A Joint Energy and Transmission Rights Auction on a Network with Nonlinear Constraints: Design, Pricing and Revenue Adequacy

Draft, March 2, 2006

Richard P. O'Neill, Udi Helman, Benjamin F. Hobbs, *Senior Member, IEEE*, Michael H. Rothkopf and William R. Stewart, Jr..

***Abstract*— This paper presents an auction model that implements a sequence of forward and spot auction markets operated by a Regional Transmission Organization (RTO) or Independent System Operator (ISO) for energy and several types of transmission rights simultaneously, including point-to-point rights as options and obligations and flowgate rights. The auction model incorporates non-linear transmission constraints and is a generalization of an earlier model with linear transmission constraints [15]. The non-linear model has several applications, including forward auctions for transmission rights conducted on an AC load flow model, the extension of real power markets to include reactive power, which would also require an AC model, and the modification of auctions for transmission rights on a DC load flow model to include nonlinear losses for the purpose of loss hedging. To demonstrate market clearing in a nonlinear case, a numerical example is given of an auction for transmission rights on a DC model with quadratic losses. A proof shows revenue adequacy of the auction sequence in the case of nonlinear transmission constraints that define a convex feasible region.**

***Index Terms*— Auctions, electric power, nonlinear constraints, revenue adequacy, transmission rights.**

I. INTRODUCTION

The forward and spot auction markets operated by Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs) allow for trade in multiple wholesale electricity products (since for our purposes here, ISOs and RTOs perform essentially the same functions, we will use ISO generically in the remainder of the paper).[1] We refer to forward markets as any market that clears prior to "real-time" or the dispatch hour, in practice typically including hourly, day-ahead, monthly and multi-month timeframes. Both in theory and increasingly in practice, the pre-day-ahead forward auction markets can include both point-to-point and flow-based financial transmission rights, energy, operating reserves, and capacity with a call option.[2] In the day-ahead and real-time, or spot, auction markets, products typically include real energy (in many regions now priced through locational marginal pricing), regulation and operating reserves, while transmission users are charged for marginal transmission usage (congestion and possibly losses) and congestion revenues accruing to financial transmission rights are settled. Hence, the ISO markets are becoming more "complete": market prices are available for a fuller range of the products and services provided by generation and transmission assets (and in the future for demand response capability), as needed for economic efficiency.

This paper presents a general auction model that implements key features of the existing ISO auction markets and provides a framework for introducing new products while maintaining revenue adequacy. In an earlier paper [15] (see also [16]), the authors introduced a joint energy and transmission rights auction (JETRA; henceforth, the "auction model" or "auction") on a network characterized by a linearized, DC load flow model.[3] The generality of the model in [15] has allowed it to be used by market designers in the United States and other countries.[4] The auction model in [15] and the analogous one presented here for the nonlinear case synthesize and extend several prior, and competing, auction models to allow for the simultaneous auction of flow-based transmission rights, point-to-point transmission rights specified as options or obligations, real energy and possibly other products.

Auctions with linear transmission constraints are used in several ISO markets. For example, forward auctions for

R.P. O'Neill and U. Helman are with the Federal Energy Regulatory Commission, Washington, DC 20426 USA (e-mail: richard.oneill@ferc.gov; udi.helman@ferc.gov). The views expressed here do not reflect an official position of the FERC.

B.F. Hobbs is with Johns Hopkins University, Baltimore, MD 21218 USA (e-mail: bhobbs@jhu.edu).

M.H. Rothkopf is with Rutgers University, Piscataway, NJ 08854 USA (e-mail: rothkopf@rutcor.rutgers.edu)

W.R. Stewart, Jr. is with the College of William and Mary, Williamsburg, VA 23187 USA (e-mail: william.stewart@business.wm.edu).

[1] As of this writing, these include include the California ISO, PJM RTO, Midwest ISO, New York ISO, and ISO New England. For a survey of the designs of these markets, see [19].

[2] For more detailed description of transmission rights, see [3], [7], [9], [15] and the discussion below. Energy is not yet a pre-day-ahead product in the ISO markets, but a forward reserves market is operated by the New England ISO. For capacity products, in current markets with such a product the call option is triggered by a reliability condition and the price paid is the locational marginal price. An alternative design would be a negotiated strike price for exercising the option. Ancillary services and capacity are not discussed in this paper. However, it is worth noting that while forward transmission and energy requires a network model, increasingly there is interest also in locational reserves and/or capacity.

[3] On the derivation of the DC load flow approximation, see, e.g., [20]).

[4] The authors are aware of the model in [15] being used and cited in market design discussions in ERCOT (Texas) and the California ISO as well as by European researchers.

transmission rights in PJM employ a DC load flow model.[5] However, other regions currently use an AC load flow model for the forward auctions of transmission rights (e.g., New York ISO).[6] The inclusion of reactive power in the auction market would also require the AC load flow model [6], [13], [14]. Moreover, proposals for forward hedging of marginal losses through unbalanced point-to-point transmission rights would require auctions with a DC load flow model and quadratic losses [8]. This paper thus generalizes the linear auction model in [15] to the case with nonlinear constraints. A theoretical result presented here is that for the auction with nonlinear transmission constraints that define a convex feasible region, the forward and spot auction sequence can be revenue adequate (the analogous proof for the linear case is shown in [15]).[7]

Following the practice in the existing ISO markets, the auction model is designed to be conducted in a sequence of forward (auction) markets culminating in a real-time or dispatch market. The various types of transmission rights are traded in the pre-day-ahead forward markets and congestion revenues accruing to those rights are determined in either a day-ahead or a real-time auction (where there is a day-ahead auction, the congestion revenues are settled using the day-ahead prices) on the basis of locational marginal prices (LMPs) or transmission shadow prices (called flowgate marginal prices in [7]). Energy could also enter the pre-day-ahead auctions for transmission rights; [15] offered examples where a forward commitment of energy was needed to support the auction of transmission rights into transmission constrained areas.

The remainder of the paper is organized as follows. Section II offers a description of the types of energy and transmission right bids in the auction. Section III presents the mathematical statement of the auction model with nonlinear transmission constraints, and provides more mathematical detail on how transmission rights are specified for the auction. Section IV discusses the settlement system and conditions for maintaining revenue adequacy. Simple examples of the full auction with the AC load flow are difficult to present in compact form; instead, Section V provides an example based on a DC load flow with quadratic losses. Section VI offers conclusions. An appendix presents the proof of revenue adequacy for a sequence of forward and spot auctions with transmission constraints that define a convex feasible region.

II. Auction Products

The types of electricity products that can be traded in this auction have been described in [19], [1], [15], [16] as well as in earlier seminal papers, such as [2], [3], [9]. The debate over the implementation of alternative transmission rights formulations is recounted in [11], [12], [15], among other sources, and will not be repeated here. This section briefly describes these products, with additional detail provided in Sections III.B and IV.

