A Bayesian optimization framework for cost-effective non-point sediment source control and research

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Abstract. Rural nonpoint sources of water pollution are particularly difficult to control, with relatively little progress having been made compared to point sources. Management choices are difficult because of large uncertainties in both the monitoring of nonpoint pollution and the effectiveness of various actions to reduce that pollution. We propose a framework for selecting the optimal combination of research, monitoring, and management actions. The approach combines Bayesian inference and multiobjective linear programming to explicitly represent uncertainty in the effectiveness and cost of controls, and quantify the value of reducing uncertainty through research and monitoring. We illustrate the framework using the problem of reducing turbidity from rural sediment sources in the Minnesota River basin. We find that a combination of research methods in different subbasins usually yields the most valuable information and is predicted to result in benefits via reduced cost and increased effectiveness of sediment reduction.

Subject Headings: Nonpoint pollution; Bayesian analysis; Sediment transport; Multiple objective analysis; Economic Factors

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Introduction

Nonpoint source pollution is the largest contributor to surface water quality impairment in the United States (U.S. Environmental Protection Agency 2003). Rural sediment is of increasing concern. Reducing sediment loads requires identification and quantification of sources, followed by selection, implementation, and monitoring of controls. Source identification can be difficult because sediment supply is often episodic and localized and derives from many unmonitored sources distributed throughout a watershed, including farmland, construction sites, and stream banks. In agricultural regions such as the Minnesota River basin, source identification can also be contentious, with various groups identifying either farmland or near-stream locations as the dominant sources (University of Minnesota Extension Service 1996; Steil 2004; Gupta et al. 2001). Without accurate source identification, management can be misdirected.

Various methods can be used to estimate sediment sources and amounts. Erosion rates for agricultural fields are often estimated by variants of the Universal Soil Loss Equation (USLE) (e.g., Renard et al. 1997). Estimates are adjusted by a sediment delivery ratio to estimate the fraction of field sediment that exits the watershed (e.g., de Vente et al. 2007). Sediment yields can be more directly estimated using flow and sediment concentration observations at stream gages, although this provides no direct information as to source. The increased use of radionuclide and isotopic exposure tracers has made sediment fingerprinting a valuable technique (Bogen et al. 1992; Belmont et al. 2011) that can indicate the proportion of sediment derived from agricultural fields. These methods can also be combined with field observations of erosion and deposition in a mass balance for all sources, fluxes, and stores of sediment for a defined watershed and period (see Reid and Dunne 1996). This approach provides the most complete evaluation of sediment sources and a basis for evaluating several types of uncertainty (Gran et al. 2009), but also involves greater effort, expense, and time. Sources of uncertainty include error inherent in specific methods, extrapolation from monitored sites, unknown future weather and land uses, and delivery ratios.

Sediment management also requires a selection among many possible controls ("best management practices" or BMPs) that differ in cost, effectiveness, and public acceptability. For farmland, alternatives include changing cultivation methods (e.g., tillage practices) and structural actions (e.g., terracing). For streambank or bluff erosion, management can include redirecting streamflow, protecting banks, or developing buffers. Finally, for gullies and ravines, controls include bank protection, runoff control, and revegetation. Uncertain performance and cost of these options further complicates decision making.

Given uncertain sources, stochastic future loadings, competing interests, and control actions of uncertain effectiveness, management decisions for sediment reduction are complex and difficult. We present an integrated approach to comparing the cost-effectiveness of investing in monitoring, research, and sediment controls. These options interact in complex ways. For example, improved information can delay, but also more effectively target controls. Investment in controls can yield better information, but only if outcomes are monitored. Using a scenario based two-stage stochastic programming framework involving Bayesian inference and multiobjective optimization, we identify the mix of research, monitoring, and controls that can minimize the weighted sum of expected cost and sediment loads. In particular, based on weights assigned to cost and loading objectives, the method identifies non-inferior research and monitoring strategies (if any) to undertake first, followed by which controls to implement, given that information. A strategy is non-inferior if no other feasible mix of information and control activities will improve one objective without worsening the other. By varying their weights, tradeoffs between the two objectives are described. We also quantify the expected value of information yielded by each research action. This combination of planning optimization and Bayesian analysis contributes to the application of systems analysis to watershed management: these two widely used methods in environmental systems are combined in a unique way to strengthen watershed management in general and nonpoint pollution control in particular. The approach is a practical and potentially highly useful means of implementing the adaptive management philosophy.

For specificity, the framework is demonstrated using sediment management in the Maple River tributary of the Minnesota River. A total maximum daily load (TMDL) for turbidity has been established for much of the Minnesota River and its tributaries, including the Maple (Minnesota Pollution Control Agency 2009). Lake Pepin, a primary sediment sink for the Upper Mississippi River, has experienced a ten-fold increase in sedimentation rates since European settlement in the mid-1800s (Belmont et al. 2011) and most of this sediment derives from rivers like the Maple. Cumulative loading is a leading concern and we focus here on annual average sediment load.

Models of Non-point Source Pollution Management

Several methodologies have been used to optimize management of non-point pollution under uncertainty. We compare our framework to existing work in terms of six features of optimization models. These include objectives, representation of sediment generation and transport, decision alternatives, uncertainty, the range of possible outcomes, and solution methodology.

