ENVIRONMENTAL SYSTEMS AND DECISION ANALYSIS MODELS FOR AIDING ENVIRONMENTAL POLICY DECISIONS UNDER DETERMINISTIC AND STOCHASTIC SETTINGS

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A dissertation submitted to Johns Hopkins University in conformity with
the requirements for the degree of Doctor of Philosophy

Baltimore, Maryland
May 2009

ABSTRACT

This dissertation presents three new environmental system and decision analysis tools, each with an environmental application. Together these studies incorporate techniques from environmental systems and decision analysis to provide decision makers with tools to aid in managing complex, real-world environmental problems.

The first study develops two novel integer programming models for identifying irreplaceable nature reserve sites. Knowing which sites are irreplaceable allows decision makers to target reserves that must be selected in order to achieve a conservation objective. The models efficiently determine irreplaceable sites, but find a general lack of trend between the number of irreplaceable sites and the number of sites available for selection. Moreover, irreplaceability at one resource level may not be a predictor of irreplaceability at a higher or a lower resource level.

The second study develops a model for estimating and correcting attribute-weighting biases that result from the use of value trees to elicit decision makers' preferences. Value trees have been used to aid decision makers selecting among alternative solutions to complex environmental problems. The model is based on the conjecture that attribute weights are influenced by tree structure and a subject's use of the "anchor-and-adjust" heuristic. Weights corresponding to environmental and economic attributes of electric system expansion alternatives are elicited from electric utility employees are used to test the model. The model results support the hypothesis that a bias exists that is consistent with the anchor-and-adjust heuristic and illustrate the value losses caused by using elicited versus model-estimated debiased weight sets.

The third study presents a framework to identify the optimal set of information acquisition and abatement actions to address environmental management when there is uncertainty in the environmental processes, the outcomes of those processes, and the effectiveness of management. The framework combines Bayesian inference with multiobjective programming to select research actions, which improve understanding of the natural system, and management actions, which reduce environmental contamination. The model is applied to the problem of reducing turbidity from nonpoint sediment sources in the Minnesota River basin. The results indicate that the economic value placed on sediment reduction influences the choice of both monitoring and management options.

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ACKNOWLEDGEMENTS

I would first like to express my thanks to the late, great Chuck ReVelle. I thoroughly enjoyed being his apprentice and miss all the stimulating conversations we had my first few years of graduate school. He is sorely missed. I would also like to thank my advisor, Ben Hobbs, who always kept me thinking. I really appreciate his helpful comments and email responses at any hour of the night.

Peter Wilcock provided numerous helpful conversations about sediment to my sedimentally challenged self. Patrick Belmont also graciously participated in crucial expert elicitation and was always quick to reply to any questions I had. I would also like to thank Pearl Zhang, Jennifer Bassman Ruch, and Carol Zuerndorfer for their help with data collection for the Minnesota research, as well as Steve Becker and John Brach for their helpful insights into management practices in Minnesota. Thanks to Karen Gran and the rest of the stream restoration team at NCED, who were always extremely helpful in pointing me in the right direction.

I would also like to thank Bob Pressey and Justin Williams for the collaboration on the first part of my dissertation. It was really enjoyable working with them. Eric Lonsdorf provided me with intellectual and emotional support in the final stretch. Kirsten Almberg kept me pushing ahead when I was sure the end would never come.

Lastly, I'd like to thank my friends and family, especially my husband Tomas. He put up with me in the darkest of times. I am forever grateful for his caring advice and understanding. And to my friend Emily, I could not have gotten through many days without her help. I cannot thank her enough for all the support over the last many years.

This work was made possible by a Clare Boothe Luce Fellowship, a National Science Foundation Graduate Research Fellowship, and tuition support from the National Center for Earth Surface Dynamics, a NSF Science and Technology Center at the University of Minnesota.

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Chapter 1

Introduction

Economic development, human population growth, and government policy play major roles in impacting the natural environment (Forester and Machlis 1996). Human interactions with the natural environment can cause a variety of negative consequences. For example, biodiversity has been significantly impacted by humans. Extinction rates of species populations are currently estimated to be 100 – 1000 times greater than pre-human rates (Pimm et al. 1995). Similarly, humans have had a profound impact on air quality. Electricity generation, while a necessity of modern society, can result in undesirable environmental impacts, particularly air pollution. Human activity has also degraded the quality of water bodies throughout the US. According to the Environmental Protection Agency, as of the year 2000, "39% of assessed stream miles, 45% of assessed lake acres, and 51% of assessed estuary acres are impaired (U.S. Environmental Protection Agency 2003)," meaning that they do not meet state or federal water quality standards. Impairments are due primarily to nonpoint source pollution, with nearly 48% of the impacts on impaired rivers and streams due to agricultural activities (U.S. Environmental Protection Agency 2003).

There are a wide variety of human activities that impact the natural environment, and their consequences can be quite diverse. However, the range of complex environmental problems presented above can each be addressed using the framework of environmental systems analysis, which assists decision makers in optimally designing and managing environmental systems.

1.1 Purpose

To further expand the breadth and utility of the environment systems analysis framework, this dissertation presents three distinct research studies in which new systems tools are developed and applied to three different environmental management problems.

The first study involves the development of two novel integer programming formulations for aiding the design of nature reserves for species preservation. A key principle of reserve design is irreplaceability. Irreplaceability of any site is defined as the site's probability of appearing in alternate optimal solution sets to a reserve selection problem (Pressey et al. 1994). If a site appears in all alternate optimal sets, the site has an irreplaceability value of 1.0 and is called irreplaceable. Consequences of omitting irreplaceable sites in a nature reserve design include the inability to achieve objectives, such as full species representation, and required increases of resources, such as additional reserve sites, to meet objectives. The particular research questions addressed in the first study are:

1. Can an efficient mathematical program be developed to determine which reserve sites are irreplaceable when requiring full species representation?

2. Can an efficient mathematical program be developed to determine which reserve sites are irreplaceable when protecting the maximum number of species with a limited number of sites?

3. What are the impacts on the number of species protected and the number of additional sites required if an irreplaceable site is omitted from the set of available sites?

While the first study identifies alternate optimal solutions to single objective problems, the second study addresses alternate solutions to problems with multiple objectives. Many decision analysis tools exist that can aid decision makers to express priorities and value judgments and ultimately select among alternative solutions to complex problems. These methods have been used extensively for environmental management problems. One common task in decision analysis is the elicitation of preference weights for multiple objective problems; however, certain techniques aimed at eliciting decision makers' preferences are prone to well-established biases. The second research study in this dissertation answers the following questions:

1. Can a model be developed to quantify and mitigate biases that appear when using value trees to aid in preference weight elicitation?

2. How does the use of different sets of preference weights affect the preferred solution?

These questions are considered in the context of selecting among electricity generation planning alternatives, each having various economic and environmental attributes.

The third research study addresses water quality impairments in the Minnesota River basin using a multiobjective, stochastic approach. Many streams and rivers within the basin are in violation of standards for turbidity due to sediment loading; however, the relative contributions of the range of possible sources to the sediment loadings are uncertain. To reduce sediment loadings to the waterways, a variety of management actions are under consideration. Researchers are also developing and implementing methods for improving the understanding of the contribution of each sediment source. In this research study, I answer the questions:

1. Can a stochastic mathematical model be developed to determine the optimal set of management actions and research actions to minimize both the expected cost and expected sediment loadings to the waterways?

2. What is the value of information that can be provided by possible research actions that can be used to reduce uncertainty about sediment sources?

The model combines Bayesian inference with linear programming to answer these questions. The following section provides a description of the environmental systems analysis framework used throughout this dissertation.

1.2 Systems Analysis and Decision Analysis Defined

The field of systems analysis, or operations research, combines economics, mathematics, statistics and other disciplines to develop mathematical models aimed at identifying optimal solutions to complex decision problems. Environmental systems analysis applies the tools of systems analysis to environmental problems.

A systems approach based upon mathematical programming (or optimization) consists of several elements. First, a set of objectives and corresponding performance measures are identified. The objectives describe the stakeholders' and decision makers' desired outcomes. Next, decision variables describe the management actions available. Their costs and effects upon important problem components are incorporated into the objectives. Constraints are included to address strict requirements that the solution must adhere to, such as describing how a physical system responds. For example, mass balance constraints might be included to accurately reflect the physical system. Constraints can also be used to enforce budget and resource limits and impose necessary social, legal, or environmental requirements. For example, if a particular water quality standard must be met, a constraint can be included in the methodology to enforce this. The goal of the systems approach is to generate a solution(s) (set of values of the decision variables) that is efficient with respect to the objectives, while meeting all the constraints. "Efficiency" means that there are no other feasible alternatives that do better in one or more of the objectives, while not performing worse with respect to any other objective.

Complementing optimization-based systems analysis, decision analysis promotes a clearer understanding of tradeoffs and uncertainties. Decision analysis is a broad framework that helps model decision problems, including the uncertainty contained within them, as well as decision maker preferences regarding priorities among objectives and willingness to take risks. First, decision analysis helps define objectives by assisting decision makers in addressing and organizing their values with a variety of techniques. These objectives correspond to the objectives described above, meaning that decision analysis can be a useful starting point for an optimization-based systems approach. Decision analysis also helps the decision maker consider uncertainty in the problem setting and identifies the decision maker's preferences and attitudes towards risk. These functions are performed by eliciting subjective probabilities regarding problem outcomes and developing utility functions for the decision maker that can be used to define preferred solutions (i.e., expected utility maximizing solutions). Together, systems and decision analysis help decision makers frame and solve problems in a more methodical manner.

1.3 A Brief History of Environmental Systems Analysis

The field of operations research (later also called systems analysis or systems engineering) arose during World War II when the British Air Ministry established the Bawdsey Manor Research Station to study how to intercept enemy aircraft with newly developed radar technology (see Gass and Assad 2005 for further details). Around the same time, two scholars, Kantorovich and Koopmans, independently solved what is now the classical transportation problem, which seeks to minimize the cost of distributing items. The types of problems Kantrorovich and Koopmans studied involved optimizing a linear objective subject to linear equality and inequality constraints and were termed linear programming problems.

In 1947, while working on a US Air Force research project, Dantzig developed the Simplex method for solving linear programs. The method was nearly identical to the ones used by Kantorovich and Koopmans to solve the transportation problem. A modified version of Dantzig's Simplex method is still used today to solve linear programming problems.

Another influential figure in operations research was Charnes. Charnes and his colleagues were very active in applying linear programming (LP) to industrial applications and developed the gasoline-blending model, which was the first LP to be used in industry (Charnes et al. 1952). In addition, Charnes and his colleagues were the first to address environmental problems with LP. Charnes and his student Lynn addressed the design and operation of reservoirs, as well as water quality problems with LP. At the same time, the Rockefeller Foundation funded the Harvard Water Program, which was a water resources seminar for graduate students and government personnel in the Graduate School of Public Administration. The Program developed a systems based approach for selecting water projects (Maass et al. 1962) that greatly influenced the current operations of the US Army Corps of Engineers.

Further developments in the field of operations research allowed a wider variety of problems to be addressed. For example, integer programming was developed by Gomory (1958) to formulate problems in which some variables must be binary. Kuhn and Tucker introduced nonlinear programming for problems with nonlinear objectives or constraints (Kuhn and Tucker 1951). Stochastic programming (Dantzig and Infanger 1999) and chance-constrained programming (Charnes and Cooper 1959), as well as other methods, were developed to address problems in which uncertainty was important.

Currently, the range of environmental systems analysis problems is vast. A quick survey of the most recent literature includes topics ranging from using decision analysis techniques to manage dairy effluent (Hajkowicz and Wheeler 2008) to the development of a chance-constrained program for managing water pollution (Liu et al. 2008). The research presented in this dissertation further expands the environmental systems analysis literature by modeling three distinct environmental problems with the use of systems analysis and decision analysis tools.

1.4 Problem Backgrounds

1.4.1 Irreplaceability in Species Protection Models

The first complex environmental problem addressed in this thesis is the preservation of species. The protection of species from human threats is important because of the many benefits that biological communities provide, including direct use, indirect use, existence, and option values. Direct use values include consumptive and productive uses. Consumptive use values are assigned to goods consumed locally and not sold in national or international markets (Primack 2002), while productive use values are assigned to products that are harvested from the natural environment and sold in national and international commercial markets. In 2001, 4.5% of the U.S. GDP depended on wild species in some way (Primack 2002). The indirect economic values of species are perhaps more important than the direct economic values. Species provide a wide range of benefits derived from non-consumptive uses. For example, the primary productivity of ecosystems is the building block for food webs. Healthy ecosystems protect watersheds, guard against the impacts of flood and drought, maintain water quality, curtail erosion, and prevent disruption of the hydrologic cycle (Primack 2002).

Ecosystems also provide humans with recreational services such as hiking, sport fishing and swimming. Lastly, ecosystems and the species within them provide existence and option values. Existence value measures the willingness to pay to prevent the destruction of a habitat or species, without the intention of using the resource in the future. The existence of many environmental advocacy groups, such as The Society for Conservation Biology, The Nature Conservancy, and the World Wildlife Fund, reflect society's desire to protect the environment. The option values of species indicate the willingness to pay to ensure the future existence of a species because they may be interested in using the resource in the future (Field 2001).

Loss of biodiversity can be prevented through the use of nature reserves (Bruner et al. 2001). Nature reserves are defined as areas under in situ protection measures (Pressey et al. 1993). Examples of nature reserves include wildlife refuges, national parks, and land protected by conservation groups. Since 1980, researchers have developed algorithms to protect species by selecting land to set aside as nature reserves (Pressey 2002). ReVelle et al. (2002) provide a review of exact and heuristic methods for solving five classes of reserve selection problems that aim to protect as many species as possible subject to a variety of constraints including resource limitations and uncertainty in the distribution and survival rates of species.

Three main principles have guided reserve selection (Pressey et al. 1993). First, when limited resources are available to devote to nature reserves, sites should be selected to complement the features of the existing reserve sites. This is called complementarity. Second, it is often true that there are many ways of combining sites to form networks of reserves that achieve the same objective. It is important to consider many (if not all) possible reserve networks so the selection of a reserve network is flexible. Flexibility allows for adaptation in the face of changes in site availability. Lastly, reserve selection must consider irreplaceability. Irreplaceability has been defined in several ways. For this problem, the irreplaceability of any site is defined as the site's probability of appearing in alternate optimal solution sets to a reserve selection problem (Pressey et al. 1994). If a site appears in all alternate optimal sets, the site has an irreplaceability of 1.0 and is called irreplaceable. If an irreplaceable site is not selected, one or more objectives will be unachievable or, in some cases, the number of sites needed to achieve the objectives will increase

The implications of identifying the irreplaceability levels of sites in a reserve design are important to decision makers. When selecting sites for a nature reserve, the decision maker can identify which sites are irreplaceable, and therefore absolutely required to achieve an objective. At the same time, the decision maker can determine which sites may be traded with other sites while still meeting the reserve selection goals. However, determining irreplaceability values for sites can be deceptively difficult. Suppose the objective is to select the fewest reserve sites necessary to ensure that each species is present in at least one selected site. It might be surmised that the irreplaceable sites could be found simply by identifying the reserve sites that contain unique species. However, due to the demands of complementarity, some irreplaceable sites do not contain unique species. In order to determine the irreplaceable value for each site, all optimal and suboptimal solutions must be examined. This is an intractable problem when the number of potential reserve sites is large (Pressey et al. 1994). However, identifying the sites with irreplaceability value of 1.0 (irreplaceable sites) can be accomplished.

In Chapter 2, two integer programming models are developed to identify irreplaceable nature reserve sites in the context of the species set covering problem and the maximal covering species problem. The improvement of these methods over previous methods for producing the set of irreplaceable sites can be measured in terms of the decreased computational burden, as well as the existence of an optimal technique that does not rely on enumeration of solutions.

1.4.2 Quantifying and Mitigating the Splitting Bias and Other Value Tree-Induced Weighting Biases

The second environmental management problem addressed in this dissertation combines behavioral decision making research with decision analysis to develop a model to quantify and mitigate biases that occur with the use of particular techniques aimed at eliciting decision makers' preferences. Many problems facing environmental decision makers are complex, involving multiple conflicting objectives. Multiattribute decision analysis provides a formal framework that helps decision makers tackle these complex problems. An important recent trend in multicriteria decision making research is combining the streams of behavioral decision making and decision analysis to devise methods to adjust expressed value judgments for known, persistent and important biases (Anderson and Hobbs 2002; Bleichrodt et al. 2001; Clemen 2008).

In general, if biases exist in elicited weights, it is desirable to address the biases and correct the weights to better represent the decision maker's preferences. When a large portion of the variation in weights is attributable to the elicitation process used (i.e., the choice of method can bias the weights), one approach is to choose a simpler, less cognitively demanding task for eliciting weights, such as deriving weights from holistic rankings (Edwards and Barron 1994). However, simple methods can result in loss of important ratio scale information and even result in incorrect rankings of objectives, and suboptimal decisions can be made (Jia et al. 1998). Therefore, it is important to address these sources of variation in developing more informative weight elicitation procedures.

In Chapter 3, a model is developed to quantify and mitigate biases that occur with the use of a value tree to assist decision makers in preference weight elicitation. A model to mitigate these biases has not been developed previously, and other methods to mitigate the biases have been unsuccessful. Weights elicited from employees of the Centerior Energy Corporation regarding environmental and economic attributes of alternative electric system expansion plans are used to illustrate the existence and correction of these biases.

1.4.3 A Bayesian Framework for Cost Effective Management of Sediment Reduction in the Minnesota River Basin

The last essay of this dissertation addresses the problem of controlling nonpoint source pollution, particularly from agricultural lands. Despite the passage of the Clean Water Act amendments to the Federal Water Pollution Control Act in 1977, many river basins are still impaired, meaning that they do not meet state or federal water quality standards. (U.S. Environmental Protection Agency 2003). One major nonpoint pollutant impairing water bodies is sediment. The loss of soil from erosion can lead to significant problems on the land itself, including depletion of nutrients and deterioration of soil structure, which decreases the productivity of the land. When the sediment reaches surface waters, the suspended sediment particles increase turbidity, which limits light penetration and causes changes in primary production. Conditions for primary producers can shift from nutrient-limited to light-limited, ultimately affecting dissolved oxygen and stream metabolism. Excess sediment can lead to habitat loss, changes in predation success, and alteration of food web dynamics. Finally, other nonpoint source pollutants such as nutrients, pesticides, and herbicides may also be associated with soil loss, leading to other important pollution problems such as eutrophication.

As a nonpoint source pollutant, the sources of sediment are diffuse, each contributing uncertain amounts of sediment loadings; however; methods exist to improve understanding of sediment loadings, thus reducing their uncertainty. In addition, a variety of management actions have been developed to reduce the sediments loadings from a variety of sources. Choosing among actions to improve the understanding of the system and actions to reduce sediment is challenging. Chapter 4 presents a Bayesian decision analysis framework that chooses the optimal set of actions that improve understanding of the system and reduce sediment. This framework is applied to the Minnesota River Basin.

The Minnesota River frequently violates federal or state water quality standards for a variety of pollutants including nutrients and turbidity (Mallawatantri 1999). In 2001, the Minnesota Pollution Control Agency published the Minnesota River Basin Plan (2001) that described the state of the river basin and identified priorities for addressing water quality impairments. The plan described a basin management approach that addressed water pollutants by aiming to achieve environmental objectives using an integrated management approach to address both point and non-point sources of pollution while encouraging the involvement of stakeholders.

One focus of the Minnesota River Basin Plan is to reduce sediment loading from non-point sources to the Minnesota River, its tributaries, and Lake Pepin. Runoff from agricultural lands is likely to be a major cause of sediment and water quality impairments. Additionally, stream bank and bed erosion may contribute significantly.

Total maximum daily loads (TMDLs) are being developed to place an upper limit on the amount of sediment reaching the waterways. Once the TMDLs have been established, it is necessary to identify an action plan to meet the standards. Various actions have been suggested including agricultural best management practices (BMPs), stream restoration and riparian buffers. The Minnesota River Basin Plan describes an action strategy for addressing sedimentation and turbidity in the Minnesota River. The strategy is concerned with both identifying the sources of sediment and implementing actions to reduce the loading to waterways. A broad list of objectives is identified; however, there is relatively little discussion about what actions will be taken to achieve the objectives, and how effective the actions will be. It appears that the plan's recommendations consist of actions that are combined in a qualitative and judgmental manner, possibly resulting in an inefficient collection of actions that may or may not reach the desired sediment reduction goal. In addition, the plan does not quantify or even acknowledge uncertainty surrounding sediment loadings.

A potentially useful way to address the sedimentation problems in the Minnesota River is to use a systems approach. The uncertainty-based methodology developed in Chapter 4 combines Bayesian inference with multiobjective linear programming. A Bayesian framework is advantageous over a deterministic approach because it allows prior information to be combined with observed data to improve understanding of the physical system. Application of the methodology identifies optimal strategies (i.e., a mix of research and source control actions) that account for what can be learned from research and how that information might alter optimal strategies. The explicit consideration of research actions in environmental management can be viewed as a quantitative implementation of adaptive environmental management (Holling 1978; Walters 1986).

1.5 Scope

The remainder of this dissertation is organized as follows. Chapter 2 presents the first essay, Irreplaceability in Species Protection Models. The chapter begins with a detailed introduction and literature review of the problem. Next, the methodology section introduces the new integer programming model. A dataset of habitat for terrestrial vertebrates in Oregon is used to test the models. The chapter concludes with a discussion and concluding remarks.

Chapter 3 presents the second research study: Quantifying and Mitigating Value Tree-Induced Attribute Weighting Biases. This chapter follows the format of Chapter 2. First an introduction and literature review are presented. The methodology section describes the development of the model. The following section presents the results of using case study data to test the model. The final section concludes the paper.

The final study, A Bayesian Framework for Cost Effective Management of Sediment Reduction in the Minnesota River Basin, is presented in Chapter 4. This chapter also begins with an introduction and literature review. The next section develops the model. Results are presented following the methodology section, and a conclusion completes the chapter. Chapter 5 discusses overall conclusions. Plans for further research are also presented.

