

# Electric Power Markets: Why is Electricity Different? Dumb Grids, the Ultimate Just-in-Time Problem, & Polar Bears

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Research Group

## Outline

- I. Why power?
- II. Model definition
- III. Model uses
- IV. The basics
  - A. Supply Dispatch
  - B. Demand Bidding
  - C. Commitment
  - D. Transmission
  - E. Investment



# I. Why Power?

## (1) Lynchpin of the Economy

- Economic impact
  - ~\$1000/person/y in US (~oil)
    - 2.5% of GDP (10x water sector)
  - ~50% of US energy use
  - Most capital intensive
- Consequences when broken
  - 1970s UK coal strikes
  - 2000-2001 California crisis
  - Chronic third-world shortages
- Ongoing economic restructuring
  - Margaret & Fred
  - Vertical disintegration
    - generation, transmission, distribution
    - Access to transmission
  - Spot & forward markets
  - Horizontal disintegration, mergers

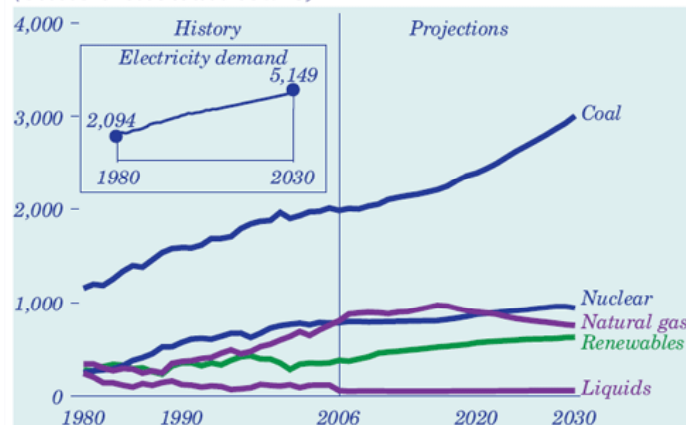


# Why Power?

## (2) Polar Bears

- Environmental impact
  - Transmission lines & landscapes
  - 'Conventional' air pollution: 3/4 US SO<sub>2</sub>, 1/3 NO<sub>x</sub>
  - 3/8 of CO<sub>2</sub> in US; CO<sub>2</sub> increasing

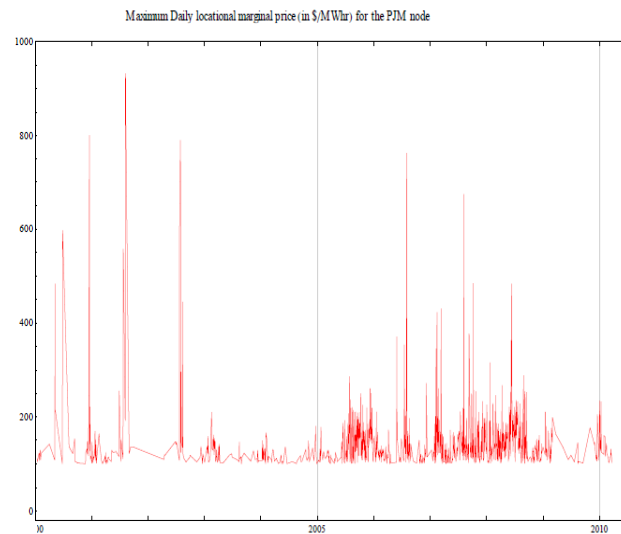
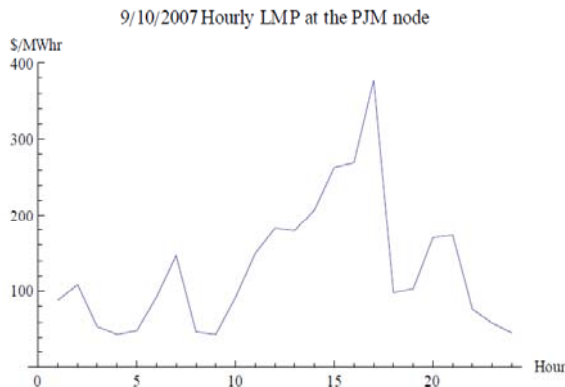
Figure 4. Electricity generation by fuel, 1980-2030  
(billion kilowatthours)



# Why Power?

## (3) The Ultimate Just-In-Time Product

- Little storage/buffering
  - Must balance supply & demand in real time
- ⇒ Huge price volatility



# Why Power?

## (4) Dumb Grids

- **Physics of networks**
  - North America consists of 3 synchronized machines
  - What you do affects everyone else ⇒ must carefully control to maintain security.
    - E.g., parallel flows due to Kirchhoff's laws
- **Valveless networks**
- **Saint Fred's dream remains just that**
  - Broken demand-side of market



## II. Definition of Electric Power Models

### ■ *Models that:*

- simulate or optimize ...
- operation of & investment in ...
- generation, transmission & use of electric power ...
- and their economic, environmental & other impacts ...
- using mathematics &, perhaps, computers