The objectives of our framework are to 1) minimize research and management costs and 2) maximize the reduction of sediment. Sediment loss is represented by loss rates per unit area or stream length, as a function of management decisions, for constant delivery ratio within the watershed. We consider the sediment delivery ratio describing the contribution of upstream watersheds to downstream watersheds to be uncertain and a focus of research. Decisions are to acquire more information or implement BMPs. Information acquisition includes research or monitoring. The framework could be extended to also explicitly address uncertainty in within watershed sediment delivery ratios and include research actions aimed at reducing this uncertainty. The uncertainties we address include parameter uncertainty (the long term average sediment released by sources in the watershed) and uncertainty in the outcomes of learning. Uncertainty is defined based upon a sediment balance for the Maple River (Gran et al., 2009) and expert judgment. The framework could be expanded to further investigate uncertainty in control costs, but we only consider their expected values here. We model risk-neutral decision makers who maximize the annual probability-weighted sediment reduction. The framework considers the full range of possible research and acquisition outcomes by defining a probability distribution for each; however, we assume that the expected costs and sediment reduction are of most importance to the user and use the expected outcomes of the Bayesian updating procedure to define our scenarios. The framework is flexible and can define the scenarios based upon other quantities from the distribution, such as the variance, to reflect the user's attitude towards risk. Our solution method combines Bayesian inference with multiobjective linear programming using scenario based two stage stochastic programming.

Most previous efforts on this subject minimized the cost of controlling agricultural soil loss. Earlier optimization models (see Seitz et al. 1979; Wade and Heady 1977; Kramer et al. 1984; Veith et al. 2003; Braden et al. 1989) did not consider uncertainty. These deterministic models assume that loadings and control effectiveness are known, even though they are uncertain. Deterministic methods cannot evaluate opportunities for learning to reduce uncertainty. Even if there are no such opportunities, considering the full distribution rather than expected values of outcomes can yield different decisions if system nonlinearities mean that the expected performance of an alternative under uncertain inputs differs from the performance under the expected inputs (Morgan et al. 1990), or if decision makers are risk averse (a form of nonlinearity).

One approach to address uncertainty is to assess robustness of decisions from a deterministic model. For instance, using sensitivity analysis, Yulianti et al. (1999) identified BMPs that were least sensitive to uncertainties in soil properties and production costs, among other uncertainties. However, insensitive decisions are not necessarily optimal when considering the probability distribution of outcomes. This motivated the application of stochastic optimization. Chance-constrained optimization has been used to consider variable pollutant loadings (Milon 1987) and soil loss (Zhu et al. 1994) due to precipitation variability. However, those papers did not incorporate uncertainty in soil loss rates, as we do here: long-term sediment loading depends on both extreme events that are the focus of chance-constrained programming as well as the cumulative loading delivered from a wide range of events.

Another stochastic approach that has been applied to non-point source control is optimal control theory (Nicklow and Muleta 2001), which considers multiple decision stages, allowing for learning and adaptation over time. Optimal control theory is limited due to the number of state variables and multiple decision stages. In order to meet study objectives without too much computational burden, it is feasible to define only two stages. Since we consider many sources distributed throughout a watershed, our framework considers only two decision stages, which prevents us from using an optimal control theory approach.

Closer to our framework is the two-stage stochastic program of Luo et al. (2006). Their first-stage decision chooses the amount of farmland to retire in the face of uncertain loadings and transport; in the second-stage, pollution is controlled from non-retired lands. In contrast, our first stage considers research actions, followed by second-stage control actions based on what is learned. Also, we use Bayesian inference to generate the scenarios we investigate.

Valuation of information for rural sediment control has been considered recently. Farzin and Kaplan (2004) use both streamflow and sediment data to update prior distributions of sediment from two sources, and then optimize logging road removals. They quantify the expected value of perfect information (EVPI), equaling the reduction of control costs that would occur if loads were known exactly. Borisova et al. (2005) also calculate EVPI for various uncertainties concerning nitrogen loads and transport from farmland to the Chesapeake Bay.

Sediment source estimates have also been updated using Bayesian inference and fingerprinting information (Small et al. 2002; Rowan et al. 2001; Caitcheon et al. 2006; Douglas et al. 2003). However, none of these consider the management value of that information. Bayesian nets have also been used to model non-point sources. Dorner et al. (2007) use the approach to represent tradeoffs among erosion rates and crop revenues.

To conclude, our framework is similar to the existing approaches in terms of three of the six problem features: objectives (cost and loadings), physical representation (constant loss and sediment delivery ratios), and consideration of a full range of outcomes. Our framework diverges from most previous approaches in the other three features. First, we consider sediment control not only from cultivated land, but also from streambanks, ravines, and bluffs. Second, we include several research actions that can reduce uncertainty about loadings, along with the value of that information. Third, our framework uses Bayesian inference and Gibbs sampling to generate discrete sediment loading scenarios that are used in a two-stage stochastic programming framework, which is a novel approach to integrated evaluation of research and control in adaptive management of non-point pollution. In stage 1, the optimal research action is selected, followed by selecting the optimal control actions in stage 2. We solve the optimization problem by decomposition and backward dynamic programming: we first solve the second stage problem separately for all possible research outcomes, and then we solve the first stage using a combination of enumeration and optimization to select the best combination of research and management. A generalization to multiple stages is possible, in which further data can be obtained after initial implementation of management actions; this would significantly increase the size of the stochastic program.

Framework

Our framework utilizes a scenario based two stage stochastic programming approach. Given a set of sediment loading scenarios, multiobjective linear programming (MOLP) is used to determine the optimal suite of controls to deploy. MOLP is also applied under a base case of no additional information. A non-inferior research and management strategy is determined based on balancing cost and sediment loss. Changing the relative weight assigned to the sediment objective yields distinct non-inferior portfolios that represent tradeoffs between the two objectives. The expected value of imperfect information (EVII) is quantified as the expected improvement in the overall objective resulting from the information provided by research.

The sediment loading scenarios are derived using expert elicitation and Bayesian inference. First, expert elicitation is used to obtain prior probability distributions characterizing current understanding of the natural system as well as the quality of information produced by research actions. Bayesian inference then quantifies the resulting improved understanding in the form of posterior probabilities, which form the scenarios used in the MOLP. The framework is a two-stage decision approach: first the optimal research action is selected, followed by the optimal management action. The decision period considered is a single year.