Energy. Several types of energy related bids are typically allowed in energy and transmission auctions: supply offers, demand bids, adjustment bids (such as incremental and decremental energy supply offers used for congestion management), "virtual" bids,[8] and congestion bids.[9] The model presented here can accommodate each of these types of bids. For purposes of this discussion, some important aspects of energy auctions are not considered, such as the inclusion of unit commitment start-up and no-load costs and restrictions on bids to control the exercise of market power.[10]

Heretofore, these types of energy offers and bids have been offered in ISO day-ahead and real-time markets. In a pre-day-ahead ISO auction market for energy and transmission, energy transactions can be used also to balance point-to-point transmission rights in a lossy system, or to increase transmission capacity for forward sale—an example of the latter being the San Francisco nomogram constraint discussed in [15]. These one-sided or unbalanced "rights" (actually, obligations) can be called "nodal revenue rights."

Simple Transmission Capacity Rights and Portfolio Combinations. As noted in the flow-based rights literature (e.g., [2], [3], there are two types of elementary transmission rights, which we call here the "simple rent collection right" and the "simple rent payment right." The simple rights are defined over transmission elements, which include lines, transformers or other grid elements whose capacity is limited by thermal, stability, or contingency considerations. These are often generically called "flowgate" rights [7]. For each element, the direction of the flows covered by the simple rights are defined initially, and arbitrarily, in either a positive or negative direction. The simple rent collection right on a transmission element confers to the *buyer* the right to *collect* the rents that would occur when that element is congested, for the capacity specified in the right. Because the flow-based right is directional, the holder of a rent collection right only collects non-negative rents. Alternatively, the simple rent payment

[5]The rules and network modeling details for the auction of obligation and option point-to-point financial transmission rights, called Fixed Transmission Rights (FTRs), are available on the PJM RTO web-site, www.pjm.com/markets/ftr.

[6]The rules and network modeling details for the auction of obligation point-to-point financial transmission rights, called Transmission Congestion Contracts (TCCs), are available on the New York ISO web-site, www.nyiso.com. The auction is conducted using an AC optimal power flow model that respects thermal, voltage and stability constraints within the New York control area.

[7]However, as with any transmission rights auction, additional rules are needed to account for revenue inadequacy due to changes in system topology.

[8]A virtual bid is a bid into the forward market (in the case of the spot market, the day-ahead energy market) that is not necessarily associated with physical energy supply or demand. It is used for purposes of financial hedging and is sometimes called a "financial" energy bid. Appropriate creditworthiness rules are required for such bids. In the current Eastern U.S. ISO markets, such bids are submitted in the day-ahead energy market, where they affect the day-ahead price and are then bought back in the real-time, or dispatch, market at the real-time price. For reliability purposes, the system operator typically uses a two-phase day-ahead market clearing, in which first both real and virtual supply bids are accepted and day-ahead prices determined, and second, a reliability commitment is conducted using bids associated with real generation only and forecast load.

[9]Congestion bids represent what a bilateral transaction is willing to pay for marginal congestion charges associated with its transmission schedule. They are typically used on the boundaries of LMP systems where there is no fully arbitraged LMP on the "other side" of the boundary.

[10]Bid restrictions for market power reasons can include a uniform, "safety net" bid cap for all generators, bid thresholds on particular generators that trigger market power mitigation, and other measures.

right commits the *seller* to *pay* any rents (to the buyer) on a transmission element, for the capacity specified in the right. The rent payment right allows a market participant to create financial ("virtual") capacity on a specific transmission element.[11] The simple rights can be aggregated into more complex rights through linear combinations or portfolios, for example, covering several transmission lines, nomograms, or constructing "point-to-point" rights on the basis of power flow distribution factors [15].[12] In general, the individual rights and the portfolios are more likely to offer an imperfect rather than a perfect hedge against congestion charges, since an exact match between a particular point-to-point transaction and a portfolio of the rights would be difficult to create and maintain (although some authors propose that the ISO provide subsidies to maintain particular portfolios as complete hedges, e.g., [3]).[13] For many holders, then, the flowgate right will be used to collect rents on heavily congested transmission elements rather than to hedge any particular power transaction.

Point-to-Point Transmission Rights. There are two types of point-to-point rights, the obligation right[14] and the option right. A point-to-point obligation transmission right is defined as the obligation to collect or pay the congestion charge rents that result from the physical flows associated with putting power into the system at a point of injection (POI) and taking power out of the system at a point of withdrawal (POW) [9]. Note that for a point-to-point obligation, flow in one direction adds an equivalent amount of "counterflow capacity" in the other direction. This can be generalized to multiple point–to–multiple point rights, which we will call network rights. These rights may simply aggregate point-to-point rights or may be "contingent" rights, when they hedge multiple possible POIs and POWs.[15]

The amount that is received (or paid, if negative) by the holder of the obligation right is the nodal price at the POW minus the nodal price at the POI multiplied by the quantity specified in the right. If the injections and withdrawals of power specified in the right are scheduled in the market in which the right is settled (and then executed in the real-time market, if different from the settlement market), then the right provides a complete congestion hedge, i.e., no additional payment for congestion will be necessary.

The point-to-point option transmission right is defined as the option to put power into the system at one or more POIs and take power out of the system at one or more POWs.[16] It can be interpreted as the right to collect congestion rents if they exceed zero, without the obligation to pay that amount if negative. This option faces considerable computational challenges in an auction model with nonlinear transmission constraints, in that a separate load flow has to be calculated for each combination of possible exercised options [11]. However, using a linearized, or DC load flow approximation, model, the computation can be reduced sufficiently, thus facilitating the implementation of point-to-point options (alternatively, portfolios of flowgate rights could be used to approximate a point-to-point option right).[17]

Point-to-point rights can be balanced or not balanced. A balanced right is one in which the quantity injected is equal to the quantity withdrawn. An unbalanced right does not have this requirement, so that an entity can approximate losses (average or marginal) by specifying a higher quantity injected than withdrawn.

Finally, as with the flowgate right, the point-to-point rights can be bought from or sold into the auction (see Section III.A).

III. The Auction with Nonlinear Constraints

A. Mathematical Statement

The types of energy bids and transmission rights described in Section II are represented in the mathematical statement of the JETRA-AC model below. Further detail about the specification of the bids follows in Section III.B. For ease of recognition, the notation used in the model borrows and extends from standard references, such as [2], [9]. All variables are assumed to be real power; however, the framework allows for the inclusion of reactive power (VARs). Units of the decision variables and right hand sides (RHS) of the constraints are in megawatts (MW), while the objective function coefficients are in \$/MWh. The JETRA-AC model is:

$$JETRA - AC : \max v(t_{K1}, t_{K2}, g, x, f^{+}, f^{-}, y)$$

$$= b_{K1} t_{K1} + b_{K2} t_{K2} + Bg,$$

[11] Moreover, if the ISO did not itself allocate rights, but simply facilitated an auction of buyers and sellers (see Section III), then all sellers would offer rent payment rights. As with virtual energy, appropriate creditworthiness rules are required for virtual transmission rights.

[12] The combination of buying a rent collection right in one direction on a transmission element and selling a rent payment right for the equivalent quantity in the opposite direction on the same transmission element creates a financial "obligation" associated with that element. That is, the holder of this portfolio collects rents when the element is congested in one direction, but owes rents when the element is congested in the opposite direction. For a set of simple rights that constructs a point-to-point right, holding this portfolio on each transmission element in the set is analogous in the linear model to the point-to-point obligation rights discussed below [3].

[13] A transmission right that offers a perfect, or complete, congestion hedge is defined as one in which the congestion charges associated with spot market transactions are equal to the congestion revenues obtained by the rights holder. An imperfect hedge is one in which the congestion charges are not equal to the revenues to transmission rights holders.