### ■ *Focus here: “bottom-up” or “process” engineering economic models*

- Technical & behavioral components
- Used for:
  - firm-level decisions
    - MIN costs, MAX profits
  - policy-analysis
    - simulate reaction of market to policy



## Process Optimization Models

### Elements:

- *Decision variables. E.g.,*
  - Design: MW of new combustion turbine capacity
  - Operation: MWh from existing coal units
- *Objective(s). E.g.,*
  - MAX profit or MIN total cost
- *Constraints. E.g.,*
  - $\Sigma$  Generation = Demand
  - Capacity limits
  - Environmental rules
  - Build enough capacity to maintain reliability



## The Supply Chain & the “Deciders”



*Fuel extractors*



*Power plant owners (GENCOs)*



*Transmission operators (TSOs)*

*Distribution companies (DISCOs)*



*Retail suppliers, Energy service companies (ESCOs)*



*Consumers*

## III. Process Model Uses Company Level Decisions

### *Real time operations:*

- Automatic protection (<1 second): auto. generator control (AGC) methods to protect equipment, prevent service interruptions.
  - TSO
- Dispatch (1-10 minutes): MIN fuel cost, s.t. voltage, frequency constraints
  - TSO or GENCOs

### *Operations Planning:*

- Unit commitment (8-168 hours). Which generators to be on line to MIN cost, s.t. “operating reserve” constraints
  - TSO or GENCOs
- Maintenance & production scheduling (1-5 yrs): fuel deliveries, maintenance outages
  - GENCOs

## Company Decisions Made Using Process Models, Continued

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### *Investment Planning*

- Demand-side planning (3-15 yrs): Modify consumer demands to lower costs
  - consumers, ESCOs, DISCOs
- Transmission & distribution planning (5-15 yrs): add circuits to maintain reliability and minimize cost
  - TSO, DISCOs
- Resource planning (10 - 40 yrs): most profitable mix of supplies, D-S programs under projected prices, demands, fuel prices
  - GENCOs

## Company Decisions Made Using Process Models, Continued

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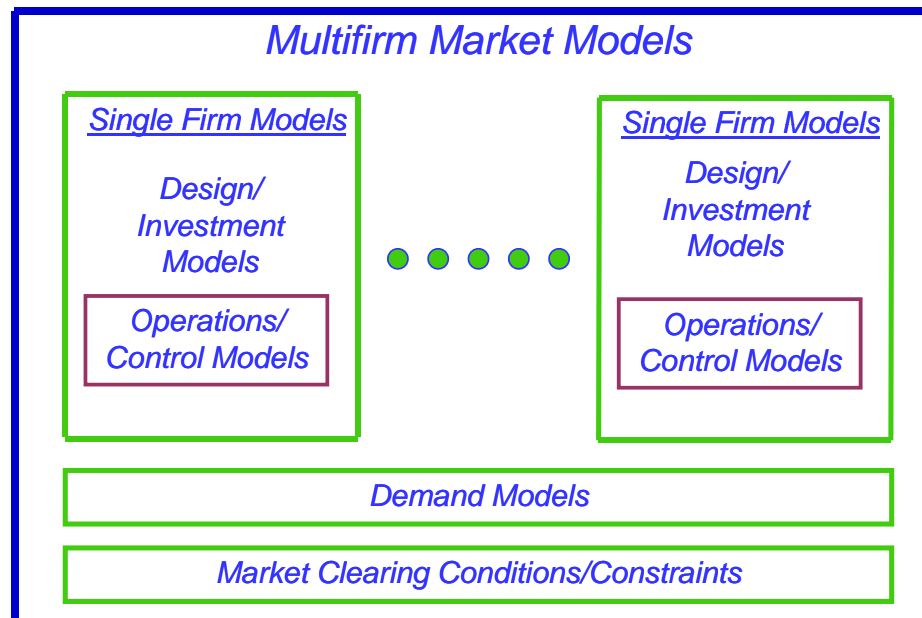
### *Pricing Decisions*

- Bidding (1 day - 5 yrs): optimize offers to provide power to MAX profit, s.t. fuel & power price risks
  - GENCOs
- Market clearing price determination (0.5- 168 hours): MAX social surplus/match offers
  - TSOs, traders

# Policy Uses of Process Models

- Use models of *firm's* decisions to simulate *market*
- Approaches
  - Via single optimization (Paul Samuelson):
    - MAX {consumer + producer surplus}
    - ↔ Marginal Cost Supply = Marg. Benefit Consumption
    - ↔ Competitive market outcome
  - Other formulations for imperfect markets
  - Attack equilibrium conditions directly
- Use
  - Effects of environmental policies / market design / structural reforms upon ...
  - ... market outcomes of interest (costs, prices, emissions & impacts, income distribution)

## Structure of Market Models



- If each firm assumes it can't affect price → competitive model
- If each assumes others won't change sales → Nash-Cournot oligopoly model
  - What did John Nash's father do for a living?