Chapter 2

Irreplaceability in Species Protection Models

This chapter introduces two new integer programming optimization models that efficiently determine the set of irreplaceable nature reserve sites out of all sites under consideration. Two existing reserve selection problems, the Species Set Covering Problem and the Maximal Covering Species Problem, are investigated with these new models to determine irreplaceable reserve sites.

2.1 Introduction

For over two decades, researchers have developed different methods to select land parcels as nature reserves to achieve adequate representation of species populations (Pressey 2002). For problems in which the spatial arrangement of parcels is not a concern, several fundamental problem statements have evolved and a number of exact and heuristic methods have been applied to these combinatorial optimization problems (ReVelle et al. 2002).

An important question has been raised about the conservation value of the parcels chosen relative to the objective(s) being optimized. The question, which is important from both a practical and research perspective, is whether or not there are sites (i.e., parcels of land) that are irreplaceable. The irreplaceability of a site has typically been defined in the context of the species set covering problem (SSCP) (Camm et al. 1996), which selects the minimum number of sites required to achieve representation targets for all species or other features in a data set. The SSCP was first defined for conservation planning and solved heuristically by Kirkpatrick (1983), and later identified as an integer programming optimization model (Cocks and Baird 1989; Possingham et al. 1993; Underhill 1994). In the context of the SSCP, the irreplaceability of a site is defined as the likelihood, varying from zero to 1.0, that the site is required as part of a set of sites that achieves all proposed targets (Pressey et al. 1994). Given that many alternative sets of sites can often be found for the SSCP, irreplaceability is measured as the proportion of these alternative sets in which each site occurs. Occurrence in all alternative sets indicates an irreplaceability of 1.0. Sites with an irreplaceability of 1.0 are defined as irreplaceable sites and possess considerable importance for the achievement of conservation targets. If these sites are not available for conservation, one or more targets will become unachievable or, in some cases, the number of sites needed to achieve targets must increase.

Similarly, the irreplaceability of reserve sites can be determined in the context of the maximal species covering problem (MCSP) (Camm et al. 1996), which seeks the maximum number of species covered given a predetermined number of reserve sites. This problem has also been approached heuristically (Vanewright et al. 1991), but was identified by Underhill (1994) as appropriate for integer programming optimization and later formulated in this way by Camm et al. (1996), Church et al. (1996), and others. As with the SSCP, a reserve site has an irreplaceability of 1.0 in the context of the MCSP if that site appears in all alternate optimal solutions to the MCSP. Sites with irreplaceability = 1.0 are important because, if they are rendered unavailable for conservation action, fewer species will be protected with the same number of sites, or a increased number of sites will be needed to protect the same number of species.

Irreplaceability has played a key role in conservation decisions. In 1995, irreplaceability became the basis for a decision support system (C-Plan) and was extensively used in negotiations over new forest reserves in New South Wales (Pressey 1998). Since its inception, C-Plan has been used throughout the world in conservation planning (Cowling et al. 2003) and presently has about 400 users. Another widely used system, MARXAN, applies a different predictor of irreplaceability and is used by hundreds of decision makers worldwide (Stewart and Possingham 2005).

Irreplaceability can be used for resolving choices between alternative sites or alternative sets of sites. Cowling et al. (2003) for example, utilized irreplaceability to avoid threatened sites and achieve compact groups of sites, where possible, while still achieving all targets. Threat and irreplaceability have been combined to recommend scheduling of protection to minimize the extent to which targets are compromised by ongoing attrition of biodiversity (Primack 2002). Cost has been used extensively to estimate irreplaceability in MARXAN to find alternative sets of sites that achieve targets and minimize costs defined as area, money, or other surrogates.

While irreplaceability has played a key role in conservation decision making, finding irreplaceability values for sites can be deceptively difficult. It might be surmised, for example, that with a target of a single occurrence of each species in a region, the irreplaceable sites could be found simply by identifying those with unique species. However, due to the demands of complementarity, some sites identified as irreplaceable do not have unique species and have lower irreplaceability values when the set of sites is increased above the optimal size (Pressey et al. 1994). Identification of these sites requires complete exploration of optimal and suboptimal solutions. Moreover, for area-based targets, characteristics other than unique features can define irreplaceable sites (Ferrier et al. 2000): occurrences of features that are scattered but have very large targets relative to their total size; and unusually large occurrences of non-unique features that would compromise target achievement if not protected.

To address the challenge of identifying irreplaceable sites, this chapter develops two integer programming models to efficiently extract the set of irreplaceable reserve sites in the context of the SSCP and the Maximal Covering Species Problem (MCSP). The remainder of this chapter is organized as follows. Section 2.2 provides a literature review. Section 2.3 presents the species set covering problem and maximal covering species problem formulations, and extends them to create two new integer programs used to determine the number of irreplaceable sites. Section 2.4 describes the data used to test the new formulations. The results are presented in section 2.5, and section 2.6 presents a discussion and conclusions.

2.2 Literature Review

Several approaches have been taken to measure or estimate irreplaceability for the SSCP. Irreplaceability can be measured exactly for very small data sets by exhaustive analysis of all possible site combinations (Pressey et al. 1994) or enumeration of all alternative representative combinations by stepwise analysis (Tsuji and Tsubaki 2004). For these and somewhat larger data sets, exact values can also be obtained by operations research methods that find all alternative optimal solutions (Csuti et al. 1997). For still larger and/or more complex representation problems, estimates of irreplaceability values have been derived from stepwise heuristic approximations (Pressey et al. 1994), statistical techniques (Ferrier et al. 2000), and sampling from the range of possible representative site combinations, either by modified heuristic selection methods (Tsuji and Tsubaki

2004; Rebelo and Siegfried 1992), randomly generated sets, (Ferrier et al. 2000) or simulated annealing (Leslie et al. 2003).

Research on conservation planning has also focused on the maximal covering species problem (Camm et al. 1996), which maximizes the number of species or other features that can be represented in a set of sites smaller than that required for the set covering problem. Multiple optimal solutions are known for the maximal covering species problem (Csuti et al. 1997; Arthur et al. 1997), although only one study has used these to measure irreplaceability. Kiester et al. (1996) used exhaustive combinatorial analysis for small sets of sites to identify two that were irreplaceable for solutions to the maximal covering species problem.

Other work in this area has focused on finding all alternate optimal solutions to both the SSCP and the MCSP, and then determining irreplaceability values from the resulting solutions. Csuti et al. (1997) identified many (and in some cases, all) alternate optima for both the SSCP and MCSP. The 144 optimal solutions found for the SSCP were examined to identify the sites that appeared in each solution. Instead of manually examining all optimal solutions for common sites, an efficient integer program that determines the set of irreplaceable sites is developed in this chapter. The methods presented here for the SSCP are faster and more tractable than an exhaustive enumeration of site combinations, especially for moderate to large data sets.

In the context of the MCSP, two additional studies have found many (and in some cases, all) alternate optima. Arthur et al. (1997) used an approach similar to the one developed in this chapter, called Explicit Exclusion, in which a constraint is added to the MCSP to prevent previous optimal solutions from being found again. As soon as all the alternate optima were found, the solutions were inspected and the sites common to all solutions were identified as the irreplaceable sites. Kiester et al. (1996) also found irreplaceable sites by comparing all optimal solutions to the MCSP. The methods presented in this chapter for the MCSP are a clear improvement on the techniques developed previously, since all irreplaceable sites are found using an efficient and optimal technique that does not rely on complete enumeration of solutions.

2.3 Methodology

In this section, integer programming formulations for the SSCP and MCSP are presented. These two formulations are then modified to determine the irreplaceable sites among a set of reserve sites. For both the SSCP and the MCSP, many alternative optimal solutions have been observed and are likely to exist in new species/population data sets. The sites that appear in every one of the multiple optimal solutions for a given set of data and parameters are recognized as irreplaceable. If only one solution exists (no alternative optima present), all sites selected in that solution are irreplaceable. If, on the other hand, several solutions exist, only the sites that appear in every one of the optimal solutions are irreplaceable – that is, they cannot be excluded without degrading the objective. For the SSCP, degrading the objective means that the representation target (e.g., at least one occurrence of each species) can no longer be achieved for one or more species, or that a larger number of sites is needed to meet the target. For the MCSP, degrading the objective means a reduction in the number of species that can be represented in a given number of sites.

2.3.1 Irreplaceability and the Species Set Covering Problem

A modification of the SSCP is developed for finding the set of irreplaceable sites that must be included in a reserve system. The irreplaceable sites are found by determining the set of sites that appear in all alternate optima to the SSCP. Two formulations are used throughout this section – the SSCP and a modified version of the SSCP called the Irreplaceable Sites Species Set Covering Problem (IS-SSCP). The two formulations are described below.

Notation

$m :=$	total number of species
$n :=$	total number of candidate reserve sites
$I :=$	$\{i \mid i = 1,...,m\}$ index set of the population (species) to be covered
$J :=$	$\{j \mid j = 1,...,n\}$ index set of candidate reserve sites
$N_i :=$	subset of J, set of candidate reserve sites that contain species i
$S_k :=$	subset of J, solution set for the SSCP when $k = 1$ and solution set for the kth instance of the IS-SSCP for $k = 2, 3, 4,...$
$x_j :=$	$\{1$, if site j is chosen for preservation; 0, otherwise$\}$
$p :=$	$\lvert S_1 \rvert$ number of sites required to cover all species. Found as the solution to the SSCP

Species Set Covering Problem

$$\text{(SSCP)} \quad \text{Min} \quad z = \sum_{j \in J} x_j \tag{2.1}$$

$$\text{subject to} \qquad \sum_{j \in N_i} x_j \geq 1 \quad \forall\ i \in I \tag{2.2}$$

$$x_j \in \{0,1\} \quad \forall\ j \in J \tag{2.3}$$

This problem determines the minimum number of parcels (2.1) subject to the constraint (2.2) that every species must be in at least one of the sites selected (written for each of the m species) and (2.3) that the variables must be binary (for n variables).

kth Irreplaceable Sites Species Set Covering Problem

$$\text{(IS-SSCP)} \quad \text{Min } z = \sum_{j \in S_k} x_j \tag{2.4}$$

$$\text{subject to} \quad \sum_{j \in N_i} x_j \geq 1 \quad \forall\, i \in I \tag{2.5}$$

$$\sum_{j \in J} x_j = p \tag{2.6}$$

$$x_j \in \{0,1\} \quad \forall\, j \in J \tag{2.7}$$

The objective of this problem is to minimize the sum of the sites that appeared in the optimal objective function of the SSCP (when $k = 1$) or the previous IS-SSCP problem. The sites that appeared in the optimal objective of the SSCP (or previous IS-SSCP) comprise the set of sites denoted by S_1 (S_k). By minimizing this sum, the model forces those sites out of the objective function if they can be excluded in a feasible solution (all species covered) to the species set covering problem. Therefore, the sites that remain in the solution set comprise the objective function to the subsequent minimization problem. The first constraint (2.5) is the same as in the SSCP – each species must be covered in at least one selected site. The second constraint (2.6) requires that the same number of sites (p) must be selected as were selected in the SSCP. Again, the decision variables must be binary (2.7).

The set of irreplaceable sites for the SSCP is found using a successive minimization approach. Figure 2-1 presents a flow chart for the method. First, an optimal solution to the SSCP is found (box 1). This result identifies the smallest set of sites that must be

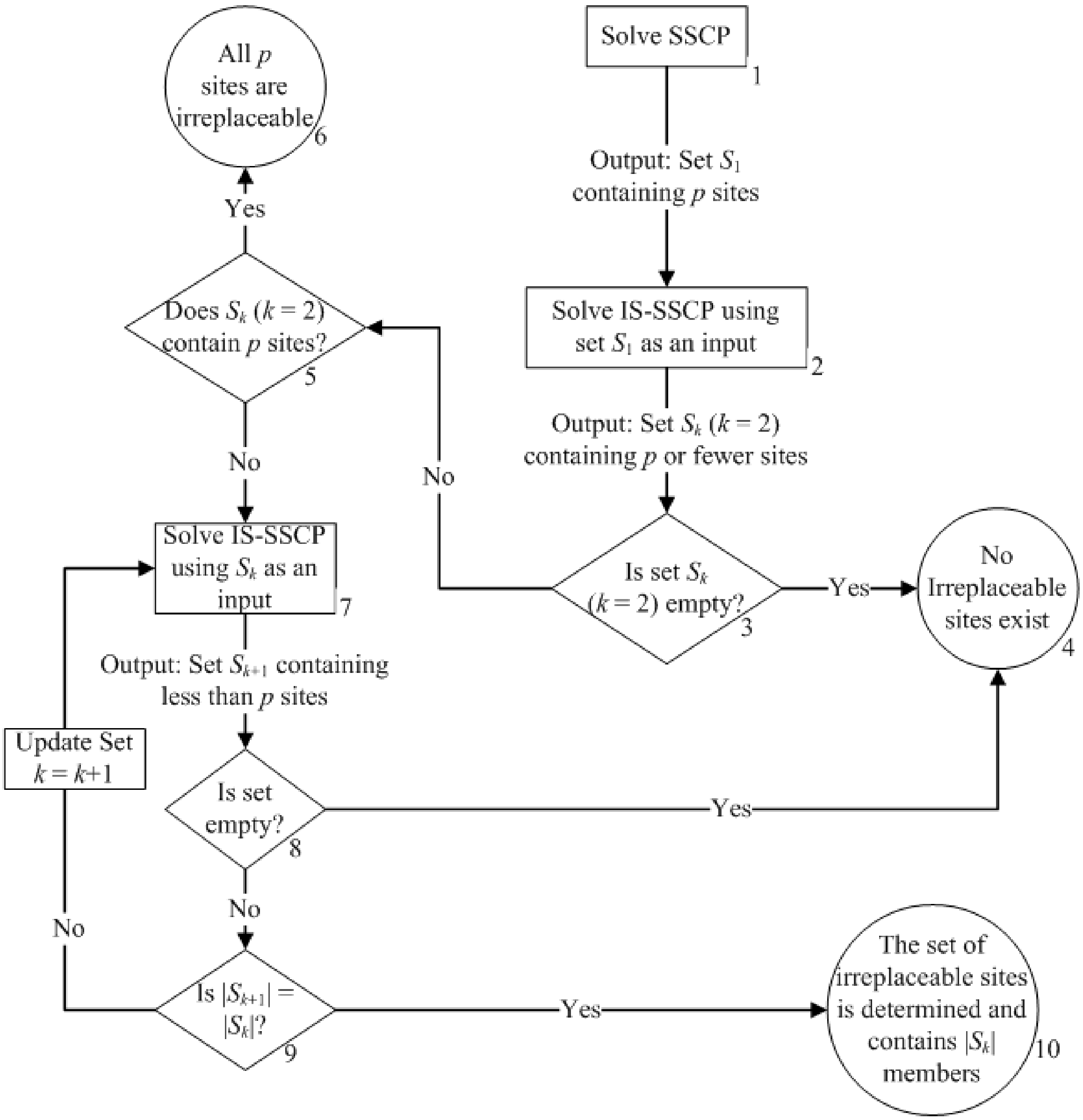

Figure 2-1: Flowchart describing the process for determining the set of irreplaceable sites when requiring full species representation

selected to represent every species at least once. These p sites form the set S_1, which might be one of many alternative optima, and which is then used as an input in the next step of the solution method (box 2). Given this set S_1, the first IS-SSCP problem is solved, which minimizes the cardinality of S_1, or equivalently finds the fewest members of S_1 absolutely needed from among the original p sites. The optimal objective of the IS-

SSCP determines how many sites from the original set S_1 cannot be replaced if it is required that only p sites are chosen that still cover all species. The IS-SSCP simply finds an alternate optimal solution to the SSCP that contains the fewest sites in common with the original solution found.

If the objective value is zero (diamond 3), then some other set of p sites, with no members in common with the original set S_1, can accomplish full species representation, and therefore, no sites are irreplaceable (circle 4). If some irreplaceable sites are still needed in all of the alternative optimal solutions, the objective value of the IS-SSCP can not be driven to zero.

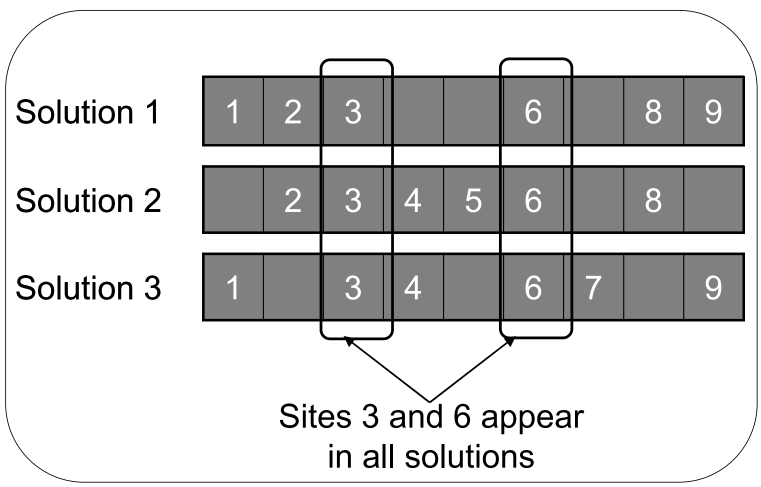

Figure 2-2: Example of Irreplaceable Sites

On the other hand, if the IS-SSCP has a nonzero optimal objective value, some of the original sites selected in the SSCP are required in order to preserve all species. However, a positive objective value of the IS-SSCP does not necessarily indicate which, or how many, sites are irreplaceable.

Consider the example illustrated in Figure 2-2. Suppose three alternate optima to the SSCP exist. Solution one consists of the sites 1, 2, 3, 6, 8, and 9. Solution two contains the sites 2, 3, 4, 5, 6, and 8. Solution three consists of the sites 1, 3, 4, 6, 7, and 9. If solution one is found, this set of six sites becomes the input to the IS-SSCP. Suppose the IS-SSCP is solved resulting in an objective of four sites: 1, 3, 6, and 9. Recall that there is a constraint (2.6) in the IS-SSCP that says that p sites must be selected. In order to cover all species with only p sites, one of the alternate optima must satisfy the constraint in the IS-SSCP. If those p sites correspond to solution three above, the sites in common (1, 3, 6, 9) are the four sites that end up in the objective of the IS-SSCP. Similarly, the IS-SSCP could have satisfied the constraint using solution two. This solution also shares four sites with the solution to the SSCP, but the four sites are different (2, 3, 6 and 8). Therefore, if solution one of the SSCP is used as input to the IS-SSCP, the IS-SSCP will have an optimal objective of four but those four sites are not fixed. Also notice that if solution two had been found as an optimal solution to the SSCP and was then used as input to the IS-SSCP, the objective to the IS-SSCP would have been three since the minimum number of sites that solution two shares with any alternate optima is three (solutions two and three share sites 3, 4, and 6). Thus, the solution to the IS-SSCP does not indicate which or how many sites are irreplaceable and depends on the solution to the SSCP; however, the process described in Figure 2-1 does identify the set or irreplaceable sites. The process is further described below.

If a nonzero objective value exists for the IS-SSCP, the following step(s) must be taken. The sites remaining in the objective function of the IS-SSCP are included in a new set called S_2. If S_2 contains p sites, all p sites are irreplaceable (diamond 5 and circle 6).

However, if S_2 contains fewer than p sites, the IS-SSCP ($k = 2$) is solved again (box 7). This time, the sum of the sites in S_2 is minimized. The constraint (2.6) requiring p sites to be selected is still included. In this step, the IS-SSCP finds an alternate optimum to the SSCP that contains the fewest sites that are members of S_2. Note that in this step, members of S_1 that do not appear in S_2 might return to the solution set. If a nonzero solution exists to the IS-SSCP, members in the objective function form the set S_3. The cardinality of S_3 represents the minimum number of sites that appear in S_2 and all of the alternate optima. If one of the alternate optima contained fewer sites in common with S_2, that solution would have been found.

Again, consider the example above. Suppose set S_2 contains sites 2, 3, 6, and 8. Solving the IS-SSCP will result in set S_3 containing sites 3 and 6, because solution three only contains sites 3 and 6 in common with the sites in S_2. It does not contain sites 2 and 8.

If the objective of the IS-SSCP is reduced to zero, none of the sites are irreplaceable. If S_3 is not empty, the size of S_3 is compared against the size of S_2 (diamond 9). If S_3 consists of fewer sites than S_2, the IS-SSCP ($k = 3$) is again solved by minimizing the sum of the members of S_3 (box 7). This successive minimization process is continued until no member can be driven out of the set over which the IS-SSCP is minimizing (diamond 9). This final set represents the sites that must appear in all alternate optima. These sites are identified as irreplaceable. It is important to note that the number of times the IS-SSCP must be run is a direct result of the solution found to the SSCP. Since many solutions may exist to the SSCP, the number of intermediate IS-SSCP steps needed depends on which solution is found to the SSCP. The final result will be the same, nonetheless, as the set of irreplaceable sites is not dependent on the starting solution.

A second approach can be used to determine the irreplaceable sites and is very similar to the method outlined above. This method adds successive "cuts" instead of using successive minimizations. Given an optimal solution to the SSCP of p sites, the SSCP is solved again with an additional constraint that limits the set of sites that can be selected. The constraint takes the form $\sum_{j \in S_1} x_j \leq p-1$, where S_1 is the set of sites that comprise a solution to the SSCP. This constraint says that, at most, only p - 1 of the p sites in the original optimal solution of the SSCP can be selected. The minimum number of sites required to cover all species that also satisfies the added constraint is then determined. If this new objective value is the same as the objective of the original SSCP, a third optimization program is run. This third program is identical to the second; however, the cut that is added allows only p - 2 of the original p sites to be selected (i.e., the right-hand side of the constraint changes to p - 2). Again, if the new objective is the same as the original, the process is repeated with another cut ($\leq p$ - 3). The cut becomes increasingly restrictive until the objective function degrades, meaning that the number of sites required to cover all species increases above p.

When the objective degrades, too many of the original p sites have been restricted from being selected. Suppose this occurs when the constraint allows at most b - 1 of the original p sites to be selected. In order to preserve all species, at least b of the original p sites must be selected. This set of b sites corresponds to S_2 noted above. Note that this set of b sites might not be unique (as in the successive minimization technique).