The decision tree in Fig. 1 represents the decision problem for our case study. Branches originating from the decision nodes (squares) correspond to available choices (defined by index *a*), while branches from the chance nodes (circles) show possible outcomes of uncertain events $\boldsymbol{\Theta}_a^s$, where *s* denotes each outcome or scenario considered. Time moves left to right, with research decisions made first, followed by the uncertain outcomes of the research. Research actions are conducted in watersheds throughout the study area, denoted as Wj, where j is the index of the watershed. Note that combinations of individual research actions can be chosen. Probabilities associated with chance node branches $P_{\mathrm{a}}(\boldsymbol{\Theta}_a^s)$ are derived from expert judgments together with Bayes' Law and specific computational methods discussed below. Learning outcomes then inform decisions about which controls to implement; the MOLP accomplishes this in the rightmost decision nodes. The value of imperfect information results from comparing the expected cost and sediment reduction for a particular research action *a* with the no information case.

Multiobjective LP for Selecting Controls

The spatial domain of the problem can be divided into multiple subwatersheds, in order to account for variations in sediment source and storage within the watershed of interest. Here, we consider annual average sediment loadings $\boldsymbol{\theta} = \{\theta_{i,j}\}$, where $i = 1,\ldots,I$ is the index of type of sediment source and $j = 1,\ldots,J$ is the watershed index, and the sediment delivery ratio, $d_{j\in WS}$, for each watershed j that is an upstream watershed (*WS* defines the set of upstream watersheds, which is a subset of J). We represent the matrix of uncertain quantities of interest as $\boldsymbol{\Theta} = \{\theta_{i,j}, d_{j\in WS}\}$. Sources in the Minnesota River include agricultural fields, streambanks, ravines and bluffs; however, the framework can be extended to include other sources. The annual loadings are uncertain due to limited scientific knowledge.

To account for the uncertainty in sediment loadings, we consider nine possible sediment loading scenarios and use multiobjective linear programming to choose from among sediment reduction actions. There are at least two competing objectives – minimizing expected cost and minimizing expected sediment loss – and stakeholders may disagree about their relative priority (Cohon 2004; Hobbs et al. 1992). It is important to generate a range of non-inferior strategies consisting of collections of recommended actions reflecting different priorities.

The selection of control actions is a complex spatial problem. Our MOLP component is simpler than some applications reviewed above, as we consider fewer locations and all relationships are linear. This simplified approach was adopted to illustrate our framework; however, the framework is flexible and more complicated optimization models can be substituted.

The MOLP chooses control actions to minimize (1) the expected cost and maximize expected sediment reduction. For each action *a*, the expected sediment loading and expected sediment delivery ratio associated with each scenario *s* ($\boldsymbol{\theta}_a^s = [\theta_{i,j}, d]_a^s$) are input to the MOLP, and a tradeoff curve of cost vs. expected sediment remaining is found by systematically varying a weight *W* placed on the sediment objective (Cohon 2004). *W* is also interpretable as the marginal worth of sediment reductions in \$/ton. Consequently, the objective has units of \$/yr (if costs are annualized and sediment is in t/yr). The decision variable $x_{i,j,k}^m$ is defined below. The MOLP describing the second stage decision is as follows:

$$\text{For each scenario and } W\text{:} \quad \min_{\{x_{i,j,k}^m \geq 0\}} E(Cost_{a,s,W}) - W * E(Sed_{a,s,W}) \tag{1}$$

$$\text{subject to: } E(Cost_{a,n,W}) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{m=1}^{M} C_{i,j,k} x_{i,j,k}^m \tag{2}$$

$$E(Sed_{a,s,W}) = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{m=1}^{M} \left(\frac{S_{i,j}^m * F_{i,k} * \boldsymbol{\theta}_{a,W}^s}{A_{i,j}} \right) x_{i,j,k}^m \tag{3}$$

$$\frac{\sum_{k=1}^{8} x_{i,j,k}^m}{A_{i,j}} \leq UP_{i,j}^m \qquad \forall i, j, m \tag{4}$$

$$\sum_m x_{i,j,k}^m \leq UB_{i,j,k} \qquad \forall i, j, k \tag{5}$$

where $x_{i,j,k}^m$ = nonnegative decision variable: the amount (in km^2 or km) of BMP type *k* addressing segment *m* of sediment source *i* in watershed *j*. $x_{i,j,k}^m$ is divided into segments to approximate the convex cost function $C_{i,j,k} x_{i,j,k}^m$ as a piecewise linear function $\sum_{m=1}^{M} C_{i,j,k} x_{i,j,k}^m$, a standard method for approximating nonlinear functions in LPs (Loucks et al. 1980). Nonlinear functions could be used instead, but that would require the use of nonlinear programming, which could limit the size of problem that can be solved.

$E(Cost_{a,s,W})$= Expected cost of BMPs under sediment weight W, given scenario s from research action a.

$E(Sed_{a,s,W})$ = Expected sediment reduction under weight W, given scenario s from research a. As with the cost function above, $-W*E(Sed_{a,s,W})$ is also a convex piecewise linear function since $E(Sed_{a,s,W})$ is concave (Fig. S2).

$A_{i,j}$ = area (km^2) or length (km) contributing sediment for source i in watershed j.

$C_{i,j,k}$ = cost coefficient ($/$km^2$ or $/km) for BMP $x^m_{i,j,k}$.

$F_{i,k}$ = reduction of sediment (fraction) from source i due to BMP type k.

$S^m_{i,j}$ = slope (dimensionless) of the m^{th} segment of the soil loss curve for source i in watershed j. The soil loss curve $S_{i,j}(y_{i,j})$ represents an ordering from highest to lowest rate of loss of particular locations of source i within watershed j, where $y_{i,j}$ is the cumulative fraction of the source's total km or km^2 and $S_{i,j}$ is the cumulative sediment loss. We assume that each sediment source i's loss rate varies among locations within a watershed j, and that BMPs can first address the highest yielding locations within the watershed. Consequently, expanded application of a BMP takes place in locations with lower yields, so that $S^m_{i,j}$ decreases with m.

$UB_{i,j,k}$ = upper bound (km^2 or km) for BMP type k addressing source i in j.