[14] This is more accurately described as a "contract" [9], since it embodies an obligation to pay congestion revenues, but is now conventionally termed a transmission right.

[15] The point-to-point obligation transmission right is equivalent to the forward transmission congestion contracts (TCCs) described in [9]. The network rights were described in FERC's proposed capacity reservation tariff [4]. The contingent rights are discussed in [16].

[16] The option TCCs discussed in [9] are equivalent to these point-to-point option rights in the linearized DC load flow model [15].

[17] In an auction with linear constraints, the point-to-point option is shown to be equivalent to setting aside capacity in each transmission constraint for positive increments of flow associated with the right but ignoring negative flows ("counterflow") in the opposite direction (e.g., [15]). This allows the auction to be run using a single set of power flow distribution factors (PTDFs), but no analogous reduction has been developed for the nonlinear case. Moreover, as shown in [15], the reduction in the linear case implies that an appropriately defined bundle of flowgate rights dominates the point-to-point option.

$$
\begin{aligned}
\text{s.t.} \quad & A_{K2}t_{K2} + A_g g - y && = 0, && (\pi) && (1)\\
& \beta^K t_{K1} + K'(x, y, f^+ - f^-) && \leq F^K, && (\mu) && (2)\\
& \beta^+ t_{K1} + f^+ && \leq F^{+max}, && (\theta^+) && (3)\\
& \beta^- t_{K1} + f^- && \leq F^{-max}, && (\theta^-) && (4)\\
& K''(x, y) - f^+ + f^- && = 0, && (\gamma) && (5)\\
& t_{K1} && \leq T_{K1}, && (\psi_{K1}) && (6)\\
& t_{K2} && \leq T_{K2}, && (\psi_{K2}) && (7)\\
& g && \leq G, && (\rho) && (8)\\
& t_{K1}, t_{K2}, g, f^+, f^- && \geq 0, && &&
\end{aligned}
$$

where the notation is defined as follows:

Index Sets

(All of the elements in the formulation are either vectors or matrices. The index sets and indices defined below are part of the definitions of these elements)

- H is the set of elements in the system on which rights can be sold, h = 1, ... , $n[H]$,where the notation $n[.]$ defines the cardinality of the set in brackets. The set H is partitioned into two subsets, H' and H''. H' is the set of elements, $h = 1, ..., n[H']$, that produce linear constraints in the auction, and H'' is the set of elements, $h = n[H'] + 1, ..., n[H]$, that produce nonlinearities in the constraint set.
- I is the set of nodes, $i = 1, ..., n[I]$, in the system.
- $K1$ is the set of transmission bids, $k = 1, ..., n[K1]$, to buy or sell rights on individual transmission elements (e.g., line, capacitor, transformer, or other transmission equipment) or flowgates.
- $K2$ is the set of transmission bids, $l = 1, ..., n[K2]$, to buy or sell point-to-point rights.
- M is the set of bids to buy or sell energy, $m = 1, ..., n[M]$.

Variables

- f^+, f^- are vector variables with elements, f_h^+ and $f_h^-, h \in H'$, representing the flow on transmission element h in the positive and negative direction respectively (defined arbitrarily). We could thus define the net real power flow as f, where $f = f^+ - f^-$. f_h is the net flow on transmission element h (and could be generalized to include reactive power).
- g is a vector, $\{g_m, m \in M\}$, where g_m represents the quantity of energy bought by or sold to the m^{th} energy bidder. This may be only generation, only consumption, or a bilateral transaction (that combines generation and consumption).
- t_{K1}, t_{K2} are vectors, $\{t_k^{K1}, k \in K1\}, \{t_l^{K2}, l \in K2\}$, where t_k^{K1} represents the quantity of rights awarded (bought or sold) to the k^{th} bid for $K1$ transmission type rights and t_l^{K2} represents the quantity of rights awarded to the l^{th} bid for $K2$ transmission type rights.
- x is the set of endogenously set variables that affect the topology and performance of the network, e.g., phase shifter settings, DC line settings, and contingency set asides on transmission elements for locational reserves. In today's practice, these variables are typically exogenously set, but the auction can accommodate bidding for these settings in the auction; see e.g. [15].
- y is a vector, $\{y_i, i \in I\}$, where y_i is the amount of real power injected at node i (withdrawn at node i if $y_i < 0$) that is induced by the type $K2$ bids and g bids that were awarded.
- $\pi, \mu, \theta^+, \theta^-, \gamma, \psi_{K1}, \psi_{K2}, \rho$ are vectors of Lagrange multipliers associated with each set of primal constraints in the auction.

Parameters and Functions

- b_{K1}, b_{K2} are vectors, $\{b_k^{K1}, k \in K1\}, \{b_l^{K2}, l \in K2\}$. The vector b_k^{K1} represents the value the bidder associates with transmission bid k. If the bid is to buy, $b_k^{K1} > 0$, and, if to sell, $b_k^{K1} < 0$. The vector b_l^{K2} represents the value the bidder associates with transmission bid l. If the bid is to buy, $b_l^{K2} > 0$, and, if to sell, $b_l^{K2} < 0$.
- A_g is a matrix, $\{a_{im}^g, i \in I, m \in M\}$, where a_{im}^g is the net injection of energy at node i associated with energy bid m. If the net injection is negative, then energy is being withdrawn.
- A_{K2} is a matrix, $\{a_{il}^{K2}, i \in I, l \in K2\}$, where a_{il}^{K2} defines the net injections at node i associated with bid l for a $K2$ type transmission right. Again, if the net injection is negative, then energy is being withdrawn.
- B is a vector, $\{B_m, m \in M\}$, where B_m represents the value associated with energy bid m. If the bid is to buy, $B_m > 0$, and if the bid is to sell, $B_m < 0$.
- F^{+max}, F^{-max}, and F^K are transmission capacity constraints–thermal, stability or contingency limits–associated with one or more transmission elements (e.g., several transmission elements grouped as a flowgate).
- G is a vector, $\{G_m, m \in M\}$, where G_m is the upper bound on the amount of energy, g_m, that the bidder is willing to generate, consume, or both at the price B_m.
- T_{K1}, T_{K2} are vectors, $\{T_k^{K1}, k \in K1; T_l^{K2}, l \in K2\}$, where T_k^{K1} is the maximum amount of $K1$ transmission rights that the bidder desires in bid k and T_l^{K2} is the maximum amount of $K2$ transmission rights that the bidder desires in bid l.
- β^+, β^- are matrices, $\{\beta_{hk}^+, h \in H', k \in K1\}, \{\beta_{hk}^-, h \in H', k \in K1\}$, where β_{hk}^+ represents the quantity in the positive direction on transmission element h that is requested in bid k (i.e., up to the quantity

T_k^{K1}), and β_{hk}^{-} represents the quantity in the negative direction on transmission element h that is requested in bid k.

β^K is a matrix, $\{\beta_{kh}^K, h \in H'', k \in K1\}$, where β_{kh}^K defines the quantity of h^{th} transmission interaction constraint (3.2) that the k^{th} bid for a $K1$ right requires. (The "K" set of interaction constraints can include all constraints not associated with particular transmission elements, such as those implied by voltages, nomograms or other constraints.)[18]

$K'(x,\ y,\ f^+ - f^-) \leq F^K$ is the set of transmission interaction constraints exclusive of Kirchhoff's laws. (We use net injections, y, and, for simplicity, ignore the characteristics of individual generators or consumption.)