## All Models are Wrong ... Some are Useful

### ■ *Very small models*

- Quick insights in policy debates
- Need:
  - transparent models to convincingly communicate implications of assumptions
  - general conclusions

### ■ *Very large models*

- Actual grid operations and planning
- Need:
  - Implementable numerical solutions
  - policy conclusions for specific systems

### ■ *In-between models*

- Forecasting and impact analyses of policies
- Need:
  - ability to simulate many scenarios
  - but still represent “texture” of actual system

## IV.A. Operations Model: System Dispatch Linear Program

### ■ In words:

- Choose level of operation  $g$  of each generator to minimize total system cost *subject to* demand level

### ■ Decision variable:

$g_{it}$  = megawatt [MW] output of generating unit  $i$  during period  $t$

### ■ Coefficients:

$CG_{it}$  = variable operating cost [\$/MWh] for  $g_{it}$

$H_t$  = length of period  $t$  [h/yr].

$CAP_i$  = MW capacity of generating unit  $i$ .

$CF_i$  = maximum capacity factor [ ] for unit  $i$

$D_t$  = MW demand to be met in period  $t$



# Operations Linear Program (LP)

$$\text{MIN Variable Cost} = \sum_{i,t} H_t \text{CG}_{it} g_{it}$$

subject to:

$$\sum_i g_{it} = D_t \quad \forall t$$

$$g_{it} \leq \text{CAP}_i \quad \forall i,t$$

$$\sum_t H_t g_{it} \leq \text{CF}_i \text{8760 CAP}_i \quad \forall i$$

$$g_{it} \geq 0 \quad \forall i,t$$

## Operations LP Exercise



- Two generators
  - A: Peak: 800 MW, MC = \$70/MWh
  - B: Baseload: 1500 MW, MC = \$25/MWh
- Demand
  - Pk: Peak: 2200 MW, 760 hours/yr
  - OP: Offpeak: 1300 MW, 8000 hours/yr
- Assignment:
  - Write down LP
  - What is best solution (by inspection?)
- What if a hydro plant?
  - 100 MW
  - But can only produce 200,000 MWh/yr?

## Operations LP Answer: Model Formulation



$$\text{MIN } 760(70 g_{A,Pk} + 25 g_{B,Pk}) \\ + 8000(70 g_{A,OP} + 25 g_{B,OP})$$

subject to:

Meet load:

$$g_{A,Pk} + g_{B,Pk} = 2200$$

$$g_{A,OP} + g_{B,OP} = 1300$$

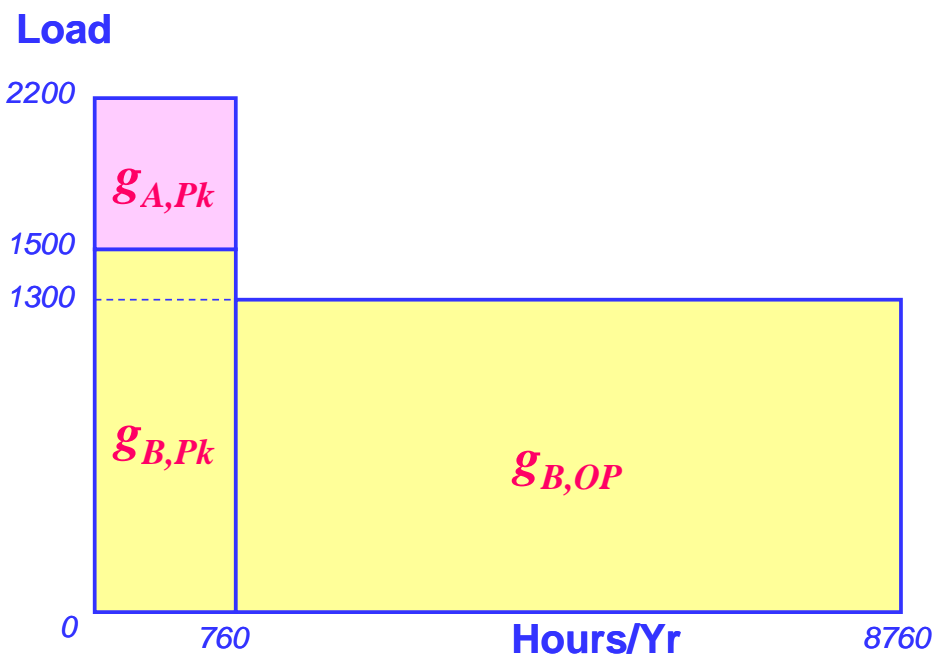
Generation  $\leq$  capacity:

$$g_{A,Pk} \leq 800; g_{A,OP} \leq 800$$

$$g_{B,Pk} \leq 1500; g_{B,OP} \leq 1500$$

Nonnegativity:  $g_{A,Pk}, g_{A,OP}, g_{B,Pk}, g_{B,OP} \geq 0$

## Operations LP Answer: Load Duration Curve



## Operations LP Answer: Model Formulation with Hydro



$$\text{MIN } 760(70 g_{A,Pk} + 25 g_{B,Pk}) \\ + 8000(70 g_{A,OP} + 25 g_{B,OP})$$

s.t.:

$$\text{Meet load: } g_{A,Pk} + g_{B,Pk} + g_{HYD,Pk} = 2200$$

$$g_{A,OP} + g_{B,OP} + g_{HYD,OP} = 1300$$

Generation  $\leq$  capacity:

$$g_{A,Pk} \leq 800; g_{A,OP} \leq 800$$

$$g_{B,Pk} \leq 1500; g_{B,OP} \leq 1500$$

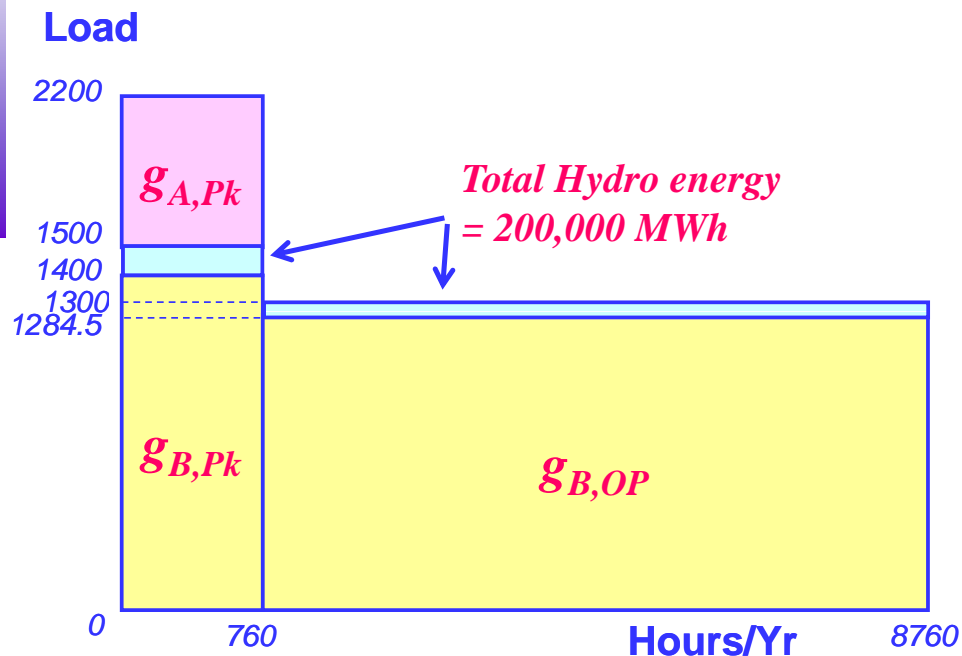
$$g_{HYD,Pk} \leq 100; g_{HYD,OP} \leq 100$$

Hydro Energy Limit:

$$760g_{HYD,Pk} + 8000g_{HYD,OP} \leq 200,000$$

$$\text{Nonnegativity: } g_{A,Pk}, g_{A,OP}, g_{B,Pk}, g_{B,OP} \geq 0$$

## Operations LP with Hydro Answer: Load Duration Curve



## IV.B. Towards a Smart Grid: Price Responsive Demand in an Operations LP

MAX Net Benefits from Market =

$$\sum_t H_t \int_0^{d_t} P_t(x) dx - \sum_{i,t} H_t \text{CG}_{it} g_{it}$$

subject to:

$$\sum_i g_{it} - d_t = 0 \quad \forall t$$

$$g_{it} \leq \text{CAP}_i \quad \forall i,t$$

$$\sum_t H_t g_{it} \leq \text{CF}_i \text{8760 CAP}_i \quad \forall i$$

$$g_{it} \geq 0 \quad \forall i,t$$

## IV.C Unit Commitment: A Mixed Integer Program

Define:

- $u_{it} = 1$  if unit  $i$  is committed in  $t$  (0 o.w.)
- $\text{CU}_i$  = fixed running cost of  $i$  if committed
- $\text{MR}_i$  = "must run" (minimum MW) if committed
- Periods  $t = 1, \dots, T$  are consecutive, and  $H_t = 1$
- $\text{RR}_i$  = Max allowed hourly change in output