$UP^m_{i,j}$ = proportion of length/area of source i in watershed j making up the m^{th} segment of the soil loss curve for i and j.

Objectives other than cost and loadings can readily be included in (1). We consider both cost and loading, even though once a water body's use has been designated under a TMDL, cost does not directly affect calculation of the permissible load. However, cost does factor into the

TMDL process at other points. First, USEPA's Use Attainability Analysis considers the practicality, in terms of cost and technology, of attaining different uses. Second, cost is usually considered when apportioning required load reductions among multiple sources. Cost curves for sediment reductions from the Maple River are therefore useful to a Minnesota River TMDL process that allocates reductions among subbasins.

Turning to the constraints, (2) and (3) define cost and expected sediment loss as a function of the BMP decisions $x^m_{i,j,k}$. Constraint (4) limits the amount of area/length of source *i* in watershed *j* treated within each segment of the soil loss curve to be no more than the area/length available. For example, if the first segment of the loss curve addresses the worst 1% of the area contributing sediment from source *i* in watershed *j*, (4) would ensure that BMPs addressing those worst areas occupy no more than that 1%. This constraint assumes that BMPs are mutually exclusive, in that at most one BMP can be placed on a particular location (an area with uniform soil characteristics, or particular stretch of bluff or stream). For instance, for streambanks, either restoration or bank stabilization (or no action) would be implemented on one length of stream. However, combinations at a single location can be considered by defining a BMP representing that combination. Finally, (5) limits the total amount of each type of BMP to be less than the upper bound on the available area (or length) for the BMP.

Given the solution of (1)-(5) for each scenario $\boldsymbol{\theta}_a^s$ and weight *W*, we can then calculate the expected *prior* cost and sediment reduction of a particular research action *a* under *W* as the expected value over the scenarios. These address the first stage (research) decision:

$$E\left(Kost_{a,W}\right) = RC_a + \sum_s P_a(\boldsymbol{\theta}_a^s) E(Cost_{a,s,W}) \tag{6}$$

$$E\left(Sed_{a,W}\right) = \sum_s P_a(\boldsymbol{\theta}_a^s) E(Sed_{a,s,W}) \tag{7}$$

where RC_a is the cost of the research itself.

In this manner, a backwards induction procedure integrates the scenarios and MOLP components of the model to derive a strategy that is non-inferior in terms of the expected cost and sediment objectives. In summary, the MOLP (1)-(5) is first solved for each scenario (i.e., for each of the branches of the chance nodes in Fig. 1) for each value of the sediment weight W considered, yielding optimal control actions $x^{m}_{i,j,k}$, their cost $Cost_{a,s,W}$, and expected sediment $E(Sed_{a,s,W})$. Then, proceeding backwards through the tree of Fig. 1, for each W, a choice of research action a is made based on minimizing the weighted sum of (6) and (7) $E(Cost_{a,W})$ - $W*E(Sed_{a,W})$ across a. This process is repeated for each W, each time leading to a different non-inferior choice of a and associated controls. The resulting expected cost and sediment can be plotted for each non-inferior a and its associated controls, tracing out a cost-sediment tradeoff curve.

Scenario Generation using Bayesian Inference

We used Bayesian inference to generate the scenarios used in the MOLP. First, available information about □ is expressed as a prior probability distribution $f_{□}(□)$. This distribution can be constructed using historical data or sample results if they exist. In sediment management, values of sediment loadings may be unavailable or disagreed upon, as is the case in the Minnesota River Basin. In that case, priors should be based on any other relevant information that is available, such as subjective expert judgment (Meyer and Booker 2001).

Once prior probabilities are chosen, the research action(s) a must be defined. These yield observations, $\boldsymbol{z_a}$, that reveal information about the uncertain quantity. The likelihood of a particular observation is described by a probability conditioned on the uncertain parameters, $f_{\boldsymbol{z_a}|\theta}(\boldsymbol{z_a}|□)$. These likelihoods can be based on statistical reasoning, error characteristics of sampling methods, expert judgment, or a combination. As with the priors, experts parameterized the likelihoods.

Bayes' Law combines the prior and likelihood function to produce a posterior distribution summarizing the new understanding of □ resulting from observation $\boldsymbol{z_a}$ (Krzysztofowicz 1983): $f_{\boldsymbol{\theta}|\boldsymbol{z_a}}(\boldsymbol{\theta}|\boldsymbol{z_a}) = \frac{f_{\boldsymbol{\theta},z_a}(\boldsymbol{\theta},z_a)}{f_z(z_a)} = \frac{f_{z_a|\theta}(z_a|\boldsymbol{\theta})f_{\boldsymbol{\theta}}(\boldsymbol{\theta})}{\int f_{z_a|\theta}(z_a|\phi)f_{\boldsymbol{\theta}}(\phi)d\phi}$. The numerator in Bayes law weights the likelihood function values by the prior probabilities. If the prior distribution is very flat (relatively equal probabilities of different $\boldsymbol{\theta}$**)**, the posterior will be nearly proportional to the likelihood function, meaning that the observations $\boldsymbol{z_a}$ will influence the posterior distribution much more than the prior knowledge. On the other hand, if the likelihood is flat, the posterior will most resemble the prior distribution .

In our computations, possible values of $\boldsymbol{z_a}$ are discretized as $\{z_{a,n}, n = 1,..,N\}$. The result, for each observation n from each research program a, is the posterior distribution $f_{\boldsymbol{\theta}|\boldsymbol{z_a}}(\boldsymbol{\theta}|\boldsymbol{z_{a,n}})$ of loadings, as well as the probability of that observation $P_a(z_{a,n})$. From the posterior, we obtain $E(\boldsymbol{\theta}|\boldsymbol{z}_{a,n})$, the vector of conditional expected loads from each source, and the expected sediment delivery ratio. These expected loads are then used to define the scenarios used in the multiobjective program. The scenario s for action a is defined as $\boldsymbol{\theta}_a^s = E(\boldsymbol{\theta}|\boldsymbol{z_{a,n}})$. Note the scenario indexing is equivalent to the observation indexing (s = 1,…,N). The details of prior distributions, likelihood functions, and the discretization process are all provided in the illustrative example below.