The next step is to determine what number of sites in the set S_2 appears in all solutions. This is done by again solving the SSCP with a cut. However, the cut here restricts the number of sites that are in the set S_2. The cut states that the sum of these sites must be less than b - 1. The constraint takes the form $\sum_{j \in S_2} x_j \leq b-1$. If the objective does not degrade at this point, not all of the sites in S_2 are essential and the cut can become more restrictive (b - 2). Again, the cut is made more and more restrictive until the cut leads to a degraded objective function. Suppose this degradation occurs when the constraint allows c - 1 sites from S_2 to be selected. That is, the constraint becomes more and more restrictive until it takes the form $\sum_{j \in S_3} x_j \leq c-1$ at which point the objective function degrades. Thus, c of the original b sites in S_2 are still necessary to cover all species. This set of c sites composes the set S_3. Again, a new cut is added which states says that the sum of the sites in S_3 must be less than c - 1 and so on. The process is repeated until the objective function degrades when only 1 of the sites is restricted from selection. At this point, all of the sites common between successive steps must be included in the set of sites selected in order to prevent the objective function from degrading. These are the irreplaceable sites.

2.3.2 Irreplaceability and the Maximal Covering Species Problem

Monetary (or other) restrictions can limit the number of selected sites, preventing full species representation. In this case, the MCSP can be solved to determine the maximum number of species that can be covered with a limited number of reserve sites.

To determine the set of irreplaceable sites in the context of the MCSP, a similar process to the one described above can be performed. First, the MCSP is solved to determine the maximum number of species, $A(q)$, that can be covered with q sites (the starting point for the value of q can be determined from the SSCP). The solution to the MCSP is used to solve successive occurrences of the Irreplaceable Sites Maximal Covering Species Problem (IS-MCSP). The process eventually identifies the sites that are irreplaceable when representing the maximum number of species with a predetermined number of sites. The formulations for the MCSP and IS-MCSP are presented below.

Additional notation

Z_k := subset of J, solution set for the MCSP when k = 1, and solution set for kth instance of IS-MCSP for k = 2, 3, 4,...

u_i := {1, if species i is represented in the reserve system; 0, otherwise}

q := user-prescribed number of sites that can be selected

$A(q)$:= optimal solution (number of species) for a given value of q

Maximal Covering Species Problem

$$\text{(MCSP)} \quad \text{Max } z = \sum_{i \in I} u_i \tag{2.8}$$

subject to

$$\sum_{j \in N_i} x_j \geq u_i \quad \forall\, i \in I \tag{2.9}$$

$$\sum_{j \in J} x_j = q \tag{2.10}$$

$$u_i = (0,1) \quad \forall\, i \in I \tag{2.11}$$

$$x_j = (0,1) \quad \forall j \in J \tag{2.12}$$

The objective (2.8) determines the maximum number of species that can be preserved with a given number of sites in the reserve system (q). The value of q is initially set equal to p-1, where p is the number of sites required for full species representation. Subsequently, q is decreased to determine the effect on the number of species protected when allowing fewer sites to be selected. The first constraint (2.9) allows species i to be included in the set of represented species if and only if at least one site containing species i is selected. The second constraint (2.10) limits the possible number of sites chosen to be

q. The last two constraints (2.11) and (2.12) require both decision variables to be binary. The set of sites that is selected in the optimal solution is defined as Z_1.

kth Irreplaceable Sites Maximal Covering Species Problem

$$\text{(IS-MCSP)} \quad \text{Min } z = \sum_{j \in Zk} x_j \tag{2.13}$$

$$\text{subject to} \quad \sum_{j \in N_i} x_j \geq u_i \quad \forall\, i \in I \tag{2.14}$$

$$\sum_{j \in J} x_j = q \tag{2.15}$$

$$u_i = (0,1) \quad \forall\ i \in I \tag{2.16}$$

$$x_j = (0,1) \quad \forall\ j \in J \tag{2.17}$$

By minimizing the sum of the sites (2.13) identified in the MCSP (or previous IS-MCSP), the IS-MCSP tries to force those sites out of the objective function. The process is continued until no more sites can be excluded. The sites remaining in the solution set are considered essential to represent the maximum number of species with only q parcels. Species i can only be counted as covered if a site containing species i is selected (2.14). The maximum number of sites selected in the MCSP (or previous IS-MCSP) must still be protected in this problem (2.15). The decision variables must still be binary (2.16) and (2.17).

Figure 2-3 presents a flowchart describing the process for determining irreplaceable sites in the context of the MCSP. The process is repeated for decreasing integer values of q beginning from the number of sites needed to cover all species down to a single site.

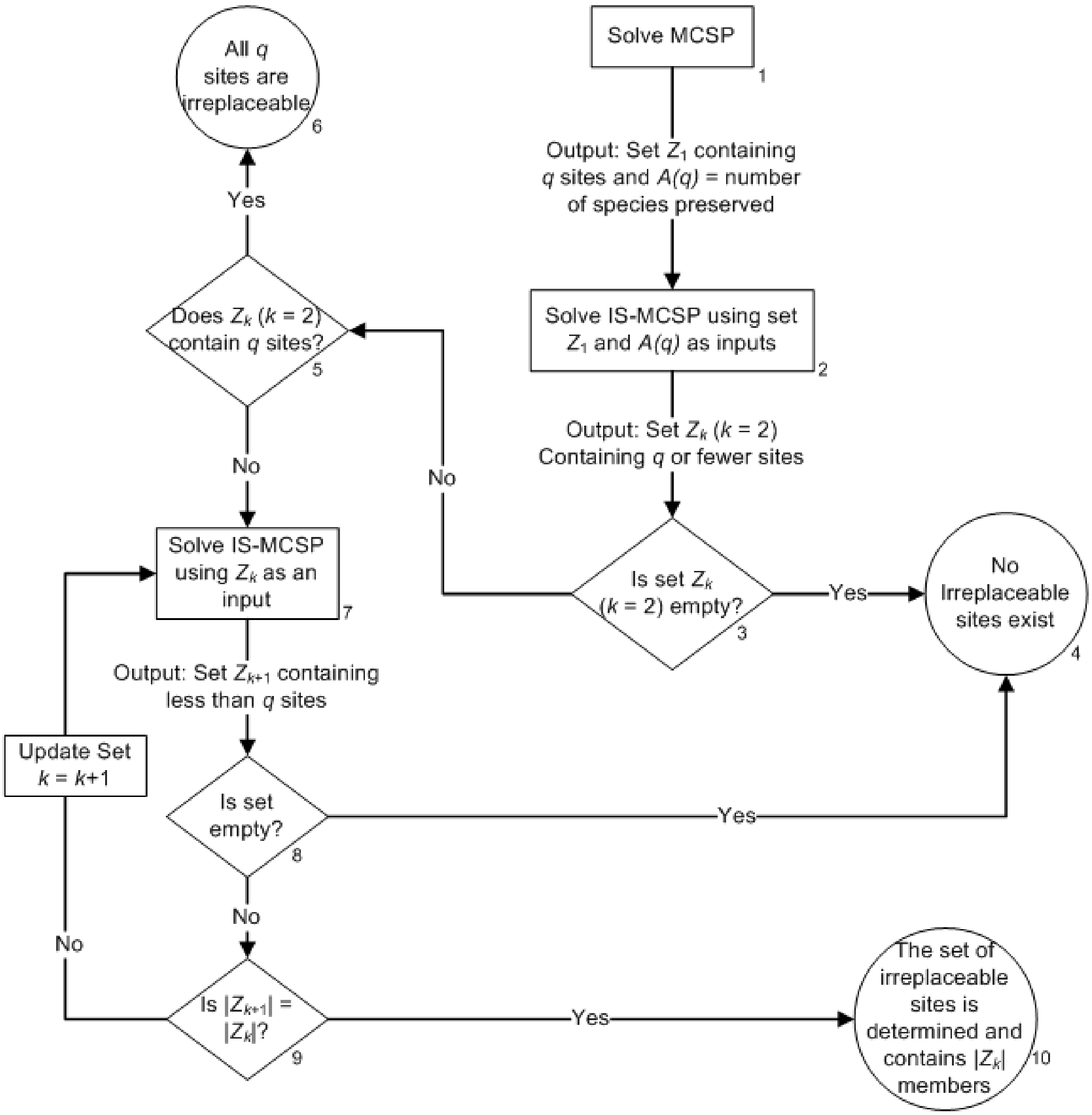

Figure 2-3: Flowchart describing the process for determining the set of irreplaceable sites when selecting the maximum number of species given a limited number of sites available for selection

2.3.3 Extension of the Species Set Covering Problem

If more resources are available, additional sites beyond the minimum number needed to protect all species could be selected. Under this scenario, the set of irreplaceable sites might change. To investigate the effects of increasing the number of sites beyond p, the following steps are taken. First, the SSCP is run and the resulting set of sites selected, S_1, is used in the IS-SSCP. However, instead of using the value of p determined by the SSCP, the right-hand side of equation (2.6) is set to $p + 1$. The number of irreplaceable sites is found by running the IS-SSCP as many times as needed, following the process described above. The problem is repeated for $p + 2$ and $p + 3$.

2.3.4 Value of irreplaceability

Two different measures of the value of an individual site are explored: species value, and economic or efficiency value. First, the effect on the number of species covered is addressed for the case when an irreplaceable site is removed from the list of candidate sites. Next, the impact on the number of sites necessary to cover a given number of species is explored for the case in which a particular irreplaceable site is not permitted to be selected.

To explore the impact of the exclusion of an irreplaceable site on the SSCP, the following test is performed. For each irreplaceable site found for the SSCP, the SSCP is run again with an additional constraint that excludes the irreplaceable site. The resulting objective indicates how many sites must be selected in order to preserve all species, also indicating the number of sites required to make up for each irreplaceable site excluded.

Two tests are conducted on the MCSP. First, for each irreplaceable site associated with a given value of q, the MCSP is run with one additional constraint that prevents the site from being selected. The resulting objective function value determines how many fewer species are covered as a result of excluding the irreplaceable site.

The second method used for assigning a value to the irreplaceable sites involves determining the number of sites needed to replace the irreplaceable site in the context of the MCSP. To determine this value, the following integer program is run.

$$\text{Min} \qquad z = \sum_{j \in J} x_j \tag{2.18}$$

$$\text{subject to} \quad \sum_{j \in N_i} x_j \geq u_i \quad \forall\, i \in I \tag{2.19}$$

$$\sum_{i \in I} u_i = A(q) \tag{2.20}$$

$$x_j^* = 0 \quad \text{where } j^* \text{ is the irreplaceable site} \tag{2.21}$$

$$u_i = (0,1) \quad \forall\; i \in I \tag{2.22}$$

$$x_j = (0,1) \quad \forall\; j \in J \tag{2.23}$$

The objective of this optimization model is to determine the minimum number of sites required to cover a certain number of species, $A(q)$, if one of the irreplaceable sites, j^*, cannot be selected. Constraints (2.19), (2.20), (2.22), and (2.23) have been described previously. Constraint (2.21) requires that irreplaceable site j^* not be selected.

2.4 Data used for Study

The data used for the application of the IS-SSCP and IS-MCSP are distribution maps for 426 species of terrestrial vertebrates in the State of Oregon, compiled by the Biodiversity Research Consortium at Oregon State University. The grid-based maps record the occurrence of each species in each of 441 hexagonal cells (sites), 635km2 in area, wholly or partly within the political borders of Oregon. Species are defined as occurring in the site if: (1) they have been verified by a sighting in that site within the past two decades; or (2) they fulfill a three-part condition, namely, (a) they have been verified by a sighting in nearby sites and (b) there exists suitable habitat in the site for the species to exist, and (c) in the opinion of a local expert, the species might occur in the site (Master et al. 1995).

2.5 Results

The formulations described in section 2.3 were coded using Xpress-MP® and run using the Oregon data. The code was run on a Hewlett-Packard® notebook computer running Windows® XP Professional with a 2.00- GHz Intel® Pentium® 4 processor. Branching and bounding took between 1 and 4.8s for the MCSP with an average of 2.2s (Table 2-1). For the IS-MCSP, branching and bounding took between 1 and 9.7s, with an average of 4s. No branching and bounding was needed for the SSCP or the IS-SSCP.

Table 2-1: Processing Times

Problem Formulation	Processing Time (s)
SSCP	0.4
IS-SSCP	0.3
MCSP	0.5 – 5.3
IS-SSCP	0.2 – 10.3

2.5.1 SSCP and IS-SSCP Results

The number of sites required for the SSCP on the Oregon data is 23. This was expected based on the results of Csuti et al. (1997). The solution of the successive IS-SSCPs shows that of these 23 sites, 19 are irreplaceable. The last column of Figure 2-4 indicates which sites are irreplaceable.

2.5.2 MCSP and IS-MCSP Results

Results of the Maximal Covering Species Problem are summarized in Figure 2-4 and Figure 2-5. The solution for the MCSP when q = 23 is the same as that of the SSCP, as expected, although the specific sites could have been different due to the existence of alternate optima. As q is decreased for the MCSP, the number of irreplaceable sites initially decreases slightly, then decreases sharply at q = 18, and then fluctuates for smaller values of q. Certain sites seem to be irreplaceable for many of the solutions (Figure 2-4). Sites 9, 428, and 440 appear in over half of the solutions. Several sites appear in the solutions only for large values of q, and then drop out.

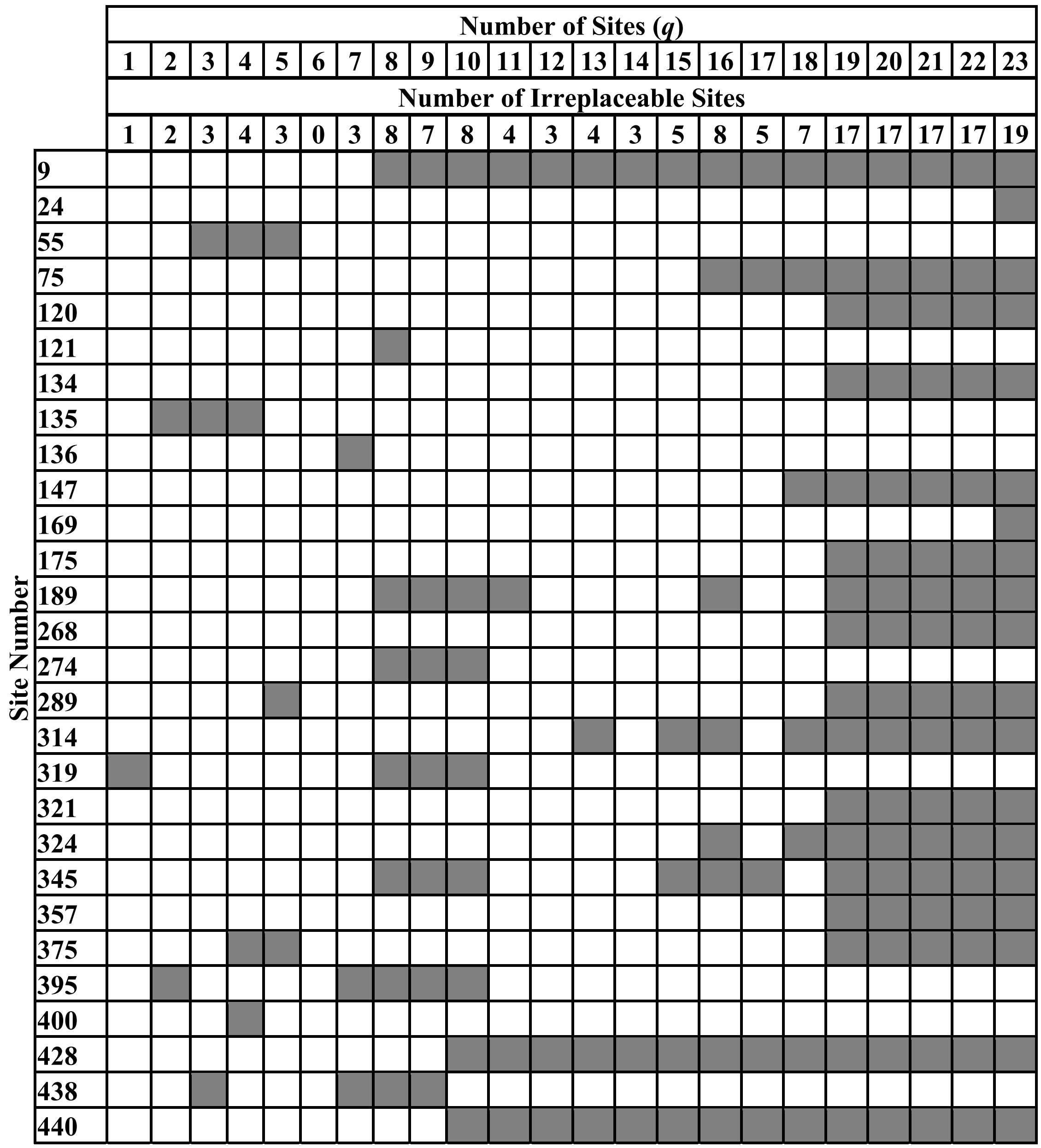

Figure 2-4: Irreplaceable sites in solutions to the IS-MCSP for values of *q* between 1 and 22, and irreplaceable sites in solution to the IS-SSCP ($q = p = 23$)

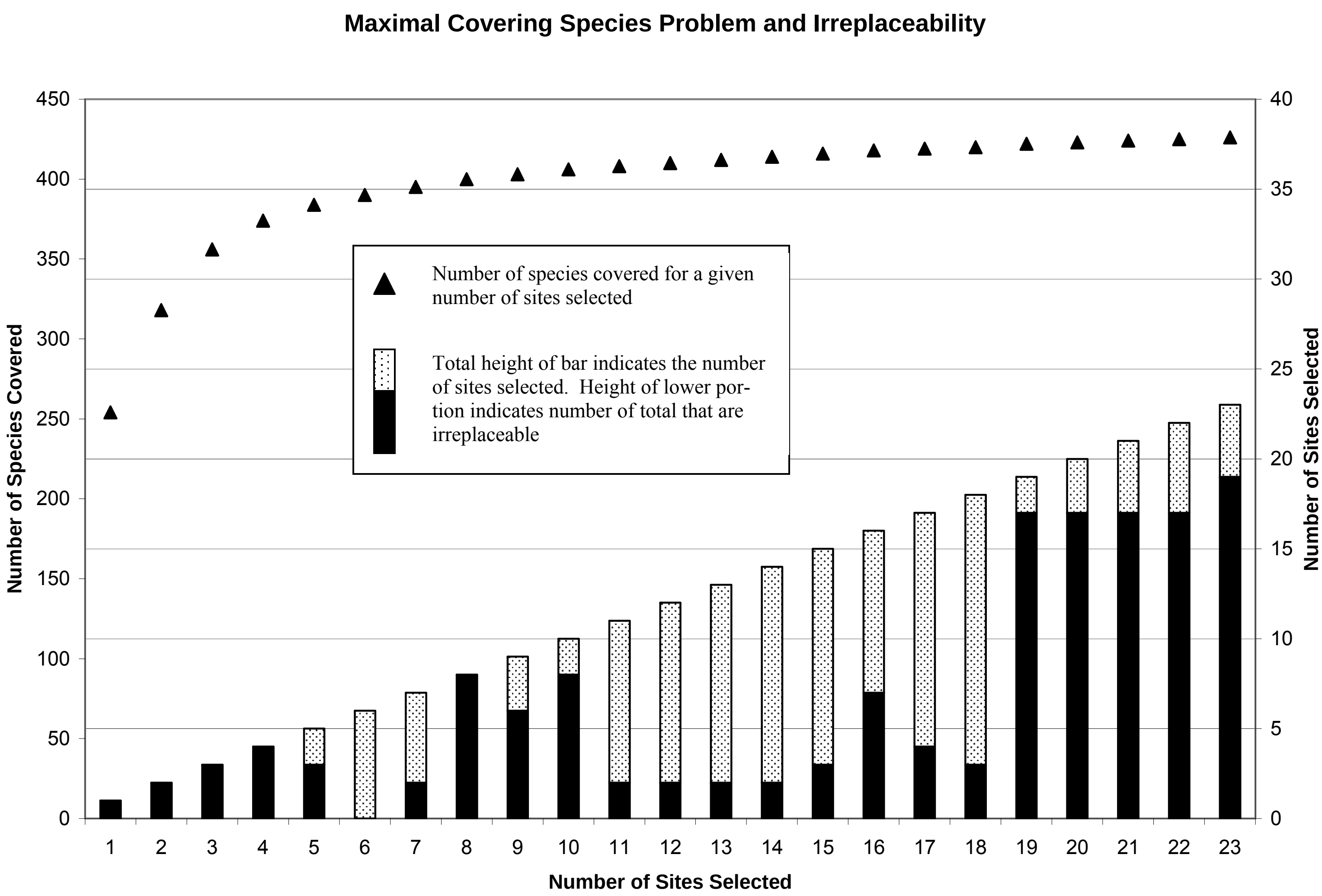

Figure 2-5: MCSP and Irreplaceable Sites

Sites such as sites 189 and 319, appear, drop out, and reappear as q varies. Other sites appear only for smaller values of q, such as site 55 and site 135. A few sites, such as 24 and 121, appear only once. When q is 4 or less, all sites are irreplaceable.

2.5.3 Extensions of the SSCP Results

For each value of p larger than 23, there are seven irreplaceable sites (Table 2-2). Investigation of the data set reveals that each of these seven irreplaceable sites contains one of seven single species occurrences.

Table 2-2: Irreplaceable Sites for Increasing *p*

p	**23**	**24**	**25**	**26**
Irreplaceable	**19**	**7**	**7**	**7**
Site 9:				
Site 24:				
Site 75:				
Site 120:				
Site 134:				
Site 147:				
Site 169:				
Site 175:				
Site 189:				
Site 268:				
Site 289:				
Site 314:				
Site 321:				
Site 324:				
Site 345:				
Site 357:				
Site 375:				
Site 428:				
Site 440:				

2.5.4 Value of Irreplaceable Sites

When an irreplaceable site is excluded in the SSCP, either the problem becomes infeasible or the number of sites required to cover all species increases by 1, from 23 to 24. When sites 9, 24, 147, 169, 314, 324, or 375 are excluded, no combination of sites can be found that cover all species. Therefore, each of these sites must contain a species that occurs only in that site, which corresponds with the results in section 0.