$K''(x,y)$ represents Kirchhoff's laws. Note that, given a particular setting of $x, \partial K''/\partial y$ are the power transfer distribution factors (PTDFs).

The set of optimal bids accepted by the auction is denoted as $\{t_{K1}^*, t_{K2}^*, g^*\}$ and the set of Lagrange multipliers that satisfy the Karush-Kuhn-Tucker (KKT) conditions for the auction is denoted $\{\pi^*,\ \mu^*,\ \theta^{+*},\ \theta^{-*},\ \gamma^*,\ \psi_{K1}^*,\ \psi_{K2}^*,\ \rho^*\}$.

Constraint (1) is a power balance equation. It requires that net injections from the energy part of the auction along with net injections implied by the point(s)-to-point(s) transmission auction equal the overall net injections, y, at each node. Constraints (2-4) require that the $K1$ rights are subject to the specified set of network interactions and upper bounds on the system (i.e., represent a feasible physical dispatch). Constraint (5) enforces Kirchhoff's laws, including losses. Constraints (6-8) enforce the upper bounds on each type of bid.

In general the set of interaction constraints, $R = \{(x, y, f^+, f^-)|$ where (x, y, f^+, f^-) satisfies constraints (2) and (5)$\}$, is non-convex. This constraint set is often represented by an energy management system combined with judgment of experienced operators and the results of contingency analyses. The set R can include relationships between power, reactive power, Kirchhoff's law, losses, voltage, phase angle regulators, DC lines and all specified contingencies. These constraints ensure the reliability/feasibility of the implied dispatch. Here we formulate the model as if all such constraints can be stated explicitly.

Several further generalizations are worth mentioning. First, the model could allow "all or nothing" bids. This can be accomplished by adding integer variables and replacing the upper bound constraints as follows:

$$t_k - T_k z_k = 0 \quad \text{or} \quad g_m - G_m z_m = 0, \tag{9}$$

where z_k are 0/1 variables. More generally, lower bounds can be specified in the model for t_{K1}, t_{K2}, and g.

The introduction of integer variables allows further for unit commitment (i.e., dynamic optimization) of generation (e.g., [10]) and transmission [6], [17] as well as for the consideration in the longer-term auction markets of entry by technologies with economies of scale, as is characteristic of large generation and transmission projects. Elsewhere we have shown that efficient market-clearing prices in auction markets with non-convexities in technology and production can be shown to be possible through a two-part pricing scheme in which the integral activity (e.g., start-up) is offered a specific ("non-anonymous") price while the associated commodity (e.g., energy) is cleared through a single or uniform market clearing ("anonymous") price [5], [18]. Most ISOs have adopted such a two-part pricing regime for the short-term energy markets.

Finally, to this point, we have assumed that the ISO is defining and selling transmission rights; for example, in the eastern U.S. ISO markets, the market operator conducts the auction as if it owns the transmission under its control, but then returns auction revenues to transmission holders. Hence, in the auction, the capacity held by the ISO would have a F_h^{+max} and/or $F_h^{-max} > 0$. However, the auction model can be adapted to a market in which the ISO is simply the auctioneer of transmission rights held by others. If $F^{+max(n)} = F^{-max(n)} = F^{K(n)} = 0$, the ISO holds no transmission rights and trading takes place among the rights holders. The initial allocation of rights can be done through an auction or by other methods.

B. Specifying the Bids for Energy and Transmission Rights

Because in some cases they modify familiar notation from prior transmission rights models (e.g., [2], [9]), this section elaborates on the product definitions and characteristics introduced in Section II, reviewing the mathematical formulation of the products as required by the auction model.

Energy. An energy bid (real or virtual) is defined by G_m, B_m and the vector a_m^g. An individual bid can be part of a stepwise function with each step a separate value of the index m. B_m then would specify the bid for a step, and G_m is the maximum quantity for the step at this bid ($g_m \leq G_m$). B_m is in value (e.g., \$) per quantity of power (e.g., MW). Adding the locational aspect, A_g is a matrix of net injection coefficients defining the net injection at each node i, with elements a_{im}^g. A bid to inject power thus requires $a_{im}^g > 0$, while a bid to withdraw power requires $a_{im}^g < 0$.[19]

Simple Transmission Capacity Rights and Portfolio Combinations. As indicated above, a bid for a transmission right of either the flow-based ($K1$) or the point-to-point ($K2$) type is defined by b and T. What differentiates the bids for $K1$ and $K2$ rights is the specification of the matrix of parameters, β, for $K1$ rights. Those parameters (extending notation introduced by [2]) indicate how much capacity on transmission element h is taken up by a unit of this type of right. In fact, any vector, $\beta_k^+, \beta_k^-, \beta_k^K$, defines a flowgate. To implement the auction, bidders would specify the βs on the basis of system information provided by the ISO about the

[18] Together, T_k^{K1}, b_k^K, and the vectors β_k^+, β_k^-, and β_{kh}^K define the k^{th} bid for a type $K1$ transmission right. Similarly, bid l for type $K2$ transmission rights is fully defined by knowing T_l^{K2}, b_l^{K2}, and a_{il}^{K2}.

[19] For example, to define a simple bid to sell one unit of energy at node '6' in a network, a_{6m}^g = 1 and $a_{im}^g = 0$ for $i \neq 6$. If a_{6m}^g = -1, then it would be a bid to buy one unit of energy just at node 6.

constraint sets $K'(x, y, f^+ - f^-)$ and $K''(x, y)$, and network flows R.

Consider first a simple rent collection transmission right, $b_{K1} > 0$, where b_{K1} is interpreted as the smallest amount a bidder is willing to pay to buy a unit of T_{K1}. A bid, $k \in K1$, for this right on transmission element j in the positive direction is defined as $\beta^+_{hk} = 1$ for $h = j$ and 0 for $h \neq j$. Similarly, for the simple rent collection right to capacity in the opposite, or negative, direction for element $j, \beta^-_{jk} = 1$ on transmission element j and 0 otherwise. This approach to specification of the simple right can be easily extended to any arbitrary bundle of such rights, by specifying appropriate values of β^+_{hk} or β^-_{hk} for each transmission element, h, in the flowgate constraint(s).

For the simple rent payment transmission right, $b_{K1} < 0$ and b_{K1} is interpreted as the smallest amount a bidder is willing to accept to sell a unit of T_{K1}. A bid, $k \in K1$, for this right on transmission element j in the positive direction is defined as $\beta^+_{hk} = -1$ for $h = j$ and 0 for $h \neq j$. Similarly, a bid on transmission element j in the negative direction is defined by $\beta^-_{hk} = -1$ for $h = j$ and 0 for $h \neq j$.