Value of Information

The framework's last component evaluates the usefulness of information produced from research. This involves computing the EVII for each research action a and each weight W as the improvement in the expected objective relative to no research. The following process is repeated for each a and W. First, the optimal objective value from the MOLP is determined for the case of no research (a = 0) by solving (2)-(6) considering the prior expected sediment loss $E(\theta_{i,j})$ instead of the posterior $E(\theta_{i,j}|z_{a,n})$. Then, for each a and observation $z_{a,n}$, the optimal objective function (2) is found using the MOLP; these values, weighted by $P_a(z_{a,n})$, yield the overall objective for *a* (omitting the cost RC_a of the research itself). The EVII for *a* is then obtained by subtracting the objective for *a* from the expected objective for *a*=0. This is the most that the user should be willing to pay for the information yielded by the research. Here, we make this calculation for 45 research actions *a* and 12 values of *W*.

Illustrative Application: Maple River Basin

The framework is used to determine a non-inferior set of research and control strategies for reducing sediment loadings in the Maple River, a subwatershed of the Minnesota River basin. Because the sediment source and storage properties of the upper and lower parts of the watershed are different, the framework is illustrated by dividing the Maple basin into two identical upper watersheds (W1, W2), each draining into a lower watershed (W3) (see Supplementary Data Fig. S1). This division allows us to explore the impact of research in a portion of the full watershed when sediment source locations are uncertain. Reflecting the basin's geomorphology, the upper watersheds each have two sources: agricultural fields and streambanks, whereas the lower watershed has four: fields, streambanks, ravines and bluffs (Belmont et al., 2011).

In order to implement the framework, the following data are needed: parameters for the prior distributions of loadings; costs and likelihood function parameters for each research action; discretized distributions of the observations resulting from research; and cost and effectiveness of each control action. Details on the numerical inputs are provided in Jacobi (2009).

Prior Information

There are several techniques for expert elicitations of probabilities (Krzysztofowicz 1983). The choice of technique depends on several factors including the type of information sought and the resources available for the elicitation. Our work takes advantage of on-going research in support of a turbidity TMDL for the Minnesota River Basin (Minnesota Pollution Control Agency 2009).

One-on-one, in-person interviews with experts working in that basin were used to (i) collect current estimates of loadings from the various sources (as joint prior distributions), (ii) inventory research actions that can be used to learn more about loadings, (iii) describe the accuracy of information from research (as likelihood distributions), and (iv) inventory applicable control actions.

The priors $f_{\square}(\square)$, characterize the current state of knowledge. An expert engaged in sediment research in the Maple River was interviewed (Patrick Belmont, National Center for Earth-surface Dynamics, personal communication, 2 July, 2008). During the interview, the expert provided an expected value for the annual average sediment load for each of the eight sources in Fig. S1, as well as a 95% credible interval around the average. These values represent the sediment exiting its watershed. This approach of eliciting summary measures and credible intervals (Winkler 2003) was used because the expert was familiar with probability and felt comfortable providing these estimates.

For each source, the correlation of average loading with loadings from other sources was also elicited. For instance, if the streambank source in W1 exceeds the prior expectation, then streambank sources in W2 and W3 are also likely to be larger. The expert was asked if the correlation of each pair of loadings was zero, low (r = 0.2), medium (0.6), or high (0.85), and to provide a physical rationale for the response. As an example of an elicited distribution, field erosion in W1 was expected to yield 9000 t/yr, with a 95% credible interval of [7000, 9900] t/yr. Field contributions in W1 and W2 were expected to be strongly but not perfectly correlated due to the similarity of their hydrologic processes and soils.

The information gathered above was used to construct the joint prior distribution of loadings. The marginal distribution for each source was assumed to be log-normal. This distribution was chosen because non-positive loadings have zero probability, and the distribution allows for easy computation of posteriors. In particular, a joint log-normal distribution of loadings is joint normal in terms of the natural logs of the loadings.

The expert also provided a distribution for the sediment delivery ratio (SDR), the percent of sediment from W1 and W2 that exits W3. Since this random variable must lie between zero and one, we used a beta distribution. SDR was also assumed to be independent from the loading variables. The full joint prior distribution of the nine parameters, $f_{\square}(\square)$, is obtained by combining the beta distribution for SDR and the joint log normal distribution for loadings, using moment matching (see Jacobi 2009 for complete details): $f_{\boldsymbol{\theta}}(\boldsymbol{\theta}) = f_{\boldsymbol{\theta}_L,d}(\theta_{F1}, \theta_{S1}, \theta_{F2}, \theta_{S2}, \theta_{F3}, \theta_{S3}, \theta_{R3}, \theta_{B3}, d) = \frac{1}{(2\pi)^4|\Sigma|^{-1}} \exp\left(-\frac{1}{2}\left(\boldsymbol{\theta}_L - E(\boldsymbol{\theta}_L)\right)^{\mathrm{T}} \Sigma^{-1}(\boldsymbol{\theta}_L - E(\boldsymbol{\theta}_L))\right) \frac{d^{40.4}(1-d)^{12.8}}{\mathrm{B}(41.4,13.8)}$,where $\boldsymbol{\theta_L}$ is the vector of loadings, θ_{ij} is the loading from source *i* (field, streambanks, ravines, bluffs) in watershed *j* (W1, W2, W3), Σ is the covariance matrix, and *d* is the SDR in W3 (following a beta distribution with parameters 40.4 and 12.8). The parameters for all distributions used are provided in Jacobi (2009).