In regards to the MCSP, for all values of q, the number of sites required to cover $A(q)$ species when any irreplaceable site is removed is $q + 1$. For all values of q where not all sites were irreplaceable, the objective function decreases by one species when an irreplaceable site is excluded. In the case where all q sites are irreplaceable, the objective function degrades by at most two species. Table 2-3 displays the results for q equal to eight sites. For most of the irreplaceable sites, the objective function only decreases by one species. For sites 319 and 395, however, the objective decreases by two. This indicates that these two sites perhaps have a larger worth than the other irreplaceable sites.

Table 2-3: Value of Irreplaceable Sites for *q* = 8 and Objective = 400 species

Removed Site	New Objective	Change in Number of Species Covered	Replacement Sites Required to achieve Objective = 400
9	399	-1	2
121	399	-1	2
189	399	-1	2
274	399	-1	2
319	398	-2	2
345	399	-1	2
395	398	-2	2
438	399	-1	2

Although some instances were observed in which removing an irreplaceable site leads to two additional species not being covered, it is always true that in order to cover all $A(q)$ species, the original set of sites needs only to be enlarged by one. For example, in Table 2-3 when site 319 is excluded, the number of species covered drops by two from 400 to 398. However, the new number of sites required to cover 400 species is 9. This is true for all the other irreplaceable sites as well.

Overall, every irreplaceable site has the same economic or efficiency value and nearly every irreplaceable site has the same species value. The only cases in which one irreplaceable site appears more valuable in terms of species value are when all q sites are irreplaceable. When q is eight, four, three, two, or one the exclusion of at least one irreplaceable site leads to a loss of two species.

2.6 Discussion and Conclusions

The results described above indicate that a large number of sites (19 of 23) are irreplaceable if full species representation is required with the fewest number of sites possible. However, if an additional site beyond that which is needed to cover all species is included, the number of irreplaceable sites drops dramatically from 19 to 7. If just a single site is added beyond the minimal number required to cover all species, the only sites that remain irreplaceable are the sites that contain unique species.

It can be expected for many data sets that, as p is increased above the size of the optimal solution for the SSCP, the number of irreplaceable sites will decrease. When p is equal to the size of the optimal solution, the need for complementarity between sites in the species they contain is extreme. Some sites without unique species, due to their composition and ability to complement other sites, will be essential members of the set. As p is increased, it is possible for additional sites to contribute species to progressively larger sets, so the demands of complementarity are relaxed. As this happens, sites that were initially irreplaceable but which lack unique species can be expected to become replaceable. At some stage, only sites with unique species remain in the irreplaceable subset. In the dataset used in this research, this change happened abruptly – as soon as p increased from 23 to 24. In other data sets and for other representation objectives, this change might be more gradual. While the reduction in irreplaceable sites with p larger than 23 was predictable, the identities of the non-unique irreplaceable sites was not. This is essentially a combinatorial problem that was solved here with a new method.

Table 2-4: Species Uncovered in MCSP and IS-MCSP

Species	Sites (Irreplaceable)	Formulation	Uncovered Species Number												
			3	11	28	101	109	114	139	143	168	188	359	374	392
423	20 (17)	MCSP				■							■	■	
		IS-MCSP1				■								■	■
422	19 (17)	MCSP				■							■	■	■
		IS-MCSP2				■							■	■	■
420	18 (3)	MCSP				■	■	■					■	■	
		IS-MCSP1	■	■		■							■	■	
		IS-MCSP2				■				■		■	■	■	■
		IS-MCSP3				■					■	■	■	■	■
		IS-MCSP4			■	■			■				■	■	■

Results of the MCSP and the IS-MCSP offer several interesting insights. While the same number of species is preserved in the MCSP and successive IS-MCSPs, the actual species that are preserved may not be the same. Table 2-4 shows an example of this phenomenon for q ranging from 18 to 20. Since the number of species covered is very large, the table represents the species not covered. As the number of selected sites decreases, the number of species covered decreases, as expected. When q is 20, the same species fail to be covered as when q is 19. In other words, the solution for $q = 19$ is contained or "nested" in the solution for $q = 20$. This does not always occur. In some instances, only some of the previously covered species remain uncovered. Another interesting point can be illustrated when $q = 19$. Although not all of the sites are irreplaceable, the MCSP and the IS-MCSP cover the exact same species. This means that in addition to there being two alternate solutions that cover 422 species; there are two alternate solutions that cover the exact same species by selecting different sites.

Additionally, there is no clear pattern between the number of irreplaceable sites and the total number of sites selected. For large values of q, many of the selected sites are irreplaceable. When q is above 19, at least 17 of the sites are irreplaceable. But, when the number of sites selected decreases by one to 18, only seven sites are irreplaceable. For values of q between 5 and 18, the number of irreplaceable sites fluctuates dramatically. For example, when $q = 8$, all eight sites are irreplaceable, meaning no alternate solutions exists. When $q = 6$, none of the six sites are irreplaceable, meaning that at least two solutions exist that do not share any sites in common but cover the exact same number of species. For values of q less than 5, all sites are irreplaceable. That is, there is only one combination of sites that can cover the maximum number of species.

One further observation can be offered and it is, perhaps, a perplexing one. Irreplaceability for the MCSP appears to be case-specific as opposed to site specific. A particular site may be irreplaceable in one setting, say $q = 10$, and replaceable in another, say $q = 14$. It is not the site's characteristics that are the sole determinant of irreplaceability, but a combination of the site's species composition and the number of sites that can be selected.

This observation is not as appealing as hoped in making the case for a site's importance to the scheme of preservation. A question that arises is whether a site that is irreplaceable at a higher resource level is also irreplaceable at a lower resource level. The results in this chapter show that there are many cases where this is not so.

However, the implications of identifying a set of irreplaceable sites in a reserve design can still be extremely important to decision makers. Understanding which sites are required to protect a maximum number of species at a given resource level allows the decision maker flexibility in the reserve design. Decision makers can begin by protecting the set of irreplaceable sites and go on to determine which combination of additional sites best complements the irreplaceable sites, possibly by considering a variety of criteria for these additional sites. The knowledge gained from identifying which sites are irreplaceable allows for improved decision making and environmental management aimed at protecting vulnerable species.

Chapter 3

Quantifying and Mitigating the Splitting Bias and Other Value Tree-Induced Weighting Biases

This chapter presents a model for estimating and correcting attribute-weighting biases (such as the splitting bias) that result from the use of value trees when structuring value function weight elicitation. The model is based on the conjecture that attribute weights are influenced by tree structure and a subject's use of the "anchor-and-adjust" heuristic, meaning that the subject starts with an equal allocation of weight among attributes in each tree partition and then adjusts the weights to reflect his or her innate preferences. Adjustments tend to be insufficient, resulting in attribute weights that are closer in value to each other than if the anchor-and-adjust heuristic was not employed.

3.1 Introduction

Multiattribute decision analysis provides a framework for helping decision makers tackle complex decisions involving conflicting objectives. The decision process often involves organizing the users' objectives into value trees or objectives hierarchies (Keeney and Raiffa 1976; von Winterfeldt and Edwards 1986). Attributes are used in value trees to quantify the extent to which an alternative achieves an objective. The relative importance of the attributes is described by attribute weights, which can be derived either non-hierarchically by simultaneously evaluating all attributes, or hierarchically by assigning weights to subsets of attributes at each level of the tree. Lower-level attribute weights are found by multiplying down the tree.

The process of constructing value trees can help decision makers identify, organize, and prioritize their objectives. In this manner, value trees have assisted private- and public-sector decision processes (Heins and Roling 1995; Marttunen and Hamalainen 1995; Keeney et al. 1996; Maniezzo et al. 1998; Kwak et al. 2002; Al-Kloub et al. 1997). However, the use of trees to structure weight elicitation in value function assessment is prone to several weighting biases, including the well-known “splitting bias.”

When priorities are uncertain, theoretically irrelevant aspects of the weight elicitation process can shape expressed values (Fischhoff et al. 1980). For example, the principle of description invariance (Tversky and Kahneman 1986) states that a value tree’s structure should not influence weights. In practice, this principle is often violated and tree structure does shape expressed weights (Stillwell et al. 1987; Weber and Borcherding 1993; Weber et al. 1988; Borcherding and von Winterfeldt 1988; Pöyhönen and Hämäläinen 1998; Pöyhönen and Hämäläinen 2000; Pöyhönen et al. 2001).

Three closely related biases can occur when value trees are used to derive attribute weights. This collection of biases will be referred to as value tree-induced attribute-weighting biases, or value tree-induced biases for short. This chapter develops a model-based method for correcting these biases in additive value function weight assessments. The model is based upon research suggesting that subjects employ an anchor-and-adjust heuristic (Kahneman et al. 1982) when eliciting weights with the aid of value trees. Section 3.2 presents a literature review of the experimental evidence and proposed causes of value tree-induced biases, as well as a discussion of methods for reducing biases. Section 3.3 develops a model-based approach to debias value tree weights elicited from subjects. Section3.4 discusses a case study in which weights are elicited from employees of the

Centerior Energy Corporation regarding environmental and economic attributes of several electric system expansion alternatives. The data are used to illustrate the existence and correction of the value tree-induced attribute weighting biases with the use of the proposed model. Section 3.5 summarizes the results of the application of that model, including the implications of using debiased weight sets from the model rather than the elicited weight sets. Changes in ranks and expected value losses from using "incorrect" weight sets (either the original biased weights or debiased weights from an incorrect model specification) are calculated. Section 3.6 concludes the paper.

3.2 Literature Review

3.2.1 Experimental Evidence of Value Tree-Induced Biases

Several experiments have provided evidence of biases occurring with the use of the value tree. First, Stillwell et al. (1987) demonstrated that hierarchically assessed weights tend to have a larger variance (or standard deviation) than weights assessed non-hierarchically. Their experiment included 37 subjects who expressed both hierarchical and non-hierarchical weights for a set of attributes. The variance of the hierarchical weights was higher than the variance of the non-hierarchical weights for 33 of 37 subjects, and was more than double the variance of the non-hierarchical weights for 30 subjects.

The second value tree-induced bias has been labeled the splitting bias in the literature. It states that decomposing an objective into multiple attributes leads to a higher overall weight for that objective when compared to a direct assessment of the objective's relative importance. Weber et al. (1988) performed an experiment in which decision makers provided weights for two different value trees. The first tree contained three objectives that each had one corresponding attribute, whereas the second tree subdivided one or more of the objectives into two attributes each, resulting in eight value trees. The first and eighth trees were the same, each having all three objectives subdivided into two attributes. The remaining six trees divided one or two of the objectives into two attributes, whereas the remaining objectives were represented by a single attribute. For those objectives that were split, the average weight (across subjects) assigned directly to the objective in the first tree was significantly lower than the sum of the weights assigned to the two attributes describing the objective in the second.

In another experiment, Borcherding and von Winterfeldt (1988) elicited weights from 200 subjects, using various value trees and weighting methods. If an objective was represented by multiple attributes, the authors discovered that when using the swing weight or ratio methods, the objective received a higher weight than if it was represented by a single attribute. This result supported the findings by Weber et al. (1988). Both Weber et al. (1988) and Borcherding and von Winterfeldt (1988) considered averages of weights over all decision makers. Alaja (1998) demonstrated that the splitting bias also occurs for individuals. She elicited weights from 30 university students and 39 stakeholders in a lake management context. All stakeholders and a majority of students displayed the bias. Pöyhönen et al. (2001) also demonstrated that the splitting bias exists on an individual level with their 180 subject experiment.

The third value tree-induced bias states that weights for an objective tend to be higher when the objective is presented at a higher level in a value tree. Borcherding and von Winterfeldt (1988) conducted an experiment involving 200 subjects demonstrating the statistical significance of the bias when eliciting weights with ratio, swing, and trade-off methods.

3.2.2 Proposed Causes of Value Tree-Induced Biases

Although the exact causes of value tree-induced weighting biases remains uncertain, research suggests that both the tree structure and a subject's use of the anchor-and-adjust heuristic can cause a subject's weights within each group of attributes in a tree to be more similar to one another than if this cognitive strategy is not used. In addition to the experimental evidence discussed previously, several other authors (Langer, T. and Fox, C.R., unpublished manuscript, December 2003; Fox and Clemen 2005; Langer and Fox 2003; Clemen and Ulu 2008) suggest that the way in which a value tree is partitioned can influence elicited weights. Turning to behavioral influences, von Nitzsch and Weber (1993) first suggested that assessing attribute weights might be affected by the anchor-and-adjust heuristic: A subject begins with an intuitive importance of the attribute as the anchor and then adjusts the attribute weight, usually insufficiently (Tversky and Kahneman 1986). Although there is agreement that the anchor-and-adjust heuristic can lead to value tree-induced biases, several authors (Langer and Fox 2003; von Nitzsch and Weber 1993; Fox et al. 2005) have conjectured that anchor weights result from equal allocation of weight among attributes within a partition in the value tree. As a result, the number of attributes representing an objective can influence that objective's weight. For example, consider two representations of an objective in a value tree: (1) the objective is represented by multiple attributes; and (2) the objective is represented by a single attribute. When eliciting non-hierarchical weights, the total weight for the objective in representation (1), as determined by summing the attribute weights, tends to be higher than the weight elicited for the objective under representation (2).

In addition to research supporting an initial equal allocation of attribute weights, there is also research supporting the adjustment stage of the anchor-and- adjust heuristic. Von Nitzsch and Weber (1993) demonstrated that subjects do not adjust their attribute weights sufficiently when attribute ranges are varied. They argue that this range effect can serve as an explanation for the splitting bias demonstrated by Weber et al. (Weber et al. 1988). Combining two subattributes into an overall attribute is similar, they argue, to increasing the range of an attribute. If the subject adjusts his or her weight insufficiently in response to the change in attribute range, an overweighting of the subattributes occurs, resulting in the splitting bias.

Pöyhönen et al. (2001) performed an experiment in which they also observed insufficient adjustment of weights when attributes are divided in a value tree. They referred to this phenomenon as the unadjustment effect.

3.2.3 Methods for Bias Correction

In general, if a bias is strongly suspected to affect expressed weights, it is desirable to address the bias and, if possible, correct the weights to better represent the decision maker's preferences. Much behavioral decision research argues that, when asked to elicit weights, subjects often construct their preferences during the elicitation process, as opposed to revealing preexisting preferences (e.g., Payne et al. 1999; Slovic 1995). Thus, the elicitation of preferences should foresee several sources of variation: systematic variation due to characteristics of the elicitation process used, systematic variation due to the decision maker's underlying values, and variation due to random errors. When a large portion of the variation in weights is attributable to the elicitation process used, one approach is to choose a simpler, less cognitively demanding task for eliciting weights, such as deriving weights from holistic rankings (Edwards and Barron 1994). However, simpler methods can result in the loss of important ratio scale information, leading to utility losses when these weights are applied (Jia et al. 1998). Therefore, it is important to address these sources of variation in order to develop more informative weight elicitation procedures.

At least three general approaches are possible for addressing sources of error and reducing biases in weight elicitation. First, subjects can be informed of the biases and encouraged to avoid them when expressing weights. Second, weights can be elicited by two or more methods that have different biases, and subjects can be asked to resolve the differences. Third, a model-based approach can be used to estimate and correct biases using two or more sets of elicitations.

A review of the literature indicates that only the first approach has been applied to value tree-induced biases. Alaja (1998) tested the effects of informing decision analysis students and community stakeholders of the splitting bias before eliciting weights. The students attended a lecture and performed class exercises relating to the bias one month prior to the experiment. Both students and stakeholders were encouraged to avoid the bias. Nonetheless, the bias persisted for all stakeholders and a majority of the students. The experiment suggests that a decision maker must be very familiar with the bias and have had experience with it in order to avoid it. This is unlikely to be the case for most real-world decisions.

Pöyhönen and Hamalainen (2000) investigated the impact of informing subjects of the splitting bias by repeating the experiment of Weber et al. (1988) with 42 student subjects. After an explanation of the bias was presented during a lecture, the students were asked to express two weight sets non-hierarchically. The first value tree divided the high-level objectives into attributes, whereas the second contained only the high-level objectives. The authors "interpreted that the student was able to avoid the splitting bias if the sum of weights given to individual attributes differed less than 20% from the weight given to the group of attributes [high-level objectives]" (Pöyhönen and Hämäläinen 2000). Only 12 of 42 students completely avoided the bias, despite the effort to educate the students to be aware of it.

The second approach to correcting biases can be applied to value tree-induced biases by assessing weights with more than one value tree and then conducting follow-up interviews with decision makers. If weight sets or rankings of alternatives differ, decision makers can be asked to resolve the conflicts. Although this general approach has been used to address inconsistencies among different methods for eliciting weights (von Winterfeldt and Edwards 1986; Payne et al. 1999; Hobbs and Horn 1997), its use to correct value tree-induced biases has not been reported.

Time or other limitations often prevent such followup interviews from being conducted. When this is the case, a model-based approach to correcting biases can be useful. This approach consists of correcting biases by statistically estimating the magnitudes of the components of the decision maker's judgments, including biases, from two or more assessments. A model-based approach is advocated by Fox and Clemen (2005) in the context of eliciting subjective probabilities: "[a]n alternative approach is to debias the judgments rather than the judge." In the same context, Fox et al. (2005) suggest that the "relevant judgment or decision making process might be formally modeled so that the extent of the bias […] might be measured and thereby subtracted from the relevant assessment."

Such a model-based approach has been used to quantify biases in value quantification, to adjust elicited weights, and to debias subjective probabilities. In a single-attribute problem involving decision making under risk, Bleichrodt et al. (2001) applied a quantitative approach for correcting two violations of expected utility. In a multiattribute context, Anderson and Hobbs (2002) proposed a Bayesian approach to quantify and correct scale compatibility bias, which tends to result in overweighting of "currency" attributes in trade-off weight assessments (Slovic et al. 1990).

A model-based approach has also been applied to the partition-dependence bias that occurs when eliciting subjective probabilities. The partition-dependence bias states that assessed probability distributions depend on the partitioning of the state space. Several experiments (Fox and Clemen 2005; Langer and Fox 2003; Fox and Rottenstreich 2003) demonstrate the existence of this bias. The results show that the assessed probability distributions are biased towards an ignorance prior distribution, which divides the probabilities equally among the events within a partition. To debias the assessed probabilities, two quantitative models have been developed. Fox and Rottenstreich (2003) introduced a multiplicative model based on support theory (Tversky and Koehler 1994; Rottenstreich and Tversky 1997), whereas Clemen and Ulu (2008) developed a model for correcting the partition-dependence bias that instead defined elicited probabilities as a linear convex combination of the ignorance prior and "untainted" subjective probabilities.

3.3 Methodology

The model developed here combines the influence of the value tree structure and the conjectured use of the anchor-and-adjust heuristic on weight elicitations. Application of the anchor-and-adjust heuristic can cause the attribute weights expressed by the subject to be affected: Subjects begin with an initial equal allocation of weights, followed by an insufficient adjustment of weights that aim to reflect the subject's innate preferences. To capture these influences, the model defines, for each value tree, the weight expressed by the subject as a linear convex combination of the equal allocation weights and the estimated debiased weights, which reflect the subject's innate preferences. In addition, a random error term is included, reflecting random variations in expressed weights.

Model-estimated weights can be considered debiased in the following sense: If the form of the model is correct, the debiased weights are the best estimate (in a least-squares sense) of the underlying weights that would be expressed if the subject suffered no value tree-induced bias. However, debiased weights may still suffer from other problems. For instance, if a subject's weights are influenced by both the splitting bias and the range insensitivity effect (von Nitzsch and Weber 1993), the model presented here will only remove the former bias.

Several possible model formulations can reflect the notion that the expressed weights are influenced by tree structure, equal allocation weights, and innate preferences. For simplicity, the convex combination model is proposed following the logic of Occam's razor: Use the simplest model that represents anchor-and- adjust behavior for each partition of the tree. More complicated model formulations were explored, but the additive model developed here had several advantages. These alternate model specifications either had restrictions on the type of data that could be input—such as the inability to include elicited weights of zero—or produced estimated debiased weights whose values greatly differed from the weights expressed by the subject. For example, several models produced estimated debiased weights in which one attribute received nearly all of the weight and the remaining attributes received weights near zero, despite a much less extreme allocation of weights by the subject.

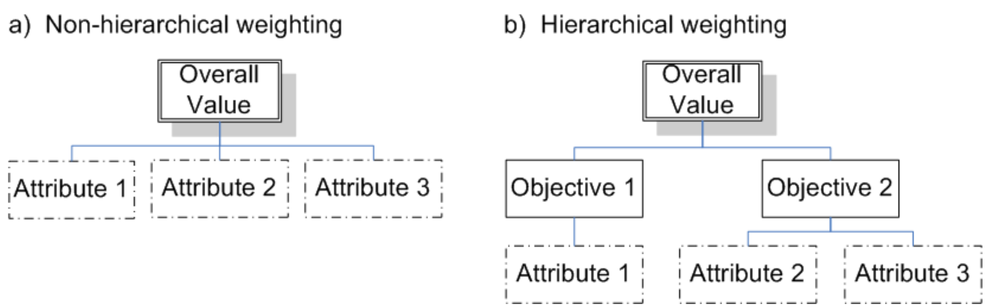

Figure 3-1: a) Non-hierarchical and b) Hierarchical Attribute Weighting

The notation used in the model is described with the aid of the two value trees in Figure 3-1. The attributes $i \in I$ are structured into tree $k \in K$ containing partition $n \in N_k$. For example, the three attributes are structured into the non-hierarchical tree (k = 1) containing one partition in Figure 3-1a. The hierarchical tree (k = 2) in Figure 3-1b contains two partitions (N_k = {1,2}). The first partition includes objectives 1 and 2, whereas the second partition includes attributes 2 and 3. Each partition n in tree k contains at least two branches $b \in B_{k,n}$. The attributes belonging to branch b of partition n in tree k are represented by $i \in I_{k,n,b}$. Similarly, the attributes belonging to partition n in tree k are represented by $i \in I_{k,n}$.