Point-to-Point Transmission Rights. As noted, the point-to-point transmission bids, $l \in K2$, are defined over one or more POIs and one or more POWs at the $n[I]$ nodes in the system (more than one POI or POW defines a network right). In the nonlinear auction, the bidder would further have to specify whether the right is desired as an option or obligation, which, as discussed above, would result in different computational methods [11]. For the buyer of the $K2$ right, $b_{K2} > 0$, where b_{K2} is interpreted as the smallest amount a bidder is willing to pay to buy a unit of T_{K2}. For sellers of the rights, $b_{K2} < 0$, where b_{K2} is interpreted as the smallest amount a bidder is willing to accept to sell a unit of T_{K2}. A_{K2} is a matrix of net injection coefficients defining the net injection at each node i in each $l \in K2$, with elements a^{K2}_{il}. For a POI (conversely, POW), $a^{K2}_{il} > 0$ (conversely, $a^{K2}_{il} < 0$). Hence, for balanced rights, $\sum_i a^{K2}_{il} = 0$.

IV. Forward and Dispatch Markets: Financial Settlement and Revenue Adequacy

ISO auction markets operate in a sequence of forward and spot auctions, with products such as transmission rights and installed capacity being traded forward (pre-day-ahead) and energy and bid-based ancillary services typically traded day-ahead and in real-time.[20] The exact timing and content of these product auctions is a matter of ongoing market design. This section provides the general mathematical procedure for financial settlement and its link to revenue adequacy, focused on the two types of transmission rights and energy. A few brief simple examples are also given.

There are different market rules that could be used for selling all or part of a set of transmission rights and/or forward energy commitments. For example, carrying the rights to the next stage could be accomplished by bidding an equal specification to the current rights with a corresponding large bid value (although this rule could conflict with market power mitigation rules). To offer to liquidate one's holdings is done in some markets by simply not submitting a new bid, in which case one's last bid in the auction sequence is rolled over into the current auction. Following the convention, transmission rights are settled finally in the day-ahead market, while virtual energy trades through the ISO auctions can be transacted day-ahead (or pre-day-ahead, if available) but not in real-time. Energy sales and purchases are settled financially in each forward market, with deviations from the pre-dispatch position settled at the final dispatch auction prices.

The notation, s, is introduced to designate the sequence of energy and transmission auctions, where $s = S, S-1, ..., 1, 0$, and the s^{th} auction as JETRA^s. JETRA^0 is the final, real-time dispatch auction. The optimal values for energy and transmission rights in the s^{th} auction are designated $t^{s*}_{K1},\ t^{s*}_{K2}$, and g^{s*}. The optimal dual values will be similarly superscripted. Define JETRA$^{s'}$, $s' > s$, as an auction that takes place prior to the s^{th} auction. This auction yields $t^{s'*}_{K1},\ t^{s'*}_{K2}$, and $g^{s'*}$ and the optimal Lagrange multipliers will be similarly superscripted.

A. Settlement System

The settlement system for the auction model is summarized in Table I. We assume a uniform clearing price rule. The table shows the market design in which transmission rights and nodal revenue rights are settled finally in the real-time market; if the market design includes a day-ahead market, then they should be settled in that market.

Row one of Table I shows that in each auction, s, holders of transmission rights are paid the auction price times their holdings from the prior auction iteration, $s+1$ (note again that incrementing by 1 is moving the auction backwards in time). Row two shows that in each auction, s, buyers and sellers are charged the auction price times the quantity of transmission rights and forward energy contracts which clear the market.

The real-time (or dispatch) market, $s = 0$, settlements shown in rows three and four follow the same logic as the forward markets with respect to holders of transmission rights or forward energy contracts, who are paid the auction price times their holdings from the prior auction iteration, $s = 1$. The table shows the market design in which no transmission rights are traded in the real-time market, and all energy transactions and congestion charges associated with transmission usage are based on the LMPs at the POI and/or POW. For instance, if two awards for bids m and m' result in $g^*_m = g^*_{m'}$ and g^*_m is an injection at node 1 ($a_{1m} = +1$) and $g^*_{m'}$ is a withdrawal at node 2 ($a_{2m'} = -1$), then the total congestion charge for these two transactions treated as a single bilateral transaction is $\pi^*_2 g^*_m - \pi^*_1 g^*_m, = (\pi^*_2 - \pi^*_1) g^*_m$.

Forward energy transactions, or nodal revenue rights, are not yet offered in ISO auctions and their settlement deserves some further explanation. Settlement takes place, as with other transmission rights, in the real-time market or in the day-ahead market if it exists. The holder of the injection right gets paid the nodal price for the energy it produces but is obligated to

[20] We assume here that the existence of ISO auctions does not preclude the operation of secondary forward markets for transmission rights or separate scheduling coordinators for energy transactions. However, all bilateral or multilateral schedules must be cleared in the ISO auctions.

pay the nodal price to the ISO for the MW represented in its nodal energy right, while the holder of the withdrawal right is obligated to pay the nodal price for the energy it actually consumes but is paid the nodal price for the energy quantity specified in its forward right. As with the two-sided, point-to-point right, executing the physical transaction specified in the right results in a net zero financial position in settlement.[21]

B. Revenue Adequacy of the Auction Sequence

Revenue adequacy of transmission rights means that, if the topology of the network remains unchanged, the ISO collects sufficient congestion revenues from the users of the grid to cover payments to holders of transmission rights, whether these are allocated directly or through an auction. There are two dimensions to revenue adequacy in the auction model: conditions on each individual auction solution and conditions on the auction sequence as a whole. Beginning with each auction clearing, a requirement for revenue adequacy is that the auction result respects the set of transmission constraints. For point-to-point rights, this is commonly known as "simultaneous feasibility," meaning that the power flow induced by the injections and withdrawals associated with the rights awarded is feasible [9]. Since awarded flowgate rights "reserve" capacity on individual transmission elements, they do not have to be simultaneously feasible with respect to other awarded flowgate rights, but the set of point-to-point rights awarded in the same auction must be simultaneously feasible given the set of flowgate rights awarded.

Turning next to the conditions on the auction sequence, we have assumed heretofore that each auction in the sequence, JETRA^s, is conducted with the same set of transmission constraints. However, an important feature of actual electricity markets is that in the forward markets for transmission rights, the transmission constraints modeled may be either more or less restrictive than the set operative in the real-time market.[22] In general, the recursion of the auction markets is revenue adequate as long as the transmission capacity constraints form a nested, expanded sequence, a restriction which is stated more formally in the proof presented in the final section below.[23] This means that in each auction in the sequence, the transmission constraint set must be no more restrictive than the prior auction. This is an obvious requirement to prevent overselling of flow-based transmission rights, yet it is rarely clearly stated in transmission rights models. Some ISOs have adopted simple rules to accommodate this requirement; for example, the California ISO sold forward transmission rights to only a small percentage of its interzonal transmission capacity. Operational experience will be required to determine what quantity of alternative types of transmission rights can be made available in each forward market (annual, monthly, weekly, etc.).

[21]There are practical issues to implementing such a forward energy auction, most notably creditworthiness.

[22]The further ahead the time considered in a forward market, the greater the uncertainty about the network topology in the dispatch. This could justify a conservative transmission constraint set in the early auctions. As the auctions come closer in time to the dispatch, some uncertainty will be resolved and this will justify increased offerings by relaxing the constraints. For example, equipment may need to be derated if it is extremely hot, but temperature is not known until a time closer to the dispatch. The uncertainty can be captured in auction models through either multi-state or chance-constrained models.