Research and Likelihood Functions

For each watershed, three single year actions are considered: gauging, sediment fingerprinting, and sediment source analysis (SSA). These actions and their expense were selected to reflect current and proposed research in the Minnesota River Basin. Gauging was specified to include placement of a gauge at the watershed outlet and 25 streamflow and sediment samples used to estimate the annual average loading, with an annual cost of $15,000 (Guy 1969). Fingerprinting was specified as twelve samples for Ce-137 and Pb-210 to estimate the proportion of field-derived sediment, with a cost of $20,000. A full sediment source analysis was specified as a year-long study combining field work, aerial photograph analysis, and literature reviews to estimate sediment contributed by each source in the watershed, with a cost of $40,000.

A research action *a* is defined as a combination of the three actions across the three watersheds. In addition to performing each action separately in each watershed, combinations of two concurrent actions are also considered, giving a total of 46 actions, including “no research.” Five actions are shown in the decision tree of Fig. 1. Combining actions allows for interesting possibilities because different actions provide distinct information. Stream gauging quantifies the total sediment passing the gauge, with no differentiation regarding source. Fingerprinting estimates the proportion of field sediment from the watershed, but provides no information on load magnitude. SSA produces estimates of the sediment loading from each sediment source.

The probability $f_{\boldsymbol{z_a}|\square}(\boldsymbol{z_a}|\square)$ of observing a particular observation $\boldsymbol{z}_a$ is defined using a log-normal likelihood function, again because the observations from the research actions must be non-negative and to facilitate computation of the posterior distributions. Each research action produces a different type of observation. Research actions that combine two individual research actions produce composite observations. For example, if both gauging and fingerprinting were performed in watershed W1 concurrently, the action would produce an estimate of the total sediment loading in W1 and the proportion of field sediment. Depending on the action, the observation produced is either a scalar or a vector. The likelihood function for each scalar observation is written as $f_{z_a|\boldsymbol{\theta}}(\boldsymbol{z_a}|\boldsymbol{\theta}) = \frac{\mathbf{1}}{z_a\sigma_a^{lik}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\ln(z_a)-\mu_a^{lik}}{\sigma_a^{lik}}\right)^2\right]$, where $\mu_a^{lik}$is the mean and $\sigma_a^{lik}$is the standard deviation of the natural log-transformed observations for research action a.For vector observations $\boldsymbol{z}_a$, the likelihood function is determined by recognizing that the natural logarithm of a log-normal variable results in a joint normal distribution: $f_{z'_a|\boldsymbol{\theta}}(z'_a|\boldsymbol{\theta}) = \frac{\mathbf{1}}{(\mathbf{2\pi})^{\mathbf{4}}|\Sigma_{\boldsymbol{a}}^{\boldsymbol{lik}}|^{-1}}\exp\left[-\frac{1}{2}\left(\boldsymbol{z'_a} - \boldsymbol{\mu_a^{lik}}\right)^{\mathrm{T}}\left(\boldsymbol{\Sigma_a^{lik}}\right)^{-1}\left(\boldsymbol{z'_a} - \boldsymbol{\mu_a^{lik}}\right)\right]$, where z'_a is the vector of natural log-transformed observations for research action a, $\boldsymbol{\mu_a^{lik}}$ is the vector of means and $\boldsymbol{\Sigma_a^{lik}}$ is the covariance matrix of natural log-transformed observations for research action a. A complete description of the means, variances and covariances is provided in Jacobi (2009, section 4.4)

For each likelihood function, the expected observation $E_{z_a|\square}(z_a|\square)$ was expressed as a function of the parameter values. Gauging estimates the total sediment exiting the watershed, thus the expected value of gauging is the expected value of the sum of the parameters representing upstream loadings. For example, gauging in W1 (a = 16) would have an expected observation of $E(z_{16}\,|\square)$ = $E(\theta_{F1}+\theta_{S1})$, where θ_{F1} is the field sediment in W1 and θ_{S1} is that from streambanks, the only sources in that watershed. In contrast, fingerprinting estimates the proportion of field sediment. The expected value of fingerprinting in W1 (a = 1) is then $E(z_1|\boldsymbol{\theta}) = E\left(\frac{\theta_{F1}}{\theta_{F1}+\theta_{S1}}\right)$. Finally, an SSA produces an estimate of each source. For example, the expected value of a SSA in W1 (a = 40) is a two dimensional vector: $E(z_{40}|\square) = \{\theta_{F1},\theta_{S1}\}$.

Measures of spread for the likelihoods were obtained as follows. Each research action is subject to several sources of error. For example, sampling is subject to both spatial and temporal variability. There are sample errors as well as measurement errors associated with the calculation of loads for each method. For each research action, the expert was asked to provide a 95% credible interval reflecting these error sources, from which standard deviations were inferred. In addition, the expert was asked if the errors of different actions were correlated. Correlations were elicited for all possible pairs of actions, as well as correlations in the errors of the individual components of SSAs. As an example of elicitation results, SSA observations were concluded to have a confidence interval of [50%,200%] around the true parameter values, while SSA observational errors in different watersheds were assumed to be moderately correlated (r = 0.5).

Discrete Observations and Posteriors

Once the priors and likelihoods are parameterized, we discretize the continuous ranges of z_a, because decision trees require discrete observations. First, a preliminary analysis was performed to determine the range of possible observations. This was accomplished by examining the range of z_a for which the likelihood function could be possible, considering the prior ranges of sediment loadings and sediment delivery ratio. For research actions for which z_a are scalar (only gauging or only fingerprinting), the range of possible values was divided into $N = 9$ mutually exclusive intervals. For the remaining actions, each of which has a vector z_a outcome, the ranges of values for each dimension were divided into 3 intervals. For actions with two observations (e.g., gauging and fingerprinting simultaneously in W1), each of the N=9 combinations were considered. For actions whose z_a have more than two dimensions, a subset of $N = 9$ discrete intervals was selected using Latin hypercube sampling (McKay et al. 1979).