MIN  $\sum_{i,t} \text{CG}_{it} g_{it} + \sum_{i,t} \text{CU}_i u_{it}$

$$\text{s.t. } \sum_i g_{it} = D_t \quad \forall t$$

$$\text{MR}_i u_{it} \leq g_{it} \leq \text{CAP}_i u_{it} \quad \forall i,t$$

$$-\text{RR}_i \leq (g_{it} - g_{i,t-1}) \leq \text{RR}_i \quad \forall i,t$$

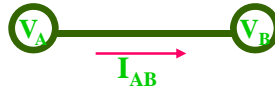
$$\sum_t g_{it} \leq \text{CF}_i T X_i \quad \forall i$$

$$g_{it} \geq 0 \quad \forall i,t; u_{it} \in \{0,1\} \quad \forall i,t$$

## IV.D Transmission-Constrained Models

### Review of DC Circuit Laws

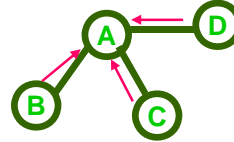
■ *Ohm's Law:*



- $V_A - V_B = I_{AB} * R_{AB}$
- Voltage difference proportional to current \* resistance

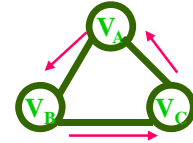
■ *Kirchhoff's Current Law:*

- No net current inflow to a node
- $\sum_n I_{A,n} = 0$



■ *Kirchhoff's Voltage Law:*

- Sum of voltage differences around any loop = 0
- $(V_A - V_B) + (V_B - V_C) + (V_C - V_A) = 0$
- Sub in Ohm's Law:  $I_{AB} * R_{AB} + I_{BC} * R_{BC} + I_{CA} * R_{CA} = 0$



## Implications of Laws

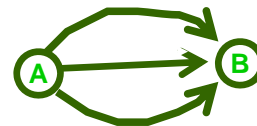
■ *Use laws to calculate flows*

- If you know generation and load at every "bus" except the "swing bus", then ...
- ..The "load flow" (currents in each line, voltages at each bus) is uniquely determined by Kirchhoff's two laws!
- = The "load flow" problem

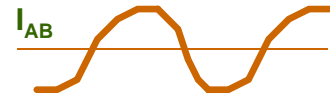


■ *Some odd byproducts of laws:*

- Can't "route" flow: "Unvalved network"
- Power follows many paths: "Parallel flows"
- Power from different sources intermingled. What you do affects everyone else:
  - 1 sells to 2 -- but this transaction congests 3's lines, increasing 3's costs
  - One line owner can restrict capacity & affect entire system
- Adding a line can worsen transmission capacity of system



# AC Load Flow is More Complex



- Sinusoidal voltage at each bus (with RMS amplitude and phase angle), as are line currents
- “Reactive” (vs. “real” power) a result of “reactance” (capacitance and inductance)
  - power stored and released in magnetic fields of capacitors and inductors as the current changes direction
- Although reactive power doesn’t do useful work, it causes resistance losses & uses up capacity

## “DC” Linearization of AC load flow

### ■ Assumptions

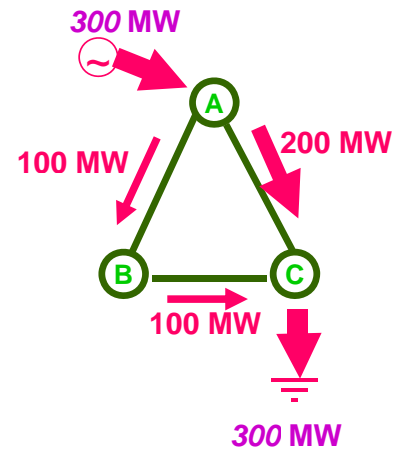
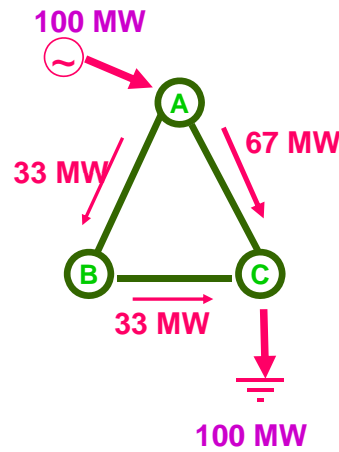
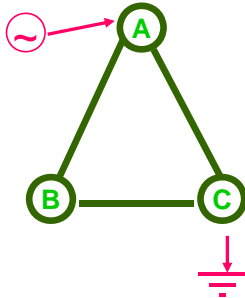
- Assume reactance  $\gg$  resistance
- Voltage amplitude same at all buses
- Changes in voltage angles  $\theta_A - \theta_B$  from one end of a line to another are small

### ■ Results:

- Power flow  $t_{AB}$  proportional to:
  - current  $I_{AB}$
  - difference in voltage angle  $\theta_A - \theta_B$
- Linear analogies to Kirchhoff’s Laws:
  - Current law at A:  $\sum_i g_{iA} = \sum_{\text{neighboring } n} t_{An} + \text{LOAD}_A$
  - Voltage law:  $t_{AB} * R_{AB} + t_{BC} * R_{BC} + t_{CA} * R_{CA} = 0$
- Given power injections at each bus, flows are unique