[23]Stated in the notation of the auction model, we can say that, for $s' > s$, if R is convex, $F^{+max(s')} \leq F^{+max(s)}, F^{max(s')} \leq F^{max(s)}$, and $F^{K(s')} \leq F^{K(s)}$ where $F^{+max(s')}, F^{max(s')}$ and $F^{K(s')}$ are the set of transmission constraints in JETRA$^{s'}$ and $F^{+max(s)}, F^{max(s)}$ and $F^{K(s)}$ are the set of dispatch' transmission constraints in JETRA^s, then $R^{s'} \subseteq R^s$ and the dispatch is revenue adequate.

V. Auction Example with Quadratic Losses

This section presents a numerical example of the auction model in a simplified network based upon a linearized DC load flow with quadratic losses (e.g., [20]). The only transmission elements considered are lines. Constraint (2) is omitted and (5) is modified to represent DC analogues to Kirchhoff's Current and Voltage Laws:

$$\text{Current Law:} \;\; -y+D(f^+-f^-)+f^{-T}L^-f^-+f^{+T}L^+f^+ \leq 0. \tag{10}$$

$$\text{Voltage Law:} \;\; R(f^+ - f^-) = 0. \tag{11}$$

where the new notation is as follows:

- D — is the arc incidence matrix, $\{d_{ik}\}$. d_{ik} = 1 if $f^+ - f^-$ represents a MW flow out of bus i through transmission line k in a positive direction; d_{ik} = -1 if the flow through k is in a negative direction; and d_{ik} = 0 otherwise.
- $L^+,\ L^-$ — are tensors of rank 3, where the only nonzero elements in $L^+(L^-)$ are $l^+_{ikk}, (l^-_{ikk})$, representing the resistance loss coefficients (decrease in imports to bus i) due to a positive (negative) flow through transmission line k.
- R — = $\{r_{vk}\}$ are line reactances used in the voltage law analogues. r_{vk} is the value of reactance for transmission line k that appears in voltage loop v. r_{vk} = $+R_k$ or $-R_k$ if line k occurs in loop v, depending on whether a positive $f^+ - f^-$ is in the same or opposite sense of flow around v. r_{vk} = 0 if link k does not occur in loop v. Consistent with the DC model, the number of independent loops v must be equal to $K - N + 1$, where K is the number of lines considered and N is the number of buses.
- T — is the transpose operator.

Note that (10) is a convex relaxation of the Current Law equality constraint. An example is given below to illustrate (10) and (11).

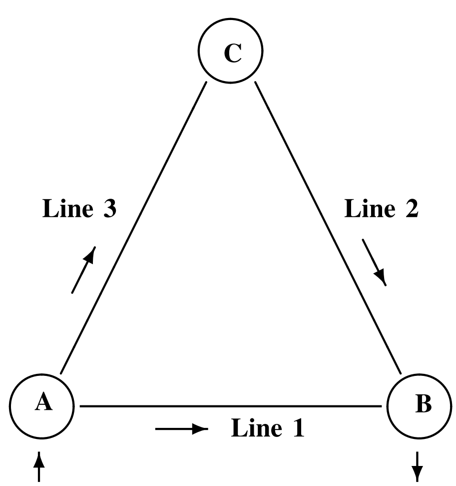
An important property, noted in [9], is that if $l_{ikk} > 0$, for some k, then in general no set of balanced $K2$ (point-to-point) rights will be feasible (revenue adequate) by themselves (except in the degenerate case of $t_{K2} = 0$). This is because of losses. Revenue adequacy is thus possible only if sufficient energy rights are also sold (in particular, "rights" that oblige the rights holder to make payments to the ISO; i.e., rights g whose coefficients in Ag are positive). A combination of such energy and balanced point-to-point rights g and T_{K1} can also be viewed as a set of imbalanced point-to-point rights.

TABLE I

CALCULATION OF SETTLEMENT PRICES, FORWARD AND DISPATCH AUCTIONS USING UNIFORM CLEARING PRICE RULE

	Flow-based (K1) Rights	**Point-to-point (K2) Rights**	**Energy Supply and Demand**
JETRA^s (forward market): payments by RTO (auctioneer) to rights/contract holders, $s \geq 1$	$\mu^{s*}(\beta^{K,s+1}t_{K1}^{s+1*})$ (interaction constraints) $\theta^{+s*}(\beta^{+,s+1}t_{K1}^{s+1*})$ (flowgates in + direction) $\theta^{-s*}(\beta^{-,s+1}t_{K1}^{s+1*})$ (flowgates in - direction)	$\pi^{s*}(A_{K2}^{s+1}t_{K2}^{s+1*})$	$\pi^{s*}(A_g^{s+1}g^{s+1*})$
JETRA^s (forward market): payments to RTO by bidders for rights/contracts, $s \geq 1$	$\mu^{s*}(\beta^{K,s}t_{K1}^{s*})$ (interaction constraints) $\theta^{+s*}(\beta^{+,s}t_{K1}^{s*})$ (flowgates in + direction) $\theta^{-s*}(\beta^{-,s}t_{K1}^{s*})$ (flowgates in - direction)	$\pi^{s*}(A_{K2}^{s}t_{K2}^{s*})$	$\pi^{s*}(A_g^{s}g^{s*})$
JETRA^0 (real-time market): payments by RTO to rights/ contract holders	$\mu^{0*}(\beta^{K,1}t_{K1}^{1*})$ (interaction constraints) $\theta^{+0*}(\beta^{+,1}t_{K1}^{1*})$ (flowgates in + direction) $\theta^{-0*}(\beta^{-,1}t_{K1}^{1*})$ (flowgates in - direction)	$\pi^{0*}(A_{K2}^{1}t_{K2}^{1*})$	$\pi^{0*}(A_g^{1}g^{1*})$
JETRA^0 (real-time market): payments to RTO by bidders for injections and withdrawals (includes physical bilateral transactions)			$\pi^{0*}(A_g^{0}g^{0*})$

FIGURE 1: THREE NODE NETWORK



The numerical example takes place on the three node network in Figure 1, in which the arrows show the direction of flow for an injection at node A and a withdrawal at node B (note that the arrows do not correspond to the direction of the flowgates). All loss factors on all lines = 0.00001 [MW/MW^2]. All reactances, R_k = 1. Then (10) for each bus or node becomes

$$\mathrm{KCL}_A : -y_A + (f_1^+ - f_1^-) + (f_3^+ - f_3^-) + 0.0001 f_1^{-2} + 0.0001 f_3^{-2} \leq 0,$$

$$\mathrm{KCL}_B : -y_B - (f_1^+ - f_1^-) + (f_2^+ - f_2^-) + 0.0001 f_1^{+2} + 0.0001 f_2^{-2} \leq 0,$$

$$\mathrm{KCL}_C : -y_C - (f_2^+ - f_2^-) - (f_3^+ - f_3^-) + 0.0001 f_2^{+2} + 0.0001 f_3^{+2} \leq 0,$$

and (11) becomes

$$\mathrm{KVL} : (f_1^+ - f_1^-) + (f_2^+ - f_2^-) - (f_3^+ - f_3^-) = 0.$$

Notice that if the only right existing is, say, a balanced t_{K2} involving an injection of 1000 MW at A (y_A = +1000) and a withdrawal of 1000 MW at B (y_B = -1000), this would be infeasible. That is, because of losses, there is no set of nonnegative flows $\{f_1^+, f_1^-, f_2^+, f_2^-, f_3^+, f_3^-\}$ that would simultaneously satisfy all four of the above constraints. This implies that there exist optimal dispatches subject to the above constraints whose nodal prices would be such that the ISO would earn less than the amount it would pay to this rights holder. For instance, if a generator at A bidding \$20/MWh was dispatched at 1076.3 MW to meet a load of 1000 MW at B, then the nodal prices at A and B would be π_A = \$20 and π_B = \$23.3. If the only rights holder had a balanced point-to-point right of 1000 MW from A to B, then the ISO (or equivalent entity) would pay her 1000(\$23.3 - \$20) = \$3280. Meanwhile, the congestion revenues that the ISO receives equal (1000 $\times$ \$23.28) - (1076.3 $\times$ \$20) = \$1754. The ISO is therefore revenue inadequate, losing \$3280 - \$1754 = \$1526.