A value $z_{a,n}$ was assigned to each of the nine intervals and the associated probability $P_a(z_{a,n})$ of the interval were estimated by Monte Carlo (MC) integration with antithetic sampling to reduce variance (Fishman 1996). We selected these discrete values $z_{a,n}$ in order to approximate the original distribution using a "moment matching" approach. This was implemented by running an optimization model that minimized the sum of squared deviations between the discrete probabilities and the MC estimates of the probabilities for their associated intervals, subject to constraints that the means and covariances of the discrete distribution matched the means and covariances of the continuous distribution as estimated by the MC integration. Table S1 presents an example of discrete observations selected by this procedure for one research action.

Based on the prior and likelihoods, Gibbs sampling (Geman and Geman 1984) using WinBUGS (Lunn et al. 2000) was used to simulate the posterior distribution $f_{\square | z_a}(\square | z_a)$ for each action a and observation n. The majority of the posterior distributions appear to be most influenced by the prior distributions, meaning that the likelihoods tend to be flat compared to the priors. A complete description of the posterior distributions is provided in Jacobi (2009, Section 4.5.2). The conditional means of the parameters $E(\square | z_a)$ are input to the MOLP in the form of the scenarios

$\boldsymbol{\theta}_a^s = E\left(\boldsymbol{\theta}|z_{a,n}\right)$. Those conditional means are adjusted to ensure that the probability-weighted posterior expectation equals the prior expectation: $\sum_{n=1}^{9} E(\boldsymbol{\theta}|\, z_{a.n})P_a(z_{a,n}) = E(\boldsymbol{\theta})$.

BMPs

The MOLP requires a set of candidate BMPs and their costs and effectiveness. These were identified through a literature review and interviews with sediment management experts in Minnesota. The cost and effectiveness for each BMP can be found in Table S2 of the online supplemental data. The field BMPs considered are critical area planting (CAP) and conservation tillage (CT). CAP involves establishing permanent vegetation, such as perennial grasses, perennial legumes, trees, or shrubs (USDA NRCS 2008). Conservation tillage allows continued production of row crops. For these field BMPs, the fractional reduction in sediment was determined using the Revised USLE, RUSLE2 (Foster et al. 2003).

BMPs addressing streambanks include stabilization (SS, including stream barbs and riprap) and restoration (SR, involving redirection of streamflows, reshaping of slopes, and bank vegetation). To reduce ravine erosion, two BMPs are considered: land retirement (LT) and tile drainage pipes (DP). Retirement replants ravine edges with perennial vegetation in order to filter out sediment and slow runoff velocities. Under DP, tile drainage pipes are laid along the bottom of the ravine to direct the flow from the ravine to the stream channel. Lastly, BMPs addressing soil loss from bluffs are toe protection (TP, including stream barbs and riprap) and complete stabilization (CS, including slope grading and retaining walls).

Load reductions (tons) per amount of installed BMP are the product of the fractional reductions (Table S2) and the uncontrolled loss θ. We assume that each source has a distribution of loss within a subbasin, and that areas with the highest relative loss would be addressed first, as described by the soil loss curves $S_{i,j}(y_{i,j})$. Fig. S2 summarizes the loss curves for all sources.

Note that the MOLP above does not address all possible sources of BMP uncertainty, such as uncertain social acceptability, cost per unit, and uncertain fractional reductions in sediment loss. Here, the uncertainties we explicitly address by Bayes' law are intentionally limited to focus on those that are most salient in public debates in the Minnesota River basin over the responsibility for sediment reduction.

The remaining data required for the MOLP are the contributing area/length of each source in each watershed and the upper bound on each BMP. This information is available in Tables S3 and S4 of the online supplemental data. As an example of procedures used to define these values (Jacobi 2009), the total field area and stream length for each watershed were determined through GIS analysis of the Maple River watershed (R. Moore, Mankato State University, personal communication). The proportion of the total area and stream length that contributes sediment in each watershed was found from the soil loss curves.

Results of the Maple River Case Study

MOLP Solutions

For each research action *a* and observation *n*, the MOLP was run using 12 values for the weight *W*, equivalent to the $/t marginal worth of sediment reductions. Then we record the optimal research action *a* for each. A wide range of *W* ($1/ton - $5000/ton) is used to generate tradeoffs between expected cost and sediment reduction. In contrast, marginal costs of $10/ton - $126/ton for sediment reduction have been reported (e.g., Moore et al. 1992; Khanna et al. 2003; Yuan et al. 2002; Yang et al. 2003).

Before examining the research and BMP decisions for different weights, we show the overall tradeoff curve (Fig. 2). This curve results from varying *W* and calculating the expected cost and loading from each resulting optimal combination of research action *a* and associated control actions. The $1/ton weight results in no actions, as would any weight less than that value. The expected total sediment loading without abatement is 90,000 t/yr. Management actions reduce this amount of sediment, up to a maximum reduction of 80%. This maximum possible abatement is achieved with a cost of over \$9.6M.

The figure shows that there are rapidly diminishing returns; spending \$1M/yr lowers sediment by 50%, but to reduce it another 25% requires another \$3M/yr. The figure also indicates that research is generally optimal if the expected cost is \$1M or above \$3M; otherwise, doing nothing (a = 0) is best. The possibility of doing research does not greatly change the costs of sediment reductions; the locations of points for a = 0 relative to the tradeoff curve show that what we learn from research does not appreciably shift the cost curve downward. That is, for the input values used in this illustration, better targeting of controls because of improved information about sediment sources yields neither large cost savings nor significantly decreased sediment.

We now turn to the optimal research and BMP choices under various values of W. Fig. 3 shows the optimal research action a and the expected sediment reduction (over the N = 9 possible observations) from subsequent implementation of BMPs in W1. To help interpret these results, Table 1 shows the implied cost/ton for each action based on the expected prior loadings. Since the use of nonlinear soil loss curves (Fig. S2) imply that costs are a nonlinear function of the extent of implementation, we list low, high and average costs. The exception is streambank stabilization and complete stabilization, for which the soil loss curve is linear, and so only an average cost effectiveness value is shown. This table shows that many actions will not be taken unless the marginal worth for sediment reduction exceeds \$100, but that some are worth doing even for a marginal value of \$10/ton. As the cheapest measure (CAP) costs \$7/ton when applied to areas contributing the most sediment, doing nothing is optimal for W = \$1 or \$5/ton.