# Example of “DC” Load Flow

All lines have reactance = 1



Kirchhoff's Current Law at C:

$$+33 + 67 - 100 = 0$$

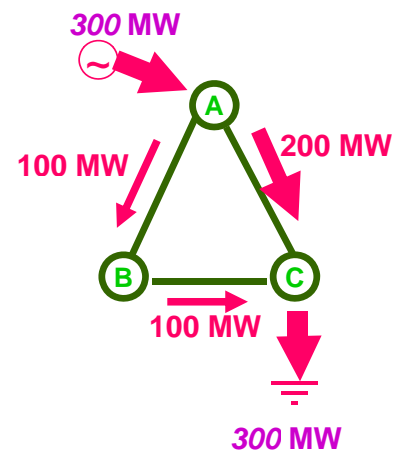
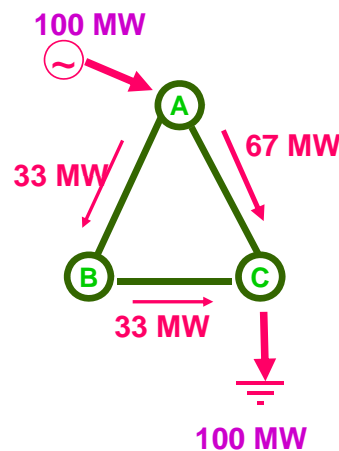
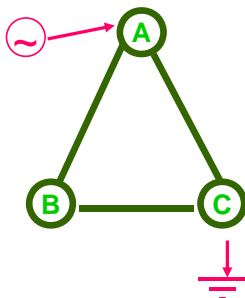
Kirchhoff's Voltage Law:

$$1 \cdot 33 + 1 \cdot 33 + 1 \cdot (-67) = 0$$

Proportionality!

## Proportionality means “Power Transmission Distribution Factors” can be used to calculate flows

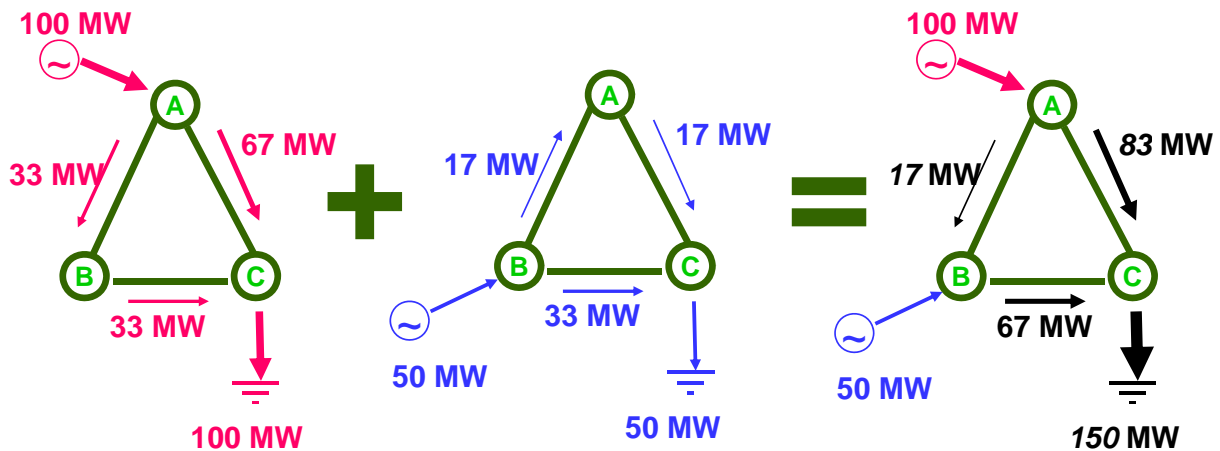
All lines have reactance = 1



$PTDF_{mn,jk}$  = the MW flowing from  $j$  to  $k$ , if 1 MW is injected at  $m$  and 1 MW is removed at  $n$

E.g.,  $PTDF_{AC,AB} = 0.33$  ( $= -PTDF_{CA,AB}$ )

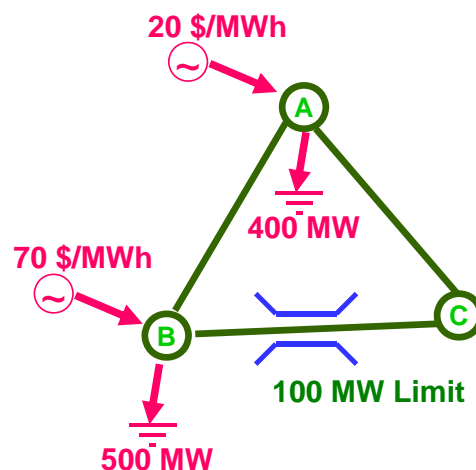
# Principle of Superposition



## Exercise in Transmission Modeling

### Assumptions

- *Equal reactances*
  - Line from B to C: 100 MW limit
- *Two plants:*
  - A:  $MC = 20 \text{ \$/MWh}$
  - B:  $MC = 70 \text{ \$/MWh}$
- *Load:*
  - A: 400 MW
  - B: 500 MW