Mutual preference independence, an assumption of the additive value function (Keeney and Raiffa 1976), is not generally satisfied if expressed weights $\hat{W}_{i,k}$ are assumed to be a simple convex combination $\left(\lambda W_{i,k}^{=} + (1-\lambda)\widetilde{W}_i\right)$ of equal allocation weights $W_{i,k}^{=}$ and estimated debiased weights reflecting innate preferences, $\widetilde{W}_i$. Instead, *fractional weights* must be used. For each branch in each partition, a fractional weight is defined as follows: The sum of attribute weights associated with the branch is divided by the sum of the attribute weights in the associated partition. For example, the fractional weight for the branch containing attribute 2 in the hierarchical value tree in Figure 3-3b is defined as $W2/(W2 + W3)$, where W_i represents the weight of attribute i.

To illustrate the need for this modification, the property of mutual preference independence can be described as follows. Consider an assessment that adheres to a model for a value tree of M attributes. Now imagine that this tree is embedded as a subtree in another value tree with $N > M$ attributes (that is, N - M attributes are added such that the value tree structure among the original M is maintained). The relative weights among the original M attributes should not change (although their total weight will, in general, be less). This relationship does not hold for the simple convex combination model just described. This can be seen with a simple example.

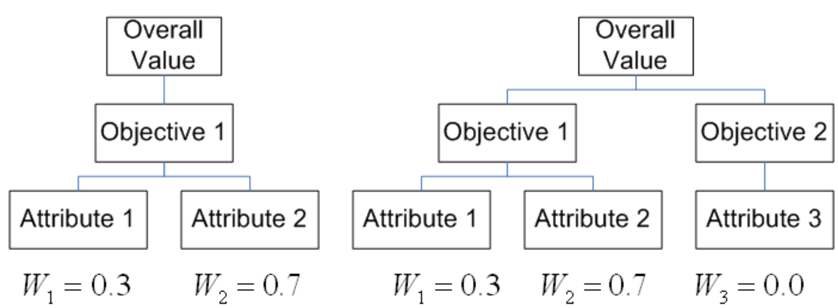

Figure 3-2: Example of Mutual Preference Independence

Figure 3-2 contains two values trees. The tree on the left is a subtree of the tree on the right. Suppose the unbiased underlying weights, W^*, are those shown in the figure and that the parameter λ has a value of 0.4. Using the simple convex combination model and assuming no random errors, the expressed weights equal 0.38 for attribute one and 0.62 for attribute two in the first tree. In the second tree, the values are 0.28 for attribute one, 0.52 for attribute two, and 0.20 for attribute three. Thus, the introduction of attribute three has resulted in a change in the relative weight between attributes one and two: The ratio of the expressed weights for tree one (0.38/0.62 = 0.61) differs from the ratio of the expressed weights for tree two (0.28/0.52 = 0.54). The use of fractional weights in the following model preserves the property of mutual preference independence.

When eliciting weights, a subject divides the total partition weight among the subset of attributes represented by the branches contained in the partition for each partition of the value tree. In the degenerate case, a subset may consist of a single attribute. Equation (3.1) models the use of the anchor-and-adjust heuristic beginning with equal allocation of weight locally in each partition of the tree. The model defines each subject's fractional expressed weight for a subset of attributes as a linear convex combination of the fractional equal allocation weights and fractional estimated debiased weights, which are presumed to represent innate preferences. The total fractional expressed weight for the subset of attributes $I_{k,n,b}$ is defined as

$$\frac{\sum_{i \in I_{k,n,b}} \hat{W}_{i,k}}{\sum_{j \in I_{k,n}} \hat{W}_{j,k}} = \frac{\lambda}{|B_{k,n}|} + (1-\lambda)\left(\frac{\sum_{i \in I_{k,n,b}} \tilde{W}_i}{\sum_{j \in I_{k,n}} \tilde{W}_j} \right) + \varepsilon_{k,n,b} \quad \forall k \in K, n \in N_k, b \in B_{k,n} \tag{3.1}$$

where $\hat{W}_{i,j}$ represents the weight expressed by the subject for attribute i in value tree k, $|B_{k,n}|$ defines the number of branches, and $1/|B_{k,n}|$ describes the fractional equal allocation weight for partition n and tree k. Three variables are estimated by the model for each subject: λ, the bias parameter; $\tilde{W}_i$, the debiased weight of attribute i estimated by the model; and $\varepsilon_{k,n,b}$, the random error associated with branch b of partition n in tree k. The expected value of $\varepsilon_{k,n,b}$ is assumed to be zero. The debiased weights do not contain a subscript k, indicating that a single set of debiased weights is estimated for each subject. Applying equation (3.1) to the hierarchical value tree (k = 2) in Figure 3-1b, for example, results in

$$\frac{\hat{W}_{1,2}}{\hat{W}_{1,2}+\hat{W}_{2,2}+\hat{W}_{3,2}}=\frac{\lambda}{2}+(1-\lambda)\frac{\tilde{W}_{1,2}}{\tilde{W}_{1,2}+\tilde{W}_{2,2}+\tilde{W}_{3,2}}+\varepsilon_{2,1,1} \qquad \frac{\hat{W}_{2,2}}{\hat{W}_{2,2}+\hat{W}_{3,2}}=\frac{\lambda}{2}+(1-\lambda)\frac{\tilde{W}_{2,2}}{\tilde{W}_{2,2}+\tilde{W}_{3,2}}+\varepsilon_{2,2,1}$$

$$\frac{\hat{W}_{2,2}+\hat{W}_{3,2}}{\hat{W}_{1,2}+\hat{W}_{2,2}+\hat{W}_{3,2}}=\frac{\lambda}{2}+(1-\lambda)\frac{\tilde{W}_{2,2}+\tilde{W}_{3,2}}{\tilde{W}_{1,2}+\tilde{W}_{2,2}+\tilde{W}_{3,2}}+\varepsilon_{2,1,2} \qquad \frac{\hat{W}_{3,2}}{\hat{W}_{2,2}+\hat{W}_{3,2}}=\frac{\lambda}{2}+(1-\lambda)\frac{\tilde{W}_{3,2}}{\tilde{W}_{2,2}+\tilde{W}_{3,2}}+\varepsilon_{2,2,2}$$

Three constraints accompany equation (3.1): first, the attribute weights estimated by the model must sum to one, $\Sigma_{i\in I}\,\tilde{W}_i=1$; second, $0 \le \lambda \le 1$; and third $\tilde{W}_i \ge 0$, for all $i \in I$. The constraint on λ reflects the prior expectation that the expressed weights are a convex combination of the equal allocation and estimated debiased weights. This parameter describes the amount of influence the equal allocation weight has on the final expressed weight. When λ is large, weight adjustments aimed at reflecting innate preferences tend to be much too small, leading to large value tree-induced biases. On the other hand, λ = 0 indicates that the subject's expressed weights are not influenced by the equal allocation weights. For simplicity, only a single value of λ is considered for each subject. However, it is possible that the equal allocation weights might influence different attributes or sets of attributes in various ways; for instance, λ might be a function of the number of branches in a partition. Future research could consider more complex models.

The unknown model parameters are estimated by using nonlinear programming to minimize the sum of squared errors (3.2) subject to the constraints (3.3). Regression is not used because of the constraints.

$$\underset{\{\lambda,\tilde{W}_i\}}{Min} \quad z=\sum_{k\in K}\sum_{n\in N_k}\sum_{b\in B_{k,n}}\left[\frac{\sum_{i\in I_{k,n,b}}\hat{W}_{i,k}}{\sum_{j\in I_{k,n}}\hat{W}_{j,k}}-\left(\frac{\lambda}{|B_{k,n}|}+(1-\lambda)\frac{\sum_{i\in I_{k,n,b}}\tilde{W}_i}{\sum_{j\in I_{k,n}}\tilde{W}_j}\right)\right]^2 \tag{3.2}$$

$$\text{subject to} \qquad \sum_{i=1}^{I}\tilde{W}_i=1; \quad \lambda,\tilde{W}_i\geq 0 \quad \forall i=1,\ldots,I \tag{3.3}$$

Model (3.2)-(3.3) is solved using the nonlinear solver CONOPT in GAMS® with multiple starting points.

Similarly structured models have been used to debias judgments for other tasks in decision analysis. Clemen and Ulu (2008) developed a model based on support theory (Tversky and Koehler 1994; Rottenstreich and Tversky 1997) to reduce the partition-dependence bias that occurs in subjective probability assessments. The model defines the partition dependent probabilities as convex combinations of "ignorance priors" (derived from the uniform distribution over events) and "untainted" subjective probabilities.

Attribute weights have also been modeled using convex combinations, but in a different context than the model proposed in this research. Srinivasan and Park (1997) created a preference model that defined attribute weights as a convex combination of partworths from two different methods (conjoint and self-explicated) and then chose the value of the parameter (equivalent to λ) that resulted in the best fit of the preference model to expressed preferences among pairs of alternatives.

To collect data to test the model, a case study was conducted in which managers from an electric utility used multicriteria decision making methods to address environmental and economic objectives of electricity generation and conservation planning. The next section describes the case study. One task in the case study involved the elicitation of two sets of additive value function weights for each subject – one using a non-hierarchical value tree and the other with the aid of a hierarchical value tree. Those weights are used in Section 3.5 to implement the model (3.1).

3.4 Case Study

During a multicriteria planning exercise, managers from Centerior Energy of Ohio (an electric utility now part of First Energy Corporation) were introduced to multicriteria decision making methods for quantifying environmental externalities and other objectives in long-run (multi-decade) electricity generation and conservation planning. During a brainstorming session, the group of eleven subjects, who were either planners or mid-level executives, identified fifteen possible planning alternatives. The alternative reflecting the status quo was defined as the reference alternative (Ref), while the other fourteen alternatives were labeled A through N.

The planning alternatives (Table 3-1) consisted of various levels of demand-side management (DSM), generator life extension (LE), new generation types, reserve margin, and alternative operating methods for generators. The operating method describes the order in which electricity generators are selected to operate and could be based either on fuel cost or a combination of emissions and fuel cost, the latter yielding lower total emissions.

Table 3-1: Alternatives for Centerior Energy Case Study

Alt.	Description	DSM (MW)	LE[a] (# units)	New Capacity (Type)[b]	Reserve (%)[c]	Operating Method[d]
Ref	Reference (Ref)	-360	11	All CT/CC	20	Economic
A	Ref + emissions dispatch (ED)	-360	11	CT/CC	20	Emissions
B	Ref with 10% reserve	-360	11	CT/CC	10	Economic
C	Ref + wind	-360	11	+200 MW Wind	20	Economic
D	Ref + coal	-360	11	+600 MW Pulverized Coal	20	Economic
E	Ref + Summit	-360	11	+200 MW Summit Plant[e]	20	Economic
F	Reduced life extension (LE)	-360	7	CT/CC	20	Economic
G	Reduced LE + ED	-360	7	CT/CC	20	Emissions
H	Reduced LE w/ 500 MW purchase	-360	7	+500 MW supply contract	20	Economic
I	Reduced LE w/ new nuc-lear	-360	7	600 MW Nuclear	20	Economic
J	Increased DSM	-720	11	CT/CC	20	Economic
K	Increased DSM, reduced LE, ED	-720	7	CT/CC	20	Emissions
L	Increase DSM, reduced LE, wind + ED	-720	7	200 MW Wind	20	Emissions
M	Load building	+360	11	CT/CC	20	Economic
N	Load building + reduced LE	+360	7	CT/CC	20	Economic

[a] LE = Lifetime extension for existing plants. The alternatives with 7 generating units + LE have 720 MW less generating capacity that is subject to LE than alternatives with 11 units.
[b] CT/CC = Combustion Turbine/Combined Cycle, which are types of generating units that burn natural gas. In alternatives with other types of new generation, CT/CC capacity is decreased to accommodate the other types in order to maintain the target reserve margin.
[c] Reserve margin equals the amount of generation capacity (and perhaps contracts) in excess of the highest MW demand.
[d] In the emissions method, a cost penalty is added to CO_2 emissions, resulting in lower emissions but higher fuel costs.
[e] Compressed air storage facility.

Several uncertainties complicated the planning process. The attribute ranges considered during the weight elicitation (Table 3-2) were based on the extremes observed across all scenarios and alternatives, although the analysis of alternative ranks is based only on attribute values in a "reference" scenario. Therefore, some of the best and worst attribute values are not associated with any alternative under the reference scenario.

Table 3-2: Attribute Definitions and Ranges Relative to the Reference Alternative

Attribute	Description	Best	Worst
x_1	Levelized annual revenue requirements, years 0-20 ($Millions/year)	-41	339
x_2	Average capital expenditures, years 0-20 ($Millions/year)	-86	220
x_3	Levelized rates, years 0-6 ($/megawatt hour)	-3.3	10.8
x_4	Levelized rates, years 0-20 ($/megawatt hour)	-3.2	24.3
x_5	Average SO_2 emissions, years 0-20 (tons/year)	-28,564	17,876
x_6	Average CO_2 emissions, years 0-20 (1000 tons/year)	-4,437	3,047
x_7	Average NO_x emissions, years 0-20 (1000 tons/year)	-10	7
x_8	Number of new sites for coal ash disposal required	0	1
x_9	Total land needed for new generation, years 0-20 (acres)	-140	1,083
x_{10}	Remotely sited new generation capacity (megawatts)	-2,637	3,272
x_{11}	Nuclear power (megawatts)	0	600
x_{12}	Job losses, region	-6,447	40,006
x_{13}	Average emergency power, years 0-20 (gigawatt hours/year)	-2	124

Table 3-3: Attribute Values for Each Alternative (Reference Scenario)

Alt.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}
Ref	0	0	0	0	0	0	0	0	0	0	0	0	0
A	2	0	0.1	0.1	-3,332	-104	0	0	0	0	0	277	0
B	-41	-56	-0.1	-1.3	0	29	0	0	-50	-1,068	0	-3,016	22
C	52	8	0	1.5	-2,108	-326	-1	0	873	-235	0	6,814	0
D	105	157	0.3	3.2	182	2	0	0	0	433	0	7,081	0
E	9	-27	0.1	0.3	718	102	0	1	-43	-835	0	2,525	0
F	57	6	0	1.7	-14,460	1,155	-2	0	34	400	0	7,601	11
G	60	6	0	1.8	-17,515	-1,335	-2	0	34	400	0	8,016	11
H	59	6	-0.4	1.7	-14,285	-642	-2	0	34	400	0	7,877	0
I	99	98	0.1	3	-26,583	-3,468	-7	0	-10	266	600	9,048	17
J	11	35	1.3	2.2	-4163	-672	-1	0	-43	-835	0	-6,447	-2
K	30	48	0.8	2.7	-16,935	-1,615	-4	0	30	334	0	-4,435	8
L	82	55	0.8	4.3	-21,449	-2,045	-4	0	913	433	0	2,426	8
M	51	22	-0.1	0.2	687	544	1	0	0	-167	0	3,342	3
N	110	28	-0.1	1.5	-11,765	-670	-1	0	34	300	0	11,220	15

The electric utility planning software MIDAS (Farber et al. 1988) computed the values of thirteen attributes for each alternative. For each alternative, the attribute values corres-ponding to the reference scenario are summarized in Table 3-3.

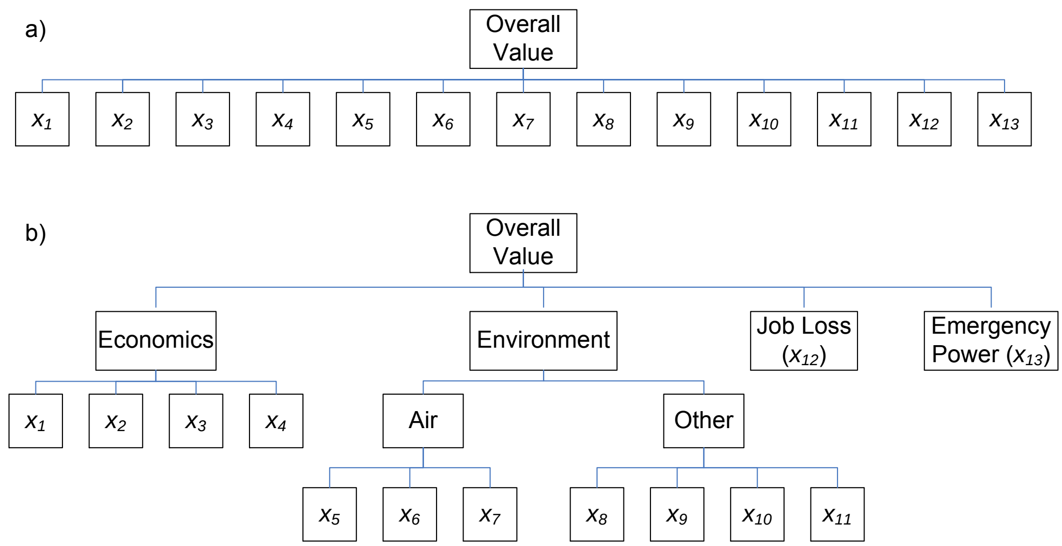

Figure 3-3: (a) Non-hierarchical and (b) Hierarchical Value Trees

The thirteen attributes were organized into two trees (Figure 3-3). To derive attribute weights, each subject completed two questionnaires in one meeting, separated in time by other value assessment tasks not considered in this research. In the first questionnaire, all attributes were evaluated non-hierarchically (Figure 3-3a), meaning that the subject considered all thirteen attributes simultaneously. The direct weighting method of point allocation was used. Subjects directly assigned a point value between 0 and 100 to each attribute such that the sum of the attribute weights equaled 100. The point values reflected the "importance" of each attribute to the subject. The subjects were presented with all the attributes and their associated ranges were displayed explicitly under each attribute in the questionnaire (Table 3-2). The subjects were told that "importance" should reflect the relative desirability of a change in the attribute value from its worst to its best value. So that the values are between zero and one, the final weight for each attribute, w_i, was determined by dividing the point value by 100.

In the second questionnaire, a separate set of weights was elicited using a hierarchical approach (Figure 3-3b). Each subject was presented with a diagram of the value tree relevant to the elicitation. The subjects evaluated subsets of attributes/objectives for each partition of the value tree (e.g., x_1-x_4) using a bottom-up approach so that the sets of attributes corresponding to each objective were known to the subjects. Again, the attributes and associated ranges were displayed clearly and the subjects were instructed to provide weights that reflected their willingness to tradeoff one attribute range for another. Point allocation was again used to divide 100 points among the subsets of attributes associated with each partition of the tree. These point values were then normalized so that the value for each attribute or subset of attributes was between zero and one. The final weight for each of the lowest level attributes was obtained by multiplying the normalized point allocation values down through the tree. The 22 weight sets elicited by the subjects (labeled S1 through S11) are presented in Table A.I- 1 in Appendix I.

It is important to note that there are well known problems with direct weighting methods. In particular, point allocation suffers from the range effect (von Nitzsch and Weber 1993). However, the aim of this research is not to advocate the use of direct weighting methods such as point allocation, but instead to derive a method for correcting the value tree-induced attribute weighting biases described in Section 3.2.1 that occur with the use of direct methods.

Attribute weights are combined with single-attribute value functions to translate each attribute into a measure of value. Due to time limitations, and because the focus of this study is on weighting, single-attribute value functions for the case study are determined by linearly rescaling each attribute on a 0-1 scale (Table 3-4). The best attribute value is given a score of one and the worst a score of zero.

Table 3-4: Rescaled Attribute Values for Each Alternative (Reference Scenario)

Alt.	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}
Ref	0.89	0.72	0.77	0.88	0.39	0.41	0.41	1	0.89	0.55	1	0.86	0.98
A	0.89	0.72	0.76	0.88	0.46	0.42	0.41	1	0.89	0.55	1	0.86	0.98
B	1.00	0.90	0.77	0.93	0.39	0.40	0.41	1	0.93	0.73	1	0.93	0.81
C	0.76	0.69	0.77	0.83	0.43	0.45	0.47	1	0.17	0.59	1	0.72	0.98
D	0.62	0.21	0.75	0.77	0.38	0.41	0.41	1	0.89	0.48	1	0.71	0.98
E	0.87	0.81	0.76	0.87	0.37	0.39	0.41	0	0.92	0.70	1	0.81	0.98
F	0.74	0.70	0.77	0.82	0.70	0.25	0.53	1	0.86	0.49	1	0.70	0.90
G	0.73	0.70	0.77	0.82	0.76	0.59	0.53	1	0.86	0.49	1	0.69	0.90
H	0.74	0.70	0.79	0.82	0.69	0.49	0.53	1	0.86	0.49	1	0.69	0.98
I	0.63	0.40	0.76	0.78	0.96	0.87	0.82	1	0.89	0.51	0	0.67	0.85
J	0.86	0.61	0.67	0.80	0.48	0.50	0.47	1	0.92	0.70	1	1.00	1.00
K	0.81	0.56	0.71	0.79	0.75	0.62	0.65	1	0.86	0.50	1	0.96	0.92
L	0.68	0.54	0.71	0.73	0.85	0.68	0.65	1	0.14	0.48	1	0.81	0.92
M	0.76	0.65	0.77	0.88	0.37	0.33	0.35	1	0.89	0.58	1	0.79	0.96
N	0.60	0.63	0.77	0.83	0.64	0.50	0.47	1	0.86	0.50	1	0.62	0.87

The single-attribute value functions were aggregated using an additive value function, the most widely applied method for amalgamating riskless preferences (von Winterfeldt and Edwards 1986)

$$V(x_1,...,x_I) = \sum_{i=1}^{I} W_i v_i(x_i) \tag{3.4}$$

The overall value for an alternative is defined as $V(x_1,...,x_I)$, which depends on the levels of the I attributes. The single-attribute value functions are represented by $v_i(x_i)$, $i= 1.., I$. The weight, W_i, for each attribute x_i is elicited from the subject as described above. The weights sum to one and are non-negative.