As W is increased, the most cost effective controls are implemented first. When research is optimal (W = 50, 200 and greater), the outcome of research generally informs the amount of each control to use; and, less frequently, a set of controls different from the initial recommendation.

Value of Information from Research

New information can change the optimal BMP. Streambank stabilization is suboptimal under no research ($a = 0$) when the marginal reduction is \$50/ton and the expected cost is \$1M, but it can become optimal if research indicates that more sediment than expected comes from streams. In particular, there are four scenarios resulting from gauging in W1 and SSA in W2 ($a = 21$) that lead to streambank stabilization being chosen in W3: $n = 3$, 6, 7, and 8. These four observations yield large posterior loadings for streambanks in W3 compared to the prior loadings (7000 t/yr).

As another example, Fig. 3 indicates that it is optimal to perform research when W is \$50/ton, but no research becomes optimal if W is increased to \$100/ton. When W is increased further, it is always optimal to perform research. This is because at \$100/ton, research would cost more to implement than the value of the information obtained (the improvement in the objective). For example, for action $a = 21$ (W1 gauging, W2 SSA), when $W =$ \$100/ton, streambank stabilization in W3 is performed for all but three observations (1, 4, and 9) which have very low posterior W3 streambank loadings. In contrast, streambank stabilization in W3 is the optimal solution under no research. The value of a=21 lies in dictating under what conditions streambanks should be stabilized in W3. The value of this information is slightly less than the cost of gauging W1 and SSA in W2. Thus research is suboptimal, and instead W3's streambanks should immediately be stabilized. In terms of expected costs, the expected sediment loss of 42 Kt/yr comes with an expected cost of just over \$3M, while it costs just over \$5M to decrease the sediment loss by an additional 6 Kt/yr. By examining the slopes in Fig. 2, the decision maker can evaluate when the marginal sediment reduction is worth the marginal increase in cost.

As W is increased further, it is always optimal to perform research; however, the marginal cost of additional sediment reduction rises sharply. Table S5 shows which research actions (highlighted in black) have an EVII exceeding the action's cost for each value of *W*.

In general, the framework indicates that are two types of controls whose adoption is highly affected by research and monitoring in advance of BMP implementation. First, when posterior streambank loadings are high and the sediment weight is sufficiently large, bank stabilization enters the solution. The choice between land retirement and drainage control, both of which address ravines, is also affected by observations. For low *W*, land retirement is preferred because it is cheaper. But it is also less effective, reducing sediment loss by 70% compared to drainage control, which reduces sediment by 90%. When expected posterior ravine loadings are high, the extra expense of drainage pipe is justified; otherwise it is not.

One trend is that EVII is nonmonotonic as sediment weight increases. At very low weights, few or no controls would be adopted, and at extremely high weights, all available controls would be implemented, no matter what is learned; consequently, EVII would be low in either case. Variations in EVII for in-between weights have more complicated causes.

For instance, as *W* increases to \$10/ton, all actions show increased EVII; however, when the weight is further increased to \$20/ton, the value of information actually drops for all but two actions. This initial increase and decrease is due to fact that better information provides value because certain observations result in the selection of different controls compared to the no research action. But when the sediment weight is increased, the information gained from research only differentiates the *amounts* of each control. In general, EVII is higher when observations yield different *types* of controls rather than just different *amounts* of the same controls as no research. Table S5 also indicates that EVII for pairs of research actions performed in the same watershed (e.g., gauging and fingerprinting in W1, a=17) tends to be lower than if the same actions are taken in different watersheds (e.g., gauging W1, fingerprinting in W2, a=18). This is because performing research in different watersheds provides more information. For example, gauging and fingerprinting in W1 provides improved understanding of just W1, whereas gauging in W1 and fingerprinting in W2 improves understanding of both watersheds.

Conclusions

We propose a framework for identifying optimal combinations of research and control actions to efficiently reduce sediment loss from rural watersheds. The framework's linear program permits consideration of a large number of combinations of research, monitoring, and BMP implementation alternatives. Bayesian inference permits integration of diverse sources of information, including data collection, engineering analysis, and expert judgment, in order to evaluate the benefits of research and monitoring. We have demonstrated that these factors can be combined in an effective framework to support decision making about monitoring and BMP implementation.

We illustrate the framework using values from the Maple River, a major sediment source to the Minnesota River. The example illustrates advantages of the framework, including direct incorporation of the value of information, calculation of joint posterior distributions of loadings from multiple sources, and evaluation of strategies that condition BMP implementation on information obtained from research and monitoring. The framework is defined for watersheds with several types of spatially dispersed sediment sources. For the illustrative case, the framework reveals that the \$/ton value (denoted as W in (1)) placed on load reduction strongly impacts the extent of BMP implementation and whether research actions are worthwhile. The complexity of the problem prevents identification of clear trends concerning which research is most valuable, indicating that several interacting factors influence the results. For example, the results indicate that the choice of where to perform gauging is not intuitive. This justifies the use of systems analysis.

When implemented for a practical application, sensitivity analyses of judgments elicited from experts will be essential. This can be accomplished by considering a range of values for each elicited parameter and then constructing several prior and likelihoods functions as a result of different elicited values. All combinations of prior and likelihood functions could then be analyzed to determine which elicited values have the most influence on the optimal research and management actions. This could lead to a second interview with experts in which the judgments that most affect the results are revisited.

The framework presented here can be expanded to other nonpoint pollutants, management measures, and additional subbasins. The larger problems that result might usefully be addressed by advanced decomposition methods (e.g., Watkins Jr et al. 2000; Cai et al. 2001).

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Supplemental Data

Figures S1 and S2, as well as Tables S1-S5 are available online.

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