■ *What's the optimal dispatch?*

■ *What are the prices?*

- *Dual variables (Lagrange multipliers) at each node*



## Linearized Transmission Constraints in Operations LP

$g_{int}$  = MW from plant  $i$ , at node  $n$ , during  $t$

$z_{nt}$  = Net MW injection at node  $n$ , during  $t$

MIN Variable Cost =  $\sum_n \sum_{i,t} H_t CG_{int} g_{int}$

subject to:

Net Injection:  $\sum_i g_{int} - D_{tn} = z_{nt} \quad \forall t, n$

Hub Balance:  $\sum_n z_{nt} = 0 \quad \forall t$

GenCap:  $g_{int} \leq CAP_{in} \quad \forall i, n, t$

Transmission:  $T_{k-} \leq [\sum_n PTDF_{nk} z_{nt}] \leq T_{k+} \quad \forall k, t$   
 $g_{int} \geq 0 \quad \forall i, n, t$

## Linearized Transmission Constraints in Operations LP: Example

MIN Variable Cost =  $20g_A + 70g_B$

subject to:

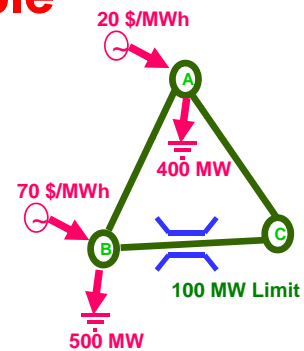
Net Injection:  $g_A - 400 = z_A$

$g_B - 500 = z_B$

Hub balance:  $z_A + z_B = 0$

Transmiss'n C→B:  $-100 \leq [0.33z_A + 0.0z_B] \leq +100$

Nonnegativity:  $g_A, g_B \geq 0$



Note: In calculating PTDFs, I assume that all injections “sink” at node B (= “Hub”)

- E.g., injection  $z_A$  at A is assumed to be accompanied by an equal withdrawal  $-z_A$  at B

# Exercise in Transmission Modeling: Answer

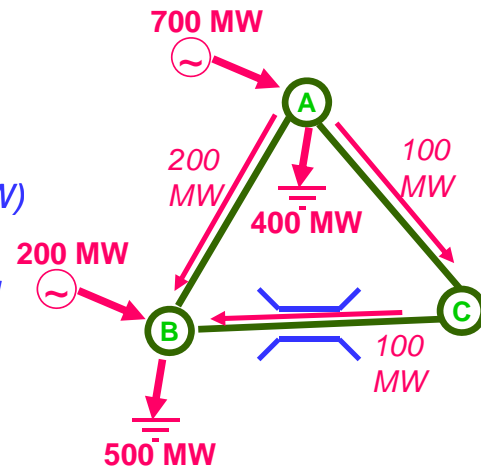
## ■ Optimal Dispatch

### • Two plants:

**A:** Meet load at A (400 MW) plus maximum amount that transmission limit allows (100 MW/PTDF = 100/.33 = 300 MW)  
= 700 MW

**B:** Serve the load at B not served by A (= 500 MW - 300 MW)  
= 200 MW

Total cost = \$28,000/hr



## ■ Marginal Costs (“LMP”) to Load:

**A:** A’s marginal cost (\$20)

**B:** Plant B’s MC (\$70)

**C:** To bring 1 MW to C, can back off 1 MW at B & expand 2 MW at A:  
=  $-\$70 + 2 \times \$20 = -\$30$  (Negative price)

## IV.E Investment Analysis: LP Snap Shot Analysis

### ■ Let generation capacity $cap_i$ now be a variable, with:

- (annualized) cost CRF [1/yr] CCAP<sub>i</sub> [\$/MW]

$$\blacksquare \text{ MIN } \sum_{i,t} H_t CG_{it} g_{it} + \sum_i \text{CRF CCAP}_i cap_i$$

$$\text{s.t. } \sum_i g_{it} = \text{LOAD}_t \quad \forall t$$

$$g_{it} - cap_i \leq 0 \quad \forall i,t$$

$$\sum_t H_t g_{it} - CF_i 8760 cap_i \leq 0 \quad \forall i$$

$$\sum_i cap_i \geq D_{\text{PEAK}} (1+M) \quad (\text{“reserve margin” constraint})$$

$$g_{it} \geq 0 \quad \forall i,t; \quad cap_i \geq 0 \quad \forall i$$

# Planning LP Exercise



- Two generation types
  - A: Peak:**
    - Operating Cost = \$70/MWh
    - Capital Cost = \$70,000 / MW/yr
  - B: Baseload:**
    - Operating Cost = \$25/MWh
    - Capital Cost = \$120,000 / MW/yr
- Load
  - Peak: 2200 MW, 760 hours/yr
  - Offpeak: 1300 MW, 8000 hours/yr
  - Reserve Margin: 15%
- Assignment:
  - Write down LP
  - What is best solution (by inspection?)