On the other hand, if someone in addition had an energy payment obligation of at least 76.3 MW at node A, then the set of rights would be simultaneously feasible and the ISO would not lose money. A feasible solution to (10) - (11) would be y_A = 1076.3 (including the balanced right of 1000 MW plus the unbalanced right of 76.3 MW), y_B = -1000, f_1^+ = 713.1, f_2^- = 350.0, f_3^+ = 363.2, and zero for the other variables. The 76.3 MW obligation equals the resistance losses in the above system resulting from a load of 1000 MW at B with all generation occuring at A.

VI. Conclusion

The auction model presented here provides a general framework for representing the more complete electricity auctions now being proposed and implemented in the United States. With all types of energy and transmission capacity bids allowed, the auction framework can be extended to most types of forward hedging, spot market transactions and marginal transmission charges. This framework facilitates the efficient operation of off-ISO forward bilateral markets, which should benefit from more liquid transmission rights, such as the rights on commonly congested flowgates or possibly hub-to-hub rights. The proof of revenue adequacy previously provided for the auction with linear constraints [15] has been extended to the auction with nonlinear constraints.

The practical obstacles to implementation are the time needed to overcome computational requirements and implementation costs (cost-benefit analysis). For these reasons, while there is now broad consensus on many elements of market design, such as LMP and financial transmission rights, both federal and regional market design proposals have allowed for phased implementation of different types of transmission rights and different spot auction products to allow for development of software and resolution of cost allocation issues.

The auction model provides an analytic framework for exploration of the properties of additional market design features. Future research being conducted by the authors within this framework includes the modeling and pricing of locational reserves, pricing of reactive power (e.g., [6]), property right awards for transmission expansion, and unit commitment of transmission elements (e.g., [17]).

Appendix
Proof of Revenue Adequacy for the Auction Sequence

This section provides a proof of revenue adequacy of the auction sequence. This proof extends the revenue adequacy proofs for transmission in [9] and [15], both of which considered the case of linear transmission constraints, to the auction with both flow-based and point-to-point rights and the cased of nonlinear transmission constraints that define a convex feasible region. To simplify the presentation, the auction model is mapped into the more compact and general non-linear program (NLP) representing an auction in the following way:

- Define g as the vector of quantities awarded to $K2$ and G-type bids (encompassing both t_{K2} and g in the JETRA-AC model) with upper bound, G_u, and lower bound, G_l.
- Define a general benefit function $B(g)$ for the bid award level, g.
- The vector y represents net injections due by g. $K^s(y)$ represents the flows induced by y.[24]
- Define t as the vector of $K1$ transmission rights (t_{K1} in the JETRA-AC model) with upper bound, T_u, and lower bound, T_l.
- Define π as the dual value on the energy balance constraint, which can be interpreted as the shadow price for energy.
- Finally, define μ as the vector of dual values associated with transmission constraints, which can be interpreted as the shadow prices for transmission rights.

Using the resulting model NLP, the s^{th} auction in the auction sequence, NLP^s, is:

$$NLP^s : v_{NLP}^{s*} = \max v_{NLP}^s(t, g, y; b^s, \beta^s, B^s, K^s, F^s, T_l^s, T_u^s, G_l^s, G_u^s) =$$

$$\begin{array}{llll} \max_{t,g,y} & b^s t + B^s(g), & & \\ \text{s.t.} & Ag - y & = 0, & (\pi) \\ & \beta^s t + K^s(y) & \leq F^s, & (\mu) \\ & T_l^s \leq \quad t & \leq T_u^s, & (\rho_l, \rho_u) \\ & G_l^s \leq \quad g & \leq G_u^s, & (\psi_l, \psi_u). \end{array}$$

The optimal solution to NLP^s is defined as $\{t^s, g^s\}$ and the corresponding optimal dual variables are $\{y^s, \pi^s, \mu^s\}$. To demonstrate revenue adequacy of the auction sequence, prices and payments must be defined for the bids for g and t that are accepted. Define π^s as the market prices for g^s, and μ^s as the market prices for t^s. The rights held from the $s+1^{st}$ auction in the sequence are g^{s+1} and $\beta^{s+1}t^{s+1}$. Financial settlements in NLP^s, analogous to those defined above for the full auction model, are:

$$\pi^s A g^s + \mu^s \beta^s t^s,$$

(Payments by buyers and sellers to auctioneer in NLP^s)

$$-(\pi^s A g^{s+1} + \mu^s \beta^{s+1} t^{s+1}),$$

[24]In the simplified model, y also includes the network parameter settings previously designated by x, hence the transmission constraints are represented as $K(y)$.

(Payments by auctioneer to holders of forward rights or charges to obligations from $s+1$).

Theorem 1: If $B^s(g)$ is concave, $K^s(y)$ is convex, $K^s(y) \leq K^{s+1}(y)$ and $F^s \geq F^{s+1}$, then the sequence of auctions $\{S-1, ..., s, ..., 1, 0\}$, is revenue adequate, such that

$$\pi^s(y^s - y^{s+1}) + \mu^s(\beta^s t^s - \beta^{s+1} t^{s+1}) \geq 0.$$

Proof. By convexity of K^s,

$$\nabla K^s(y^s)y^s \geq \nabla K^s(y^s)y^{s+1} + K^s(y^s) - K^s(y^{s+1}).$$

Multiplying by $\mu^s \geq 0$,

$$\mu^s \nabla K^s(y^s)y^s \geq \mu^s \nabla K^s(y^s)y^{s+1} + \mu^s K^s(y^s)\mu^s K^s(y^{s+1}). \tag{12}$$

From the KKTs to NLP^s,

$$\mu^s(\beta^s t^s + K(y^s)) = \mu^s F^s. \tag{13}$$

Since $K^s(y) \leq K^{s+1}(y)$ and $F^s \geq F^{s=1}$ and $(\beta^{s+1}t^{s+1}, y^{s+1})$ is a feasible solution to NLP^{s+1}, $(\beta^{s+1}t^{s+1}, y^{s+1})$ must also be a feasible solution to NLP^s:

$$\beta^{s+1}t^{s+1} + K^s(y^{s+1}) \leq F^s.$$

Multiplying both sides by $\mu^s \geq 0$,

$$\mu^s(\beta^{s+1}t^{s+1} + K^s(y^{s+1})) \leq \mu^s F^s. \tag{14}$$

Combining (12) and (13),

$$\mu^s(\beta^s t^s + K^s(y^s)) \geq \mu^s(\beta^{s+1}t^{s+1} + K^s(y^{s+1})). \tag{15}$$

Adding (11) and (14), eliminating terms that cancel and rearranging,

$$\mu^s \beta^s t^s + \mu^s \nabla K^s(y^s)y^s \geq \mu^s \nabla K^s(y^s)y^{s+1} + \mu^s \beta^{s+1}t^{s+1}.$$

Substituting $\pi^s = \mu^s \nabla K^s(y^s)$ from the KKTs (A.8) for NLP and rearranging,

$$\pi^s(y^s - y^{s+1}) + \mu^s(\beta^s t^s - \beta^{s+1}t^{s+1}) \geq 0.$$

Finally, in $NLP^s, Ag = y$, which establishes the desired result. □

Acknowledgment

The authors would like to thank R. Baldick, H.-p .Chao, R. Entriken, W. Hogan, D. Mead, and S. Oren for helpful comments.