3.5 Results

Section 3.5.1 examines the differences between the elicited weight sets and investigates the existence of the value tree-induced bias in which the hierarchically derived weights have a higher standard deviation than the non-hierarchically derived weights. The results of fitting the model (3.1) to the Centerior Energy case study data are then explored in section 3.5.2. The steps to implement the model are presented and the parameters estimated by the model are analyzed. The changes in the rankings of alternatives are then examined in section 3.5.3 for each model and the expected value losses resulting from the use of "incorrect" weights (either original biased weights or estimated debiased weights from an assumed incorrect model specification) are investigated (section 3.5.4).

3.5.1 Existence of Bias

Of the eleven case study subjects, nine have a higher standard deviation for the hierarchically elicited weights than non-hierarchically derived weights – consistent with the hypothesis that a value tree-induced bias exists. The mean standard deviation for the non-hierarchical weights was 0.07, while the mean for the hierarchical weights was 0.10. A Wilcoxon signed-rank test was performed (Rice 1995). The Wilcoxon test does not assume any underlying distribution for the data. The null hypothesis assumes the mean difference is zero, while the alternative hypothesis states that the mean difference is greater than zero, reflecting the belief that the hierarchical weights will have higher standard deviation. The test produces a p-value of less than 0.005, indicating that the larger standard deviations associated with the hierarchical weighs are statistically significant.

The larger standard deviations associated with the hierarchically elicited weights as compared to the non-hierarchically derived weights are consistent with the discussion in section 3.2.2 in which it was hypothesized that this value tree-induced bias may have been produced by an anchor-and-adjust heuristic in which subjects start with equal allocation weights as anchors and then adjust the weights insufficiently.

Another way to examine the data for an attribute weighting bias is to compare the weights of attributes 12 and 13 from the two assessments. Based on the anchor-and-adjust heuristic, the sum of weights for attributes 12 and 13 should be higher in the hierarchical tree when compared to the sum of these weights elicited using the non-hierarchical tree, despite the fact that the attributes are at the same level in both trees. This was indeed the case; all eleven subjects expressed a higher sum of weights for attributes 12 and 13 when considering the hierarchical tree, which is consistent with the alternative hypothesis. The model (3.2) - (3.3) will be used next to estimate debiased weights.

3.5.2 Model Implementation

To fit the model to the data, the anchors must first be determined for each value tree. As described previously, it is assumed that the anchor weights in each elicitation result from equal allocation of weight among subsets of attributes associated with each partition of the tree. For the non-hierarchical weight assessment (Figure 3-3a), the fractional equal allocation weight for each attribute is simply 1/13. For the hierarchical weight assessment, the fractional equal allocation weights are found by determining the number of branches at each tree partition. For example, attributes 1 through 4 each receive a fractional equal allocation weight of ¼, because there are four branches in the partition containing these attributes.

These fractional equal allocation weights are then combined with the fractional expressed weights for each subject so that the debiased weights and the bias parameter λ can be estimated by solving (3.2)-(3.3). Because directly assessed weights, such as point allocation weights, can be poor estimates of a subject's willingness to trade off different attributes, the claim cannot be made that the estimated debiased weights correctly represent rates of substitution (Keeney and Raiffa 1976). Rather, those estimates are assumed to represent the weights that would be expressed in the absence of value tree-induced biases.

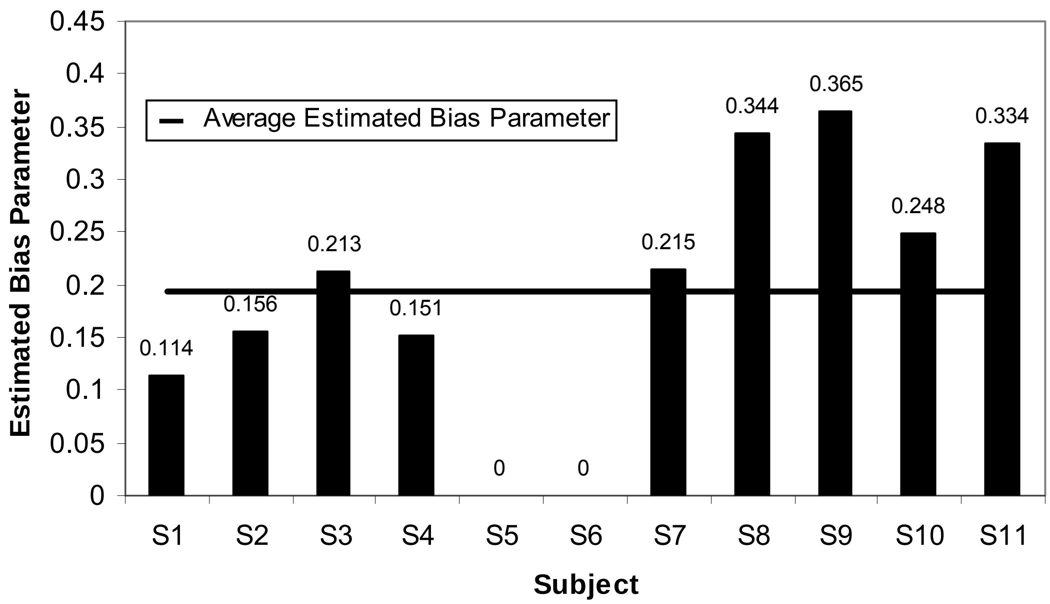

Figure 3-4: Estimated Bias Parameter (λ) for Each Subject

For each subject, the model is applied to the two weight sets elicited during the Centerior Energy case study, producing an estimate of λ and debiased weights for each subject. For nine of the eleven subjects, the model estimates a positive bias parameter (see Figure 3-4), indicating that the equal allocation weights may have an influence on the expressed weights for these nine subjects. The average value for the bias parameter is 0.195, with the largest value being 0.365. Thus, on average for this group, the elicited weights are a convex combination of the equal allocation weights and the estimated debiased weights, where the influence of the equal allocation weights is about twenty percent, and the influence of the innate preferences is about eighty percent. Table 3-5 displays each subject's estimated debiased weights.

Table 3-5: Estimated Debiased Weights for Each Subject

Attribute	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
x_1	0.151	0.000	0.054	0.025	0.011	0.043	0.067	0.121	0.053	0.046	0.022
x_2	0.008	0.000	0.008	0.000	0.014	0.131	0.129	0.131	0.241	0.117	0.017
x_3	0.295	0.497	0.424	0.214	0.397	0.502	0.256	0.256	0.326	0.203	0.352
x_4	0.056	0.243	0.016	0.170	0.158	0.062	0.191	0.175	0.129	0.255	0.098
x_5	0.108	0.000	0.008	0.100	0.067	0.067	0.025	0.050	0.003	0.117	0.000
x_6	0.013	0.000	0.006	0.075	0.023	0.019	0.018	0.018	0.011	0.004	0.045
x_7	0.051	0.000	0.006	0.039	0.023	0.010	0.008	0.000	0.011	0.034	0.003
x_8	0.027	0.000	0.048	0.025	0.034	0.003	0.028	0.012	0.016	0.000	0.101
x_9	0.027	0.000	0.009	0.071	0.023	0.001	0.015	0.021	0.001	0.000	0.191
x_{10}	0.000	0.000	0.009	0.010	0.045	0.001	0.007	0.050	0.016	0.008	0.030
x_{11}	0.012	0.000	0.085	0.025	0.011	0.019	0.000	0.042	0.016	0.040	0.000
x_{12}	0.020	0.000	0.174	0.016	0.100	0.028	0.128	0.020	0.105	0.021	0.034
x_{13}	0.231	0.260	0.155	0.229	0.095	0.115	0.128	0.104	0.072	0.154	0.109

A one-tailed binomial test (Conover 1999) with 11 trials is used to determine the statistical significance of the results. The null hypothesis, H_0: $q = 0.5$, states that no value tree-induced biases exist, meaning that the true value of λ is equal to zero. Under the null hypothesis, half of the estimated λs would be expected to be positive, and the other half zero. The alternative hypothesis, H_a: $q > 0.5$, states that the true value of λ is greater than zero. The result that 9 of 11 subjects have a positive bias parameter is significant with a *p*-value of 0.03. Thus, an influence of the equal allocation weights on the expressed weights is supported by the results.

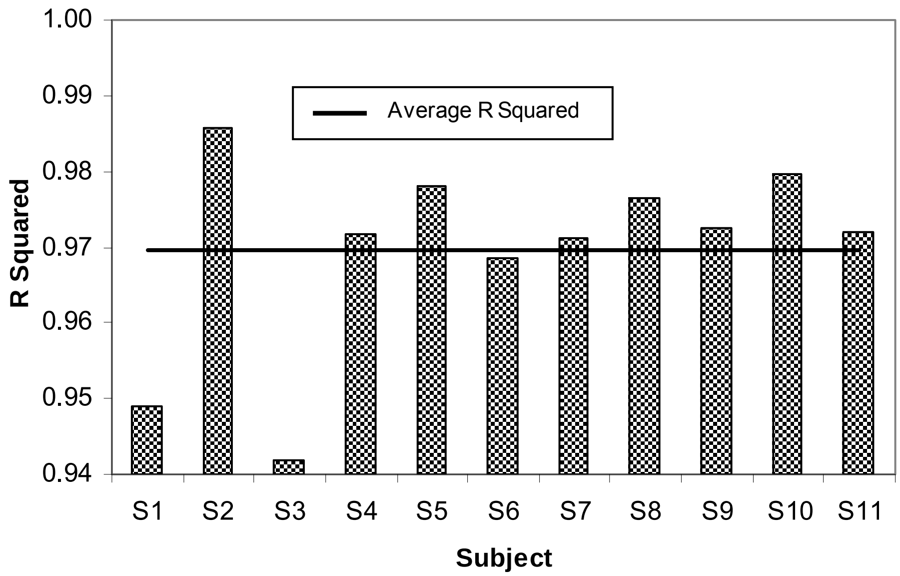

Figure 3-5: R^2 Values for Each Subject

To determine how well the variability in the data can be explained by the model (3.2)-(3.3), a coefficient of determination, R^2, is calculated for each subject as

$$R^2 = 1 - \frac{\sum_{b=1}^{30}\left(\hat{W}_b^{frac} - \hat{\hat{W}}_b^{frac}\right)^2}{\sum_{b=1}^{30}\left(\hat{W}_b^{frac} - \overline{\hat{W}}_b^{frac}\right)^2} \tag{3.5}$$

$\hat{W}_b^{frac}$ represents the fractional expressed weight for a subset of attributes at a branch (b = 1,…,30), as shown in (3.1). The term $\hat{\hat{W}}_b^{frac}$ represents the fractional expressed weight predicted by the model, and $\overline{\hat{W}}_b^{frac}$ is the mean fractional expressed weight, which is the same for each subject - i.e., number of partitions divided by number of branches = 1/5.

Figure 3-5 depicts the R^2 values for each subject; the model performs well with an average R^2 of 0.97.

3.5.3 Rankings

The ultimate purpose of eliciting weights from subjects is to rank alternatives by combining the weights with single attribute value functions. Suppose two weights sets are elicited from a subject and the values of the attribute weights differ between the sets. It is then possible for the rankings produced by each weight set to be different as well. This can occur if no single alternative dominates all the others. Tables III-3 and III-4 show that all alternatives are non-dominated; therefore, changes in ranks are indeed possible.

Table 3-6: Top Ranked Alternative for Each Subject and Each Weight Set

Weight Set	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
NH	B	Ref	B	I	B	B	B	B	B	B	B
H	H	Ref	Ref	H	B	H	B	B	B	H	Ref
Debiased	H	Ref	Ref	I	B	B	B	B	B	B	B

Three weight sets were used to derive three sets of ranks for each subject – non-hierarchical (NH) weights, hierarchical (H) weights and estimated debiased weights (Debiased). For six subjects, the top ranked alternative depended on the weight set used. For the other five subjects, all weights sets produced the same top-ranked alternative – alternative B for four subjects and the reference alternative for the other subject. Yet, for these subjects, there were inconsistencies in the ranks of the remaining 14 alternatives. That is, the ranks of the other alternatives were dependent on the weight set. Among subjects, there was some consensus. Of the 15 alternatives, only four – B, H, I and Ref – were ever ranked first by any of the subjects. The top ranked alternatives for each subject and the corresponding weight set are presented in Table 3-6. The complete set of ranks is available in Table A.I- 2 in Appendix I.

Interestingly, of the five subjects that had consistent top-ranked alternatives (S2, S5, S7, S8, S9), four of the subjects always ranked alternative B highest. Alternative B performed best with regards to three of the four economic attributes (x_1, x_2, x_4), and several "environmental-other" attributes (x_8, x_9, x_{10}, x_{11}). However, alternative B performed rather poorly with respect to the "environmental-air" attributes (x_5, x_6 and x_7). Thus, the high ranking of alternative B reflects that many decision makers were more concerned with the economic impacts of the projects than the air pollution produced by the various alternatives.

Table 3-7: Spearman's Correlations between Rankings from Different Weight Sets for Each Subject

Weights	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	*Mean*
NH-H	0.804	0.850	0.907	0.736	0.839	0.943	0.764	0.893	0.861	0.936	0.696	*0.839*
NH-Debiased	0.836	0.975	0.950	0.889	0.918	0.950	0.746	0.964	0.775	0.946	0.718	*0.879*
H-Debiased	0.925	0.907	0.939	0.918	0.971	0.993	0.996	0.886	0.925	0.893	0.939	*0.936*

While alternative B was always top ranked for five of the subjects, the ranks of the other alternatives were inconsistent and differed depending on the weight set used. The correlations between ranks derived from different weight sets are presented in Table 3-7. The largest correlations tend to be between the hierarchical weights and the model-estimated debiased weights, $\tilde{W}_i$. The smallest correlations tend to be between the two elicited weight sets; however, the two elicitation methods are nonetheless highly correlated, consistent with results of other experiments in which subjects applied two or more direct weight assessment methods (e.g., von Winterfeldt and Edwards 1986). Indeed, because of the possibility of a carryover bias, each subject's expressed weight sets tend to be more similar than they would be if the subject could forget, in the second assessment, what weights he or she chose in the first assessment. This artificial similarity will tend to decrease the estimated degree of the value tree-induced attribute weighting bias. Future research should attempt to control for this potential downward bias in the estimate of λ.

3.5.4 Value Losses

One way to evaluate the impact of differences in weights is to consider the loss of value caused by using "incorrect" weights. The loss of value incurred from using the "incorrect" weight set, w, instead of the correct weight set, c, is determined as

$$\text{Value Loss}_{\text{c,w}} = V_c(A_w^1) - V_c(A_c^1) \tag{3.6}$$

where $V_c(A)$ represents the result of the additive value function (equation III.4) for alternative A using weight set c. A_w^1 represents the top ranked alternative using weight set w *and* A_c^1 is the top ranked alternative using weight set c. Because, by definition, $A_c^{\,1}$ maximizes $V_c\,(\cdot)$, (3.6) is non-positive. By the definition of the attribute value functions and weights (all in the range 0-1), (3.6) also cannot exceed 1.

Table 3-8: Value Losses Incurred Using Different Weight Sets

Correct/ Used	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
NH/H	-0.038	0	-0.007	-0.03	0	-0.013	0	0	0	-0.010	-0.001
NH/ Debiased	-0.033	0	-0.003	-0.01	0	-0.003	0	0	0	-0.001	0
H/NH	-0.025	0	-0.042	-0.03	0	-0.030	0	0	0	-0.049	-0.041
H/Debiased	0	0	0	0	0	0	0	0	0	0	-0.009
Debiased/ NH	-0.025	0	-0.042	-0.03	0	-0.030	0	0	0	-0.049	0
Debiased/H	0	0	0	0	0	0	0	0	0	0	-0.001

Table 3-8 displays the value loss from using a weight set other than the one that is assumed to be correct. Table 3-9 shows the value losses expressed, instead, in terms of a single attribute, x_3, which describes levelized rates for years 0-6 (also called short-term rates) in dollars per megawatt-hour ($/MWh). Converting the value losses into dollar terms allows for a more tangible interpretation of the losses. This particular attribute is chosen because none of the subjects who incurred a value loss expressed a zero value weight for x_3, and all but three subjects assigned the highest weight to x_3 in both value trees. Subjects preferred lower short-term rates, so a worse rate is one that has a higher value.

Table 3-9: Value Losses Measured in Terms of Increase in Short-term Rates ($/MWh)

Correct/Used	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
NH/H	-$2.35	0	-$2.98	-$2.68	0	-$1.29	0	0	0	-$3.44	-$2.97
NH/Debiased	-$2.35	0	-$2.98	-$2.68	0	-$1.29	0	0	0	-$3.44	0
H/NH	-$1.77	0	-$0.26	-$1.77	0	-$0.28	0	0	0	-$0.95	-$0.04
H/Debiased	0	0	0	0	0	0	0	0	0	0	-$0.04
Debiased/NH	-$1.56	0	-$0.09	-$0.34	0	-$0.07	0	0	0	-$0.06	0
Debiased/H	0	0	0	0	0	0	0	0	0	0	-$0.35

The value losses can be converted to short-term rates using the following general procedure (Keeney and Raiffa 1976). Suppose that there are two alternatives, A_1 and A_2, and that A_1 is preferred to A_2. Each alternative is described by I attributes. Suppose $X^{(i*)}$ is a vector of length I that contains all zeroes except for the i^{th} element, which is x_i*. In order to calculate the value of x_i*, the equation

$$V_j(A_1) = V_j(A_2 + X^{(i*)}) \tag{3.7}$$

is solved for $X^{(i*)}$. The term V_j $(\cdot)$ represents the results of the additive value function (equation III.4) using weight set j. The term x_i* is interpreted as the amount that alternative A_2 would need to improve in attribute i for the subject to be indifferent between the two alternatives. Thus, x_3* is the decrease in short-term rates needed for options 1 and 2 to be equally desirable. The value of x_i* can be calculated by solving

$$V_{loss} = W^*_{i,c}\left(x_i^*\right) \Big/ \left(x_i^{best} - x_i^{worst}\right) \tag{3.8}$$

where V_{loss} represents the value loss due to the use of the (assumed) incorrect weights. $W^*_{i,c}$ indicates the subject's (assumed) correct weight for short-term rates (i = 3). The variables x_i^{best} and x_i^{worst} define the best and worst values of x_3. Table 3-2 shows that x_i^{best} = -3.3$/MWh and x^{worst} = 10.8 $/MWh. This range corresponds to the range used by subjects when performing weight elicitations.

Converting value losses to short-term rates using the procedure described above is equivalent to performing even swaps on all attributes until only short-term rates remain (Hammond et al. 1998). Suppose that there are two alternatives – A_1 and A_2 – and that attribute x_1 performs better under alternative A_1 and x_2 performs better under A_2. The even swap procedure begins by changing the value of x_1 under A_2 to the value of x_1 under A_1. Then, the subject determines how much the value of x_2 must change under A_2 for the subject to be indifferent between the two alternatives in regards to attributes x_1 and x_2. Even swaps are made until only the attribute of interest remains.

Overall, the value losses range from -$3.44/MWh to zero. Of course, the value loss is zero when there is agreement on the top-ranked alternative determined by both the assumed correct weights and the weights that are used. The most extreme values losses occur for each subject when the hierarchical weights are used but the non-hierarchical weights are assumed to better reflect the subject's preferences. It is interesting to note that, except for subject eleven, there are no value losses incurred when the debiased (or hierarchical) weights are used and the hierarchical (or debiased) weights are assumed to be correct. The results in Table 3-9 indicate that the value tree-induced bias can have a large effect on both the rankings and the loss of value due to the use of incorrect weights. Thus, it is important to develop a method to reduce the influence of the bias to prevent these value losses from occurring.

3.6 Conclusion

This chapter developed a model for addressing biases that occur when using value trees to elicit weights. Previous research suggests that the use of the anchor-and-adjust heuristic during direct weight elicitations may be responsible for value-tree induced biases. The model-based approach developed in this study aims to debias attribute weights by assuming that expressed judgments of relative weights are a convex combination of the equal allocation weights and a set of weights reflecting the subject's innate preferences. This model formulation incorporates the influence of the value tree structure, as well as both components of the anchor-and-adjust heuristic: the amount of influence that the anchor (equal allocation weights) has on the elicited weights is reflected by the bias parameter, which is simply the weight in the convex combination; and the adjustment step is modeled as the influence that the innate preferences have on the elicited weights.

Summarizing the empirical results, nine of the eleven case study subjects displayed the hypothesized bias of flatter, less varied weights for a non-hierarchical assessment. Additionally, the model fit the data well, with an average R^2 of 0.97. While the results of the model-fitting support the conjecture regarding the anchor-and-adjust heuristic, the small sample size means that generalizations cannot be made about how the model will perform in other situations. Additional experiments with larger samples are desirable; nevertheless, case studies in a realistic context, such as ours, can be valuable for empirically testing models (Adelman 1991; Yin 2003). A case study "allows investigators to retain the … meaningful characteristics of real-life events" (Yin 2003). The use of real decision makers with realistic alternatives and objectives provides support for the existence of value tree-induced biases in practice, and for the feasibility of the proposed debiasing method.

In addition to a larger dataset, future work should examine the effect of elicitation method (e.g., top-down vs. bottom-up) on the bias, and should consider and control for the presence of other biases as well. In particular, a carryover bias may result in an increased similarity in the weights elicited using two different value trees. Decision makers may bias the weights in a second assessment by "carrying over" the weights that were expressed with the first value tree. Lastly, future work should characterize the statistical properties of the least-squares estimates. Because the estimation is an inequality-constrained optimization, numerical methods are promising for this purpose (Geweke 1996), and they could be employed to quantify the uncertainty due to sample error in the estimates of splitting biases and debiased weights.

Despite the above limitations, this study provides a theoretically-based framework for debiasing attribute weights that suffer from value tree-induced biases, such as the splitting bias. When time or other limitations prevent follow-up interviews with subjects, the proposed model-based approach provides a practical means of debiasing weights so that they better reflect decision maker's innate preferences.

