of Market Rules and Market Structure on the Price Determination Process in the England and Wales Electricity Market," Dept. Economics, Stanford Univ., June 1996.
[34] C.D. Wolfram, "Strategic Bidding in a Multi-Unit Auction: An Empirical Analysis of Bids to Supply Electricity in England and Wales," School of Business, Harvard University, Cambridge, MA, July 1996.
[35] A.J. Wood and B.F. Wollenberg, *Power Generation, Operation and Control*, 2nd ed, N.Y.: J. Wiley, 1995.
[36] Z. Younes and M. Ilic, "Generation Strategies for Gaming Transmission Constraints: Will the Deregulated Electric Power Market Be an Oligopoly," in *Proc., Hawaii International Conference on System Sciences*, Jan. 6-9, 1997.
[37] J.-Y. Yuan and Y. Smeers, "Spatially Oligopolistic Models with Cournot Producers and Regulated Transportation Prices," Oper. Res., to be published.

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