References

[1] R. Baldick, U. Helman, B. F. Hobbs, and R. P. O'Neill. 2005. "Design of Efficient Generation Markets," *Proceedings of the IEEE* (Special Issue on Power Technolgy & Policy), 93, 11, 1998-2012.

[2] H.-P. Chao and S. Peck. 1996. "A Market Mechanism for Electric Power Transmission," *Journal of Regulatory Economics*, 10, 1, 25-59.

[3] H.-P. Chao, S. Peck, S. Oren and R. Wilson. 2000. "Flow-based Transmission Rights and Congestion Management," *The Electricity Journal*, 13, 8, 38-58.

[4] Federal Energy Regulatory Commission (FERC). 1996. "Capacity Reservation Open Access Transmission Tariffs: Notice of Proposed Rulemaking." Docket No. RM96-11-000 (April 24). [Online] Available: www.ferc.gov.

[5] W. Elmaghraby, R.P. O'Neill, M.H. Rothkopf, and W.R. Stewart Jr. 2004. "Pricing and Efficiency in "Lumpy" Energy Markets, *Electricity J.*, June, 54-64.

[6] Federal Energy Regulatory Commission. 2005. "Principles for Efficient and Reliable Reactive Power Supply and Consumption," Staff Report, Docket No. AD-05-1-000, Washington, DC, February 4. [Online] Available: www.ferc.gov.

[7] Federal Energy Regulatory Commission (FERC). 2002. "Remedying Undue Discrimination through Open Access Transmission Service and Standard Electricity Market Design, Notice of Proposed Rulemaking," Docket No. RM01-12-000, (July 31). [Online] Available: www.ferc.gov.

[8] S. M. Harvey and W. W. Hogan. 2002. "Loss Hedging Financial Transmission Rights," Note. Tech. Rep., John F. Kennedy School of Government, Harvard Univ., Cambridge, MA (January 15). [Online] Available: ksghome.harvard.edu/ .whogan.cbg.Ksg.

[9] S. M. Harvey, W. W. Hogan, and S. L. Pope. 1997. "Transmission Capacity Reservations and Transmission Congestion Contracts," Tech. Rep., John F. Kennedy School of Government, Harvard Univ., Cambridge, MA (Revised March 8). [Online] Available: ksghome.harvard.edu/ .whogan.cbg.Ksg.

[10] B.F. Hobbs, M.H. Rothkopf, R.P. O'Neill, and H.-p. Chao, eds. 2001. *The Next Generation of Electric Power Unit Commitment Models*, Boston: Kluwer Academic Publishers.

[11] W. W. Hogan. 2002. "Financial Transmission Right Formulations," Tech. Rep., John F. Kennedy School of Government, Harvard Univ., Cambridge, MA (January 15). [Online] Available: ksghome.harvard.edu/ .whogan.cbg.Ksg.

[12] W. W. Hogan. 2000. "Flowgate Rights and Wrongs." Tech. Rep., John F. Kennedy School of Government, Harvard Univ., Cambridge, MA (August 20). [Online] Available: ksghome.harvard.edu/ .whogan.cbg.Ksg.

[13] W. W. Hogan. 1993. "Markets in Real Electric Networks Require Reactive Prices," *The Energy Journal*, 14, 3, 171-200.

[14] E. Kahn and R. Baldick. 1994. "Reactive Power is a Cheap Constraint," *Energy Journal*, 15, 4.

[15] R. P. O'Neill, U. Helman, B. F. Hobbs, W. R. Stewart, Jr., and M. Rothkopf. 2002. "A Joint Energy and Transmission Rights Auction: Proposal and Properties," *IEEE Transactions on Power Systems*, 17, 4, 1058-1067.

[16] R. P. O'Neill, U. Helman, R. Baldick, W. R. Stewart, Jr., and M. Rothkopf. 2003. "Contingent Transmission Rights in the Standard Market Design." *IEEE Transactions on Power Systems*, 18, 4, 1331-1337.

[17] R. P. O'Neill, R. Baldick, U. Helman, M. Rothkopf, and W. R. Stewart, Jr. 2005a. "Dispatchable Transmission in RTO Markets," *IEEE Transactions on Power Systems*, 20, 1, 171-179.

[18] R. P. O'Neill, P. M. Sotkiewicz, B. F. Hobbs, M. Rothkopf, and W. R. Stewart, Jr. 2005b. "Efficient Market-Clearing Prices in Markets with Nonconvexities," *European Journal of Operational Research*, 164, 269-285.

[19] R. P. O'Neill, U. Helman, B. F. Hobbs, and R. Baldick. 2006. "Independent System Operators in the USA: History, Lessons Learned, and Prospects," in F. P. Sioshansi and W. Pfaffenberger, eds.,*International Experience in Restructured Electricity Markets: What Works, What Does Not, and Why?* London: Elsevier.

[20] F. C. Schweppe, M. C. Caramanis, R. D. Tabors, and R. E. Bohn. 1988. *Spot Pricing of Electricity*. Boston: Kluwer Academic Publishers.

Richard P. O'Neill is Chief Economic Advisor in the FERC Office of Energy Markets and Reliability. He received his Ph.D. in Operations Research from the University of

Maryland, and previously was on the faculty of the Department of Computer Science at Louisiana State University.

Udi Helman is an Economist in the FERC Office of Energy Markets and Reliability. He has worked extensively on U.S. ISO market design issues. He has a Ph.D. in energy economics from The Johns Hopkins University.

Benjamin F. Hobbs (SM '01) received the Ph.D. degree in environmental systems engineering from Cornell University, Ithaca, N.Y. He is a Professor in the Department of Geography and Environmental Engineering and the Department of Mathematical Sciences, Johns Hopkins University, Baltimore, MD. Dr. Hobbs is a member of the California ISO Market Surveillance Committee.

Michael H. Rothkopf is a Professor in the MSIS Dept. and in RUTCOR at Rutgers University. He has written extensively on modeling auctions. He is a Fellow and past President of INFORMS, the Institute for Operations Research and the Management Sciences.

William R. Stewart, Jr. is the David L. Peebles Professor of Operations and Information Technology in the School of Business Administration at the College of William and Mary. He earned his doctorate in Operations Research from the University of Maryland.