Chapter 4

A Bayesian Framework for Cost Effective Management of Sediment Reduction in the Minnesota River Basin

Water quality impairments remain a pressing concern in the United States. Rural non-point sources have been particularly difficult to control, with relatively little progress compared to point sources since the passage of the Federal Water Pollution Control Act Amendments of 1972. Selecting appropriate management actions (i.e., best management practices) to control pollution is made difficult by large uncertainty in the location and magnitude of pollution sources as well as in the effectiveness of different management actions for reducing pollutant discharge from the watershed. To address these concerns, this chapter presents a framework to select the optimal combination of research actions, which improve our understanding of the natural system, and management actions, which reduce pollutant loading. The method uses a combination of Bayesian inference and multiobjective linear programming to explicitly consider uncertainty in both research and management actions. The usefulness of the model is illustrated using the problem of reducing turbidity from rural nonpoint sediment sources in the Minnesota River basin.

4.1 Introduction

Nonpoint source pollution is the largest contributor to water quality impairments of surface waters in the United States (U.S. Environmental Protection Agency 2003). Sediment is one of the major nonpoint source pollutants and results from erosion. Agricultural lands are the most wide-spread source of sediment reaching rivers and lakes, while streambanks, construction sites, and runoff from urban, residential and forested areas also contribute sediment (ibid.). Soil loss can deplete nutrients in the soil and deteriorate soil structure, which decreases the productivity of the land. Sediment can also have a detrimental effect on aquatic ecosystems. When the sediment reaches a water body, it increases turbidity, which limits the amount of sunlight reaching aquatic plants. Sediment can cover fish spawning and nursery habitats, interfere with fish respiration by clogging gills, and inhibit growth of submerged aquatic vegetation. The result can be decreased productivity of the water body receiving the sediment due to decreased primary and secondary production, leading to reduced fish abundance. On the other hand, the nutrients associated with sediment loss may increase algal productivity, increasing eutrophication problems.

Reductions in sediment loading are recommended for improving many impaired water bodies; however, as a nonpoint source pollutant, sediment reaches the water body from many diffuse sources and the contribution from each source is difficult to quantify with certainty. Despite the difficulty of determining sediment contributions from various sources, a large number of management practices have been developed for reducing sediment. For agricultural lands, managing sediment includes changes in cultivation methods, such as different tillage practices, and structural actions, such as terracing. For sediment produced from streambank or hillside erosion, management practices often aim to redirect streamflow, protect banks from erosion, or develop riparian buffers. Sediment produced from gullies and ravines is often addressed through bank protection or soil stabilization and revegetation.

Choosing among the wide range of available management options is further complicated by the many parties involved including farmers, various land owners, soil & water conservation districts and regulators at the local, state, and federal levels. Furthermore, the uncertainty surrounding the sediment contribution from each source means that measures taken to reduce sedimentation can be ineffective if the wrong sources are addressed. Applied research can reduce the uncertainty in the estimates of sediment sources. There is a tradeoff between research and management: further studies may improve estimates of sediment sources and support more accurate targeting of management measures, but can be time-consuming and costly, resulting in delays in clean up.

A variety of research methods can be used to estimate sediment loadings. Erosion rates for agricultural fields are often estimated by the Universal Soil Loss Equation (USLE) or the Revised Universal Soil Loss Equation (RUSLE) version 1 (Renard et al. 1997) and 2 (RUSLE2) (Foster et al. 2003). These estimates are most appropriate for edge-of-field sediment supply, and application to larger areas brings in additional sources of sediment, as well as the opportunity to deposit field-derived sediment. The USLE tends to overpredict soil losses in areas with low erosion rates and underpredict soil losses in areas with high erosion rates (Risse et al. 1993; Kinnell 2005). Sediment yields can be measured using flow and sediment concentration observations from stream gages. An emerging methodology is sediment fingerprinting ((Walling and Woodward 1992), which uses chemical or isotopic information to trace sediment sources and can typically provide an estimate of the proportion of sediment exposed at or near the ground surface, and is thus used to identify the proportion of sediment derived from agricultural fields. Sediment loadings and yield can also be estimated in the context of a sediment budget analysis, which combines a range of field and remote sensing methods to identify sources, sinks, and fluxes of sediment (see Reid and Dunne 1996). By invoking mass conservation of sediment for a defined watershed and time period, a sediment budget allows the uncertainty in estimates of sediment sources to be evaluated (Gran et al. 2009). The sources of uncertainty are many, including error inherent in specific methods, extrapolation from monitored sites, unknown future weather and land use conditions, and the proportion of eroded sediment that is subsequently redeposited and remains in the watershed.

In addition to the uncertainty surrounding sediment loadings, other sources of uncertainty further complicate the management of sediment. Management actions subject to uncertainties include the effectiveness of an action in reducing sediment loading, the extent of voluntary adoption rates, and the cost to implement and maintain the actions. The framework developed in the next section directly addresses the problem of choosing among sediment reduction management actions in rural watersheds in the face of several of these sources of uncertainty, namely uncertainty in sediment loadings, predictive abilities of research methods, and the cost and effectiveness of management action.

Using Bayesian inference and multiobjective linear programming, the optimal set of information acquisition, or research actions, and management actions are chosen to minimize expected cost and maximize expected sediment reduction. By placing different weights on costs and sediment, alternative research and management plans can be derived and the resulting tradeoffs between those two objectives can be described. The value of information of each research action is also quantified.

To test the applicability of the framework, the problem of reducing sediment in the Minnesota River basin is addressed, with a focus on the Maple River subbasin. The Minnesota River frequently violates federal or state water quality standards for a variety of pollutants including nutrients and turbidity (Mallawatantri 1999). In 2001, the Minnesota Pollution Control Agency published the Minnesota River Basin Plan (2001), which described the state of the river basin and identified priorities for addressing water quality impairments. Regulations based upon the concept of "total maximum daily loads" (TMDLs) are being developed to place an upper limit on the amount of sediment reaching the waterways (Minnesota Pollution Control Agency 2009). However, for sedimentation, the relevant time scale is seasonal or yearly, since the concern is not so much with instantaneous concentration of sediment but with accumulated loadings over time upon sensitive ecosystems. Nevertheless, the sediment reduction planning process is still generally referred to as a TMDL. How the TMDLs will be met has yet to be determined. The framework developed here will provide a tool to assist in developing a TMDL implementation plan to meet sediment reduction goals.

The remainder of this chapter is organized as follows. Section 4.2 provides a literature review. Section 4.3 presents the framework for evaluating research and management actions for reducing sediment. This framework has two main components – a Bayesian inference model that determines the impact of each research, or information acquisition action, and a multiobjective linear program for selecting among management actions. Section 4.4 presents the Minnesota River Basin case study. The results are detailed in section 4.5 and section 4.6 presents a discussion and conclusions.

4.2 Literature Review

Previous environmental systems approaches for sediment reduction began with deterministic optimization approaches to nonpoint source pollution reduction dating back to the

1970s. That research sought to determine the effects of different soil erosion control policies on the agricultural industry. Wade and Heady (1977)evaluated five alternative policies to control sediment to rivers and streams in the United States. The authors used a linear programming (LP) model that minimized agricultural production costs subject to constraints on crop and livestock production, commodity demands, sediment delivery, and soil losses per acre. The decision variables described the crop rotation, tillage, and conservation practices needed to meet different policy requirements. The problems were modeled as deterministic linear programs. Sediment transport was described by fixed transport coefficients derived from the Universal Soil Loss Equation (USLE) with constant sediment delivery ratios describing the amount of eroded sediment that ultimately reaches a downstream targeted area. Uncertainties were not explicitly incorporated in the modeling.

Seitz et al. (1979) extended the findings by Wade and Heady (1977) by summarizing several analyses that investigated the economic impacts of soil erosion control policies. The authors used a comparative static linear programming (LP) model of the production and marketing of several corn-belt crops to evaluate the effect of various taxes, subsidies and soil loss constraints. The objective of the LP was to maximize consumer and producer surpluses in the corn and soybean markets, less the variable cost of producing small grains, hay and pasture. Different crop production activities were modeled using various crop rotation and conservation practices, such as terracing and straight-row planting. Various tillage methods, such as chisel plowing, were also included in the model. Again, uncertainties were not incorporated in the model and soil loss was estimated by fixed soil loss coefficients derived from the USLE.

Kramer et al. (1984) also developed a linear program (LP) to evaluate agricultural policies for reducing nonpoint pollution. Their model, like the one I develop here, included BMPs not tied to specific agricultural activities, such as grassed waterways, as well as land cultivation practices. Braden et al. (1989) improved on the models described by Seitz et al. (1979) by incorporating spatial sediment movement functions that allow variable sediment delivery ratios and account for interim storage of sediment. The problem was solved using a dynamic programming nonlinear model to generate a full abatement cost frontier, which identifies the least costly management regime for each possible sediment loading level. Bouzaher et al. (1990) also developed a dynamic programming model. Their model minimized the cost of achieving a sediment standard by identifying the best set of management alternatives for each sediment delivery path.

Schleich and White (1997) developed an LP that minimizes total phosphorus reduction costs such that a target total phosphorus standard is met. The model also considered total suspended solids (TSS) reduction; however, the cost of reducing TSS was not available, so TSS reductions were considered by recognizing that a reduction in total phosphorus will also reduced TSS. The decision variables in the model were management actions associated with five source categories generating phosphorus and TSS in 41 watersheds. The sources examined were municipal treatment plants, industrial point sources, construction runoff, urban storm runoff, and nonpoint agricultural sources. The authors found that it was most cost-effective to address agricultural sources.

Veith et al. (2003) developed a deterministic integer-programming model for locating BMPs at the watershed level. The authors represented the BMPs with integer variables to reflect that the BMP was either fully implemented or not. They combined a genetic algorithm with a geographic information system to determine the most cost effective placement of BMPs. MILPs could not be used to find optimal solutions because the number of combinations of BMPs at the watershed scale was prohibitively large, resulting in an intractable problem that was not guaranteed to solve to optimality in a finite amount of time. Gitau et al. (2004) built upon the work by Veith et al. (2003) by combining an upgraded version of the genetic algorithm developed by Veith et al. (2003) with a watershed level nonpoint source model and a BMP tool that summarizes literature on BMP costs and effectiveness. The authors used this framework to determine optimal BMP placement for nonpoint source phosphorus reduction from agricultural lands.

The deterministic models described above assume that loadings and control effectiveness are known with certainty. However, the actual values are highly uncertain and even controversial, as in the Minnesota basin (University of Minnesota Extension Service 1996; Steil 2004; Gupta et al. 2001). To address such uncertainties, several stochastic programming models have been developed to address sediment reduction. Chance-constrained programming models (based on the Charnes-Cooper framework) were developed by several authors. Milon (1987) and Zhu et al. (1994) maximized net economic returns to landowners subject to constraints requiring environmental standards be met with a user-specified reliability. The loadings of pollutants (Milon 1987) or soil loss (Zhu et al. 1994) to receiving waters were assumed to be uncertain due to variability in precipitation. However, average soil loss rates were assumed to be known with certainty. Kampas and White (2003) combined a geographical information system (GIS) to classify land classes based on soil properties with biophysical models to simulate the nitrogen cycle in agricultural systems, and a chance-constrained optimization model to analyze different policy approaches to controlling nitrogen pollution from agriculture. Lacroix et al. (2005) also combined chance-constrained programming models with simulation models to address nonpoint source nitrogen pollution. The authors evaluated several nitrogen reduction scenarios using an approach similar to Kampas and White (2003) that coupled chance-constrained programming with biophysical nitrogen models; however Lacroix et al. (2005)used Monte Carlo simulation to evaluate the scenarios due to limitations that prevented analytical derivation of a deterministic equivalent constraint.

Instead of chance constrained programming, Luo et al. (2006) developed an interval two-stage stochastic program that minimized total system costs by choosing cropland to retire. The model they developed combined an interval two-stage stochastic program with a distributed water quality simulation model to quantify the randomness and spatial variation in agricultural nonpoint source pollution control through land retirement. The first-stage decision was the quantity of agricultural land to retire, which was traded off with the second-stage or recourse decision of the amount of nonpoint source pollution to be discharged from the non-retire lands.

Rather than considering variability in streamflows and runoff, Yulianti et al.(1999), considered the effects of uncertainty in input parameters for a simulation/optimization model for selecting agricultural management actions to control sediment transported to surface waters. The authors considered two uncertain inputs – uncertainty in soil and land characteristics due to insufficient data to represent the spatial variability, and uncertainty in meteorological data, cost of production, price of produce, and soil loss and crop yield parameters that fluctuate with time and are unknown in the future. Monte Carlo simulation, generalized sensitivity analysis and regret analysis were used to determine management practices that were most and least sensitive to uncertainties considered.

Nicklow and Muleta (2001) addressed sediment reduction using a discrete-time optimal control framework. The research determined farm management decisions over a three-year period. Sediment yields over the time period were simulated with a process-based distributed routing model, Soil and Water Assessment Tool (SWAT), using spatially and temporally variable environmental factors like climate variables and topography. A genetic algorithm was used to determine the set of land cover and tillage practices that result in the minimum sediment yield from the watershed.

To improve understanding regarding the contribution of various sediment sources, Kaplan and Howitt (2002) developed a sequential entropy filter to update sediment loading parameters from streamflow data. This work is related to ours in that knowledge is revised in response to acquired data, unlike the previously mentioned stochastic optimization papers. However, their research is descriptive, not prescriptive, in that it does not use the improved sediment loading information to influence management policies. As a result, they could not quantify the value of better information in terms of improved outcomes from management.

More recently, the question of value of information in rural nonpoint pollution control has begun to be analyzed. Kaplan et al. (2003), Farzin and Kaplin (2004), and Borisova et al. (2005) considered the value of information with regards to sediment loadings.

Kaplan et al. (2003) developed a method to determine the optimal combination of data collection frequency and abatement efforts for reducing nonpoint source pollution subject to a budget constraint. The authors used a sequential entropy filter that provides improved estimates of sediment loadings. It was assumed that managers know the location and size of sediment sources, but there is uncertainty about the quantity of sediment generated from each sediment source. To reduce this uncertainty, daily stream flow and ambient sediment load data is collected with the use of stream gages. Two scenarios of data collection frequency were tested and the updated information from each scenario was then combined with a model to choose the optimal abatement level, defined in the paper as the miles of logging road to remove in Redwood National Park.

Farzin and Kaplin (2004) also developed a framework to address sediment loading from forestland in northwestern California, but use a Bayesian approach. During rain events, runoff can overflow stream channels at road crossings, causing sediment to enter tributaries. The authors develop a theoretical framework to determine the optimal combination of data collection frequency and abatement efforts for reducing sediment loadings subject to a budget constraint. Data collection frequency was defined as the number of samples of stream flow and ambient sediment loads collected daily, and abatement was defined as removing haul roads. The streamflow and sediment data was used to update the manager's prior probability of sediment loading from two hypothetical sources using a Bayesian approach. The authors also considered the expected value of perfect information (EVPI). The value of perfect information is defined as the difference in environmental damages incurred if the manager knows the sediment loads from each source with certainty and accordingly makes optimal decisions about road removal.

Similarly, Borisova et al. (2005) calculated the EVPI of various types of information for reducing nitrogen loads from agricultural land to the Chesapeake Bay. The objective was to maximize net benefits of pollution control subject to a variety of policy constraints. A simulation model, composed of an economic agricultural production submodel and a physical submodel, considered three sources of uncertainty: 1) uncertainty about economic costs of changes in agricultural production to reduce nitrogen loads; 2) uncertainties in nitrogen loads due to changes in agricultural practices; and 3) uncertainty in economic benefits from nitrogen load reductions. Both quantity controls and price control policies were examined. Quantity control policies placed limits on nitrogen fertilizer applications and the total amount of land in nitrogen fertilizer intensive crops. Price control policies placed a per unit cost on land and fertilizer. The authors quantified the value of perfect information for these quantity and price controls; they did not consider the value of imperfect information provided from research efforts.

Bayesian inference has also been used to reduce uncertainty in sediment source estimates in the context of sediment fingerprinting. Sediment fingerprinting uses physico-chemical properties of sediment as tracers to identify sediment sources, but is subject to several sources of uncertainty including variability within each sediment source group (i.e. agricultural lands, bluffs, ravines, etc.), variability due to limited sample size, and uncertainty about the tracer's ability to distinguish between source groups. Several studies have used Bayesian statistical models to update prior beliefs about proportions of sediment from various sources using observed sediment fingerprinting data (Small et al. 2002; Rowan et al. 2001; Rowan et al. 2001; Douglas et al. 2003; Caitcheon et al. 2006). While these papers use Bayesian inference to reduce uncertainty about sediment sources, the information is not used to inform management decisions.

The research proposed here is most closely related to the research by Farzin and Kaplin (2004) and Kaplan et al. (2003). Like their work, the model developed in the next section determines the optimal combination of information acquisition and abatement. The impacts of uncertainty in sediment loads are explicitly considered and the benefits from considering this uncertainty are calculated. However, the research here considers a range of sediment sources, not just a single source such as agricultural lands or logging roads. This work considers four possible sediment sources: agricultural lands, streambanks, ravines and bluffs. In addition to agricultural BMPs, the work here considers additional management actions including the stabilization of streambanks, ravines, and bluffs. Lastly, the most significant difference between the work proposed here and previous work is the inclusion of various uncertainty reduction measures. In particular, three types of learning actions are considered for reducing uncertainty regarding sediment loads: stream gauging, sediment fingerprinting and a modified sediment budget. Previous studies have considered at most only one type of uncertainty reducing action. The work here combines information gathered through learning with an optimization program to choose among different types of management actions to reduce sediment loadings. Bayesian inference and Gibbs sampling is used to improve understanding of sediment loads from various sources and multiobjective linear programming is used to choose among management actions with explicit consideration of tradeoffs between cost minimization and sediment reduction. This work contributes to the literature by synthesizing previous work through the development of a novel sediment reduction framework and by considering a more realistic representation of sediment reduction management incorporating uncertainty.

4.3 Methodology

The framework presented here combines expert elicitation, Bayesian inference, and multiobjective linear programming to determine the optimal combination of research actions and management actions to address sediment reduction.

4.3.1 Expert Elicitation

First, expert elicitation is used to obtain prior information regarding scientists' current understanding of the natural system as well as the quality of information produced by research actions used to improve this understanding. Bayesian inference is then used to investigate the impact research can have on improving the understanding of the system, quantified as posterior probabilities. Lastly, multiobjective linear programming (MOLP) is used to determine the optimal suite of management actions to employ based on the expected levels of loadings, as inferred from the research actions. MOLP is also applied under the base case of no additional information. Overall, the optimal portfolio of research and management actions is selected based on the objectives laid forth in the linear program, balancing their cost and effectiveness in limiting sediment loss. The three components of the methodology are detailed below.

The first component of the framework involves eliciting expert judgment, which is useful when data is either unavailable in any other format, or too costly to obtain (Meyer and Booker 2001). When managing sediment, it may be the case that scientifically derived values of sediment loadings are not yet readily available or are disagreed upon by different researchers, as is the case in the Minnesota River Basin. In addition to estimates of sediment loadings, information describing the accuracies of various methods aimed at improving the understanding of the system may be lacking. Eliciting expert judgment can help inform both the state of the system in terms of current sediment loadings, as well as describing the accuracy of research methods.

There are a variety of techniques for eliciting probabilistic information from experts (see Meyer and Booker 2001 for review), as well as a large body of literature investigating biases that can occur during the elicitation process (see Clemen 2008 for a review). The choice of expert elicitation technique depends on a variety of factors including the type of information sought and the amount of resources available for the elicitation. This work takes advantage of active research being conducted to identify sediment sources in support of a turbidity TMDL for the Minnesota River Basin (Minnesota Pollution Control Agency 2009). One-on-one, in-person interviews with experts are used to collect information about current estimates of sediment loadings from a variety of sediment sources (as joint prior distributions); to provide an inventory of research actions used to learn more about the sediment loadings; to describe how accurate the information from these actions are (in the form of likelihood distributions); and to provide an inventory of applicable management actions to reduce sediment loadings. The details of the elicitation and data collected as it relates to the case study are presented in the next section.

The second component of the methodology is Bayesian inference. Due to limited scientific knowledge, sediment loadings are uncertain. To manage sediment, decisions must be made in the face of this uncertainty. Research actions allow decision makers to learn more about the system, and thus reduce uncertainty. In order to select the best research and management actions to employ, Bayesian inference can be used to update prior distributions of sediment loadings using information obtained from the outcomes of the research actions, which result in posterior distributions that may lead to different management decisions.

A Bayesian approach to address uncertainty makes use of all available information. In particular, new information is combined with previous information using Bayes' theorem. Previous information regarding an uncertain quantity of interest θ, for example annual sediment loading (or a vector of loadings from different sources), is summarized with a prior probability distribution, $f_\theta(\theta)$. Research actions, or data generating processes, result in observations, z, that tell us something about the uncertain quantity. The probability or likelihood of observing a particular observation is summarized using a likelihood function, or conditional probability distribution, that describes the probability of an observation conditioned on the uncertain quantities of interest, $f_{z|\theta}(\boldsymbol{z}|\boldsymbol{\theta})$. Bayes' theorem combines the prior distribution with the likelihood function to produce a posterior distribution that summarizes the new information about $\boldsymbol{\theta}$ based on the observation from the research action:

$$f_{\theta|z}(\boldsymbol{\theta} \mid \boldsymbol{z}) = \frac{f_{\theta,z}(\boldsymbol{\theta}, \boldsymbol{z})}{f_z(\boldsymbol{z})} = \frac{f_\theta(\boldsymbol{\theta}) f_{z|\theta}(z \mid \boldsymbol{\theta})}{f_z(\boldsymbol{z})} \tag{4.1}$$

Information from the posterior distribution can then be used to inform management decisions. The prior distribution and likelihood functions are parameterized based on the information derived during the expert elicitation step described previously. The next section provides further details on how Bayesian inference combines expert judgment about the state of the natural system with the outcome of research actions to better inform management decisions aimed at reducing sediment loadings.

The last component of the methodology is multiobjective linear programming, which is used to select among the possible sediment reduction management actions. Multiobjective programming is used because there are two competing objectives – minimizing expected cost and minimizing expected sediment remaining in the system.