## Planning LP Answer: Model Formulation



$$\text{MIN } 760(70 g_{A,Pk} + 25 g_{B,Pk}) + 8000(70 g_{A,OP} + 25 g_{B,OP}) \\ + 70,000 cap_A + 120,000 cap_B$$

subject to:

Meet load:  $g_{A,Pk} + g_{B,Pk} = 2200$

$$g_{A,OP} + g_{B,OP} = 1300$$

Generation  $\leq$  capacity:

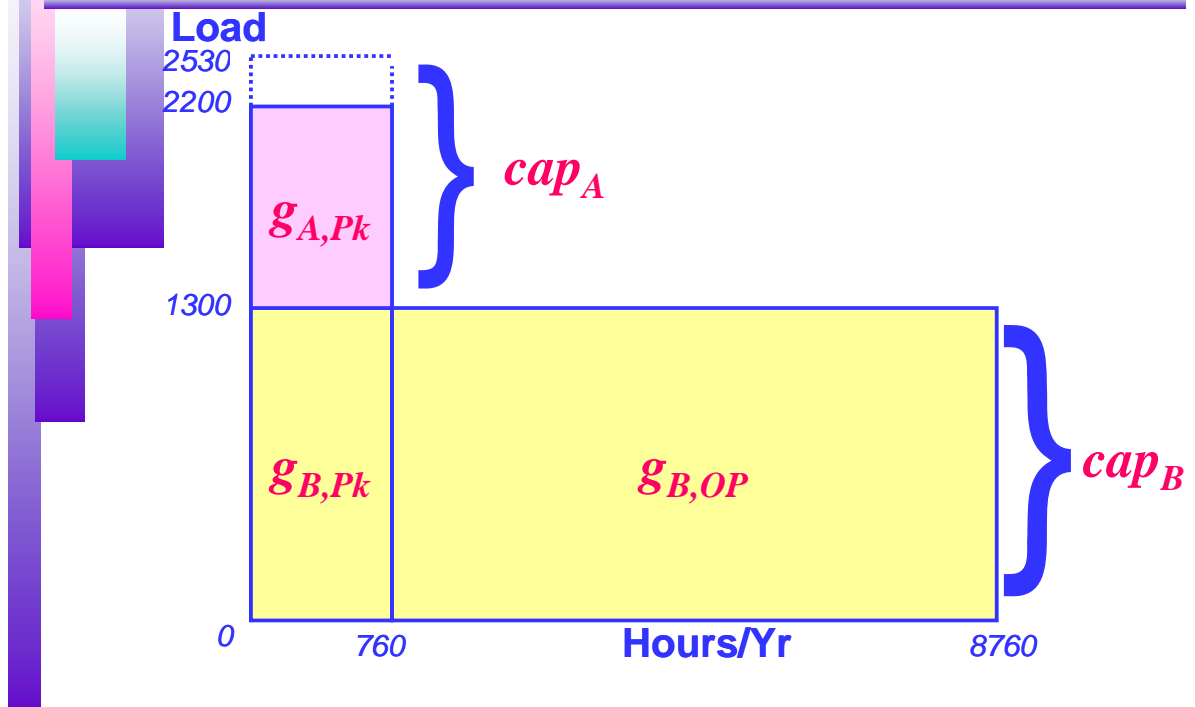
$$g_{A,Pk} - cap_A \leq 0; g_{A,OP} - cap_A \leq 0$$

$$g_{B,Pk} - cap_B \leq 0; g_{B,OP} - cap_B \leq 0$$

Reserve:  $cap_A + cap_B \geq 1.15 * 2200$

Nonnegativity:  $g_{A,Pk}, g_{A,OP}, g_{B,Pk}, g_{B,OP} \geq 0$

## Planning LP Answer: Load Duration Curve



## A Complication: Uncertain future (demands, fuels,...)

- **Math programming with recourse**
  - scenarios  $s=1,2,\dots,S$ , each with probability  $PR^s$
  - Considers how the system is operated in each realization.
- **Simplest: Assume 2 decision stages:**
  1. Choices made “here and now” before future is known
    - E.g., long-lead time plants (nuclear, hydro).
    - These are  $x^1$
  2. “Wait and see” choices, which are made after the future  $s$  is known.
    - E.g., dispatch/operations, short-lead time plants (combustion turbines).
    - These are  $x^{2s}$  (one set defined for each scenario  $s$ )
- **Model:**

$$\begin{aligned} \text{MIN} \quad & C^1(x^1) + \sum_s PR^s C^{2s}(x^{2s}) \\ \text{s.t.} \quad & A^1(x^1) = B^1 \\ & A^{2s}(x^1, x^{2s}) = B^{2s} \quad \forall s \end{aligned}$$