The problem of selecting management actions is a quite complex spatial problem. Previous research has addressed sediment reduction under uncertainty with a variety of approaches, as discussed in the literature review, section 4.2. The multiobjective linear program component of this framework is simpler than many of the more advanced techniques discussed above. This simplified approach for selecting management actions was selected to illustrate the framework in a more tractable way; however, the framework is flexible and it is possible to replace the multiobjective linear program with a more complicated technique such as chance-constrained programming to select among management actions.

The multiobjective linear program (MOLP) seeks the optimal set of management actions that minimize a weighted sum of expected cost and expected sediment loss remaining after application of controls. For each research action and observation, the expected loadings from the posterior distribution is input into the MOLP and a tradeoff curve of expected cost versus expected sediment remaining is found using the weighting method (Cohon 2004) by varying the weight placed on the sediment remaining objective. Constraints are placed on the amounts of each management action that can be selected, as well as the amount of sediment addressed by each management action. The details of the multiobjective linear program in the context of the case study are presented in the section 4.4.4.

4.3.2 Integration of the methodology components

In order to determine the optimal combination of research and management actions, a backwards induction procedure is used to integrate the expert judgment, Bayesian inference, and MOLP components of the model. For a given sediment reduction objective weight in the MOLP, there is an optimal choice of research action, and for each research action observation, an optimal portfolio of management actions. First, for each observation that occurs for each research action, the optimal management actions are determined from the MOLP and the resulting optimal value of the weighted sum of expected cost and expected sediment remaining is calculated. For each research action, the expectation of the optimal objective is calculated by weighting the optimal objective for each observation, by the probability of the observation occurring. The optimal research action (if any) is chosen by minimizing the sum of this expectation and the cost of the research action.

To evaluate the usefulness of the information produced from performing research, the expected value of imperfect information is computed for each action and each sediment objective weight as the improvement in the expected objective function relative to the no-information solution. First, the optimal objective function value from the MOLP is determined for the case when no research action is performed. In this case, the expected values from the prior distribution are used as sediment loading inputs into the MOLP. Then, for each research action and observation, the optimal objective function value is found based on the expected sediment loadings from the posterior distribution. The expected value of imperfect, or sample, information is found by subtracting the optimal objective function value under the no research scenario from the probability weighted objective function value under the research action. The details of the value of information study as applied to the case study are presented in the next section.

As discussed in the literature review section (4.2), previous work has focused on determining the optimal combination of management actions to reduce sediment from agricultural lands, and on evaluating the value of information from data collection in the context of sediment loading from logging roads. The methodology presented here expands the literature in several ways. First, this methodology considers sediment originating from four main sources, agricultural lands, streambanks, ravines and bluffs, and the management actions to reduce this sediment. To my knowledge, no previous work has considered sediment reduction management of streambank, ravine or bluff erosion in conjunction with soil losses from agricultural lands. Secondly, this work considers a variety of research actions used to reduce uncertainty about sediment loadings. Previous work has considered either the use of gauging data to reduce uncertainty, or sediment fingerprinting, not both together. In addition, the sediment fingerprinting research has not incorporated uncertainty reduction with management actions. Third, this research combines Gibbs sampling with multiobjective linear programming, which to my knowledge has not been combined in the context of sediment reduction management. Overall, this framework better addresses the complexity of managing sediment by considering multiple sediment sources prone to uncertainty, multiple research actions, and a broader range of management actions to reduce sediment.

4.4 Case Study

The methodology described in the previous section is used here to determine the optimal set of information acquisition and management actions for controlling sediment loadings in the Minnesota River Basin. Because of the large number of possible combinations of information and management actions, the framework is applied to a simple three watershed model. Input values for the model watershed were developed for the Maple River watershed, a subwatershed of the Le Sueur River watershed within the Minnesota River Basin (Figure 4-1).

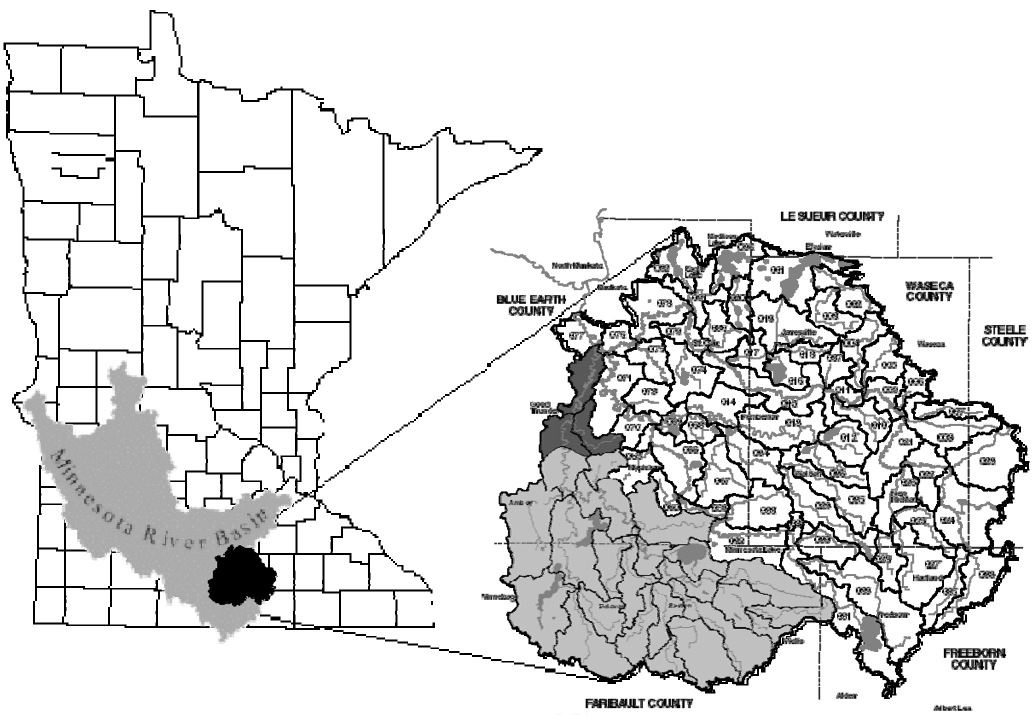

Figure 4-1: Maple River Watershed (gray) within Le Sueur River Watershed. Source: Minnesota River Basin Data Center

The watershed is divided into an upper watershed, shown in lighter gray, and a lower watershed, shown in darker gray (Figure 4-2). The upper watershed is divided into two duplicate watersheds, W1 and W2, which drain into watershed W3. This division was done to investigate the impact of actions applied to a portion of the full watershed when the location of sediment loadings is uncertain. The upper two watersheds each have two possible sources of sediment: agricultural fields and streambanks, whereas the lower watershed has four sources: fields, streambanks, ravines and bluffs.

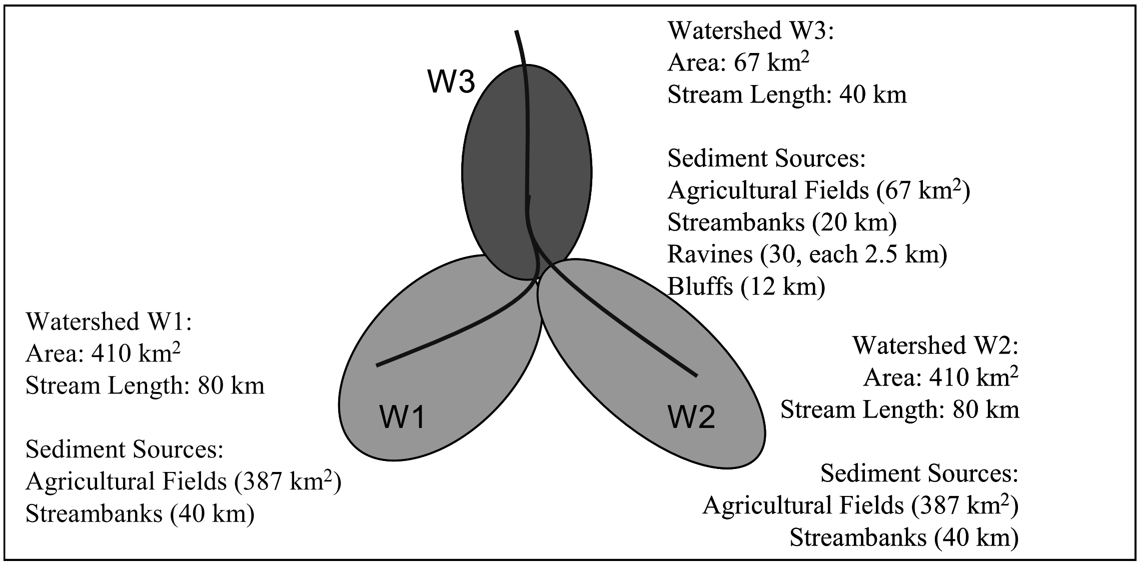

Figure 4-2: Stylized three watershed model. Watersheds W1 & W2 are upstream of watershed W3

There are two categories of actions that decision makers must choose among – research actions that provide information about the physical system, and management actions that reduce sediment loading. The framework for selecting actions is summarized in the decision tree in Figure 4-3. The squares in the diagram indicate decision nodes and the circles indicate chance nodes, in which the outcome is uncertain. The branches originating from the squares correspond to choices available to the decision maker and the branches from the circles show outcomes of uncertain events. Time proceeds from left to right, with the information acquisition decisions being made first, followed by the uncertain outcomes of the actions. The outcomes of the information acquisition actions are then used to inform decision about which management actions to choose. While not displayed explicitly in the figure, the outcome of the management actions are uncertain, due to uncertainty in the sediment loadings, upon which management decisions are made.

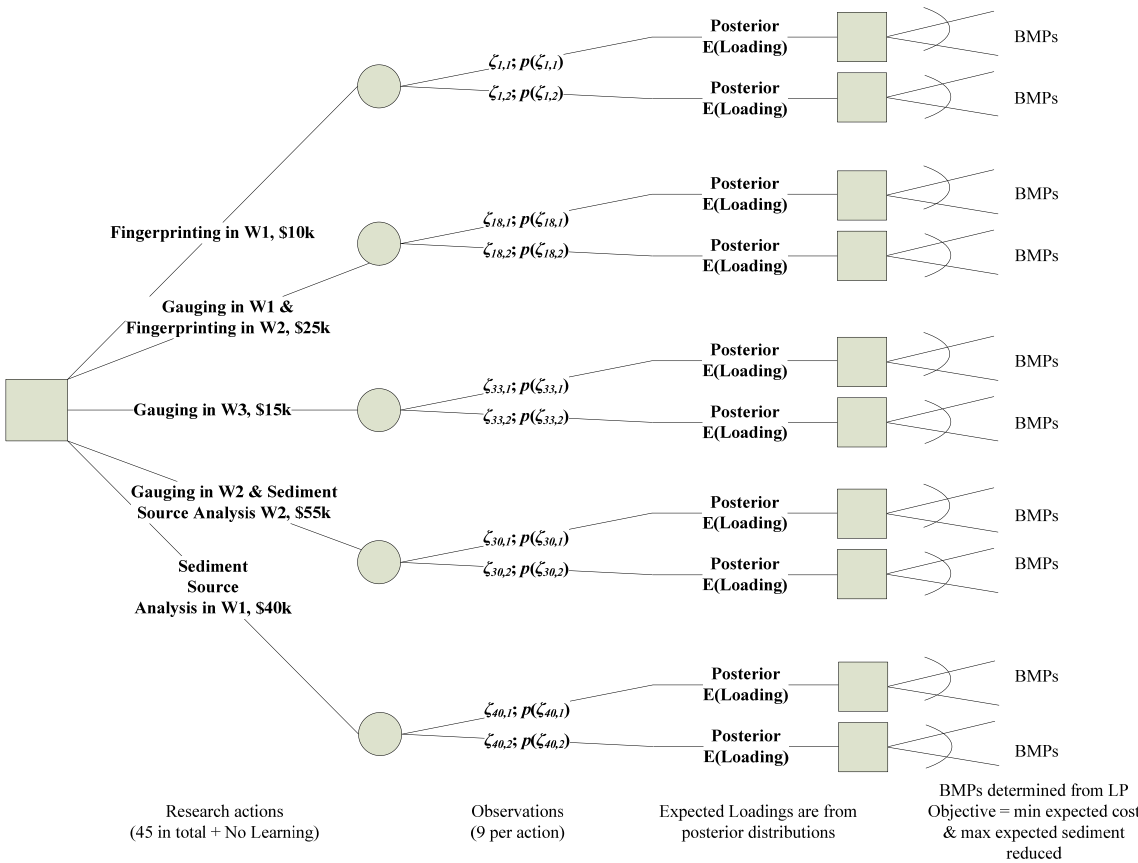

Figure 4-3: Sample of Decision Tree Illustrating Decision Framework

The methods described in the methodology section (4.3) – expert elicitation, Bayesian inference, and multiobjective linear programming – are used to produce an optimal research and management strategy. In order to implement this methodology, the following information is needed: parameters for the prior distribution describing the sediment loadings, the costs and likelihood function parameters for each research action, the discretized distributions of the observations resulting from the research actions, and the cost and effectiveness of each management action.

4.4.1 Prior Information

The first step in applying the framework presented in section 4.3 is to interview experts to identify the current state of knowledge about the system. A one-on-one in-person interview was conducted with Dr. Patrick Belmont (personal communication, July, 2, 2008), a postdoctoral research associate with the National Center for Earth Surface Dynamics (NCED), whose work includes developing a sediment budget for the Le Sueur River watershed. During the interview, the expert was asked to provide a value for the long-term annual average sediment load for each of the eight sediment sources shown in Figure 4-2. In addition to the average loading for each source, the expert was also asked to provide a range describing the 95% confidence interval around the average. The expert's response was then read back, and the expert had a chance to modify the response. It was assumed that the sediment loadings provided by the expert represent sediment exiting the watershed. Since a portion of the sediment exiting watersheds W1 and W2 may be stored in W3 before exiting the system, the expert also provided an estimate of the sediment delivery ratio (SDR), which is the proportion of sediment entering the top of W3 that exits W3.

For each source, the correlation of the loading from that source compared with each other source was also elicited. This was done by asking Dr. Belmont if the correlation in loadings was zero, low, medium or high. The qualitative correlations were converted into the following numbers: low was set to 0.2, medium to 0.6, and high to 0.85. The responses from the expert elicitation are presented Table 4-1 and Table 4-2. The expert (P. Belmont, personal communication, May 6, 2009) felt that field contributions in watersheds W1 and W2 were strongly correlated due to the fact that field erosion in both watersheds

Table 4-1: Sediment Loadings and Sediment Delivery Ratio from Expert Elicitation

Watershed	Source	Average Annual Sediment Loading or SDR	95% CI
W1	Field	9000 (tons/yr)	7000-9900 (tons/yr)
	Streambanks	1000 (tons/yr)	100-3000 (tons/yr)
W2	Field	9000 (tons/yr)	7000-9900 (tons/yr)
	Streambanks	1000 (tons/yr)	100-3000 (tons/yr)
W3	Field	7000 (tons/yr)	3500-10500 (tons/yr)
	Streambank	7000 (tons/yr)	0-14000 (tons/yr)
	Ravines	28000 (tons/yr)	14000-42000 (tons/yr)
	Bluffs	28000 (tons/yr)	14000-42000 (tons/yr)
	SDR	75%	65-95%
Total Expected Annual Loading		90,000 (tons/yr)	

is driven by similar processes such as hydrology and woody debris presence. The correlations of field contributions in each upper watershed with field contributions in the lower watershed are only moderately correlated, reflecting that the upper and lower watersheds are different in their connectivity to the river system and topographic gradient. In general, contributions that are correlated tend to have similar hydrologic, soil, and land-use controls of sediment production. Contributions with no correlation typically generate sediment based on different processes. For example, field erosion in the upper watersheds is driven by precipitation and surface runoff, whereas streambank erosion in the lower watershed driven by a mix of factors that control the configuration and hydraulics of the stream channel.

Table 4-2: Correlations from Expert Elicitation

	Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3
Stream W1	Low						
Field W2	High	Low					
Stream W2	Low	Low	Low				
Field W3	Med	Low	Med	Low			
Stream W3	Zero	Med	Zero	Med	Zero		
Ravine W3	Med	Zero	Med	Zero	Med	Low	
Bluff W3	Med	Low	Med	Low	Med	Low	Med

The information gathered from the expert interview was then used to construct a joint prior probability distribution of sediment loadings. The loadings from each sediment source were assumed to follow a log-normal distribution. This distribution was chosen because it assigns a probability of zero to non-positive loadings and allowed for easier computations of posterior distributions. Using the annual average sediment loadings and confidence interval ranges provided by the experts, the parameters of the marginal log-normal distributions were calculated for each source and watershed by solving the following system of equations for $\mu_{i,j}$ and $\sigma_{i,j}$

$$\int_L^H \frac{1}{\tilde{x}_{i,j}\sigma_{i,j}\sqrt{2\pi}} \exp\left[\frac{-(\ln(\tilde{x}_{i,j})-\mu_{i,j})^2}{2\sigma_{i,j}^2}\right] d\tilde{x}_{i,j} = 0.95 \tag{4.2}$$

where

$I :=$ $\{i \mid i = F, S, R, B\}$ is the index set of sediment sources (Field, Streambank, Ravine, Bluff)

$J :=$ $\{j \mid j = W1, W2, W3\}$ is the index set of watersheds

$\tilde{x}_{i,j} :=$ long term annual average sediment loading from source i in watershed j

$\mu_{i,j} :=$ mean of the natural log-transformed sediment loading from source i in watershed j

$\sigma_{i,j} :=$ standard deviation of natural log-transformed sediment loading from source i in watershed j

$E\left(\tilde{x}_{i,j}\right) :=$ average annual sediment loading in Table 4-1

$L :=$ lower bound of the 95% confidence interval

$H :=$ upper bound of the 95% confidence interval

The joint distribution of sediment loadings was found based on the marginal distribution parameters, the correlations elicited from the experts, and by recognizing that taking the natural logarithm of a log-normally distributed random variable results in a normally distributed random variable. Thus, the joint distribution of sediment loading was found to be joint normal in terms of the natural log of the original sediment loading random variables.

$$f_{\tilde{\mathbf{y}}}(\tilde{y}_{FW1}, \tilde{y}_{SW1}, \tilde{y}_{FW2}, \tilde{y}_{SW2}, \tilde{y}_{FW3}, \tilde{y}_{SW3}, \tilde{y}_{RW3}, \tilde{y}_{BW3}) = \frac{1}{(2\pi)^4 |\Sigma|^{-1}} \exp\left(-\frac{1}{2} (\tilde{\mathbf{y}} - \boldsymbol{\mu})^{\mathrm{T}} \Sigma^{-1} (\tilde{\mathbf{y}} - \boldsymbol{\mu}) \right), \tag{4.3}$$

where

$\tilde{\mathbf{y}} :=$ vector of natural log-transformed long term annual average sediment loadings with elements $\tilde{y}_{i,j}$

$\tilde{y}_{i,j} :=$ $\ln(\tilde{x}_{i,j})$ defines log-transformed loading from source i in watershed j

$\boldsymbol{\mu} :=$ vector of means of natural log-transformed sediment loadings, with elements defined as $\mu_{i,j}$ for source i in watershed j

$\rho_{i,j,i',j'} :=$ correlation coefficient between natural log-transformed sediment loading of source i in watershed j and source i' in watershed j' (see Table 4-2)

$\Sigma :=$ covariance matrix of natural log-transformed sediment loadings with elements $\Sigma_{i,j,i',j'}$

$\Sigma_{i,j,i',j'} :=$ $\rho_{i,j,i',j'}\sigma_{i,j}\sigma_{i',j'}$ defines covariance between source i in watershed j and source i' in watershed j'

The values of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are calculated as

$$\begin{aligned}\boldsymbol{\mu} &= [\mu_{FW1}, \mu_{SW1}, \mu_{FW2}, \mu_{SW2}, \mu_{FW3}, \mu_{SW3}, \mu_{RW3}, \mu_{BW3}] \\ &= [9.10,\ 6.56,\ 9.10,\ 6.56,\ 8.82,\ 8.73,\ 10.21,\ 10.21] \quad \text{and}\end{aligned}$$

$$\Sigma = \begin{bmatrix} 0.004 & 0.010 & 0.003 & 0.010 & 0.009 & 0.000 & 0.009 & 0.009 \\ 0.010 & 0.687 & 0.010 & 0.137 & 0.043 & 0.247 & 0.000 & 0.043 \\ 0.003 & 0.010 & 0.004 & 0.010 & 0.013 & 0.000 & 0.009 & 0.009 \\ 0.010 & 0.137 & 0.010 & 0.687 & 0.043 & 0.247 & 0.000 & 0.043 \\ 0.009 & 0.043 & 0.013 & 0.043 & 0.067 & 0.000 & 0.040 & 0.040 \\ 0.000 & 0.247 & 0.000 & 0.247 & 0.000 & 0.246 & 0.026 & 0.026 \\ 0.009 & 0.000 & 0.009 & 0.000 & 0.040 & 0.026 & 0.067 & 0.040 \\ 0.009 & 0.043 & 0.009 & 0.043 & 0.040 & 0.026 & 0.040 & 0.067 \end{bmatrix}$$

The expert also provided the percentage of sediment exiting W3. Since this random variable is constrained to be between zero and one, a beta distribution was chosen and the SDR in W3 was assumed to be independent from the sediment loading variables. The full joint prior distribution, $f_{\tilde{\boldsymbol{\theta}}}(\tilde{\boldsymbol{\theta}})$, is found by combining the beta distribution for the sediment delivery ratio and the joint normal distribution for the sediment loadings

$$\begin{aligned} f_{\tilde{\boldsymbol{\theta}}}(\tilde{\boldsymbol{\theta}}) &= f_{\tilde{\mathbf{y}},\tilde{d}}(\tilde{y}_{FW1}, \tilde{y}_{SW1}, \tilde{y}_{FW2}, \tilde{y}_{SW2}, \tilde{y}_{FW3}, \tilde{y}_{SW3}, \tilde{y}_{RW3}, \tilde{y}_{BW3}, \tilde{d}) = \\ & \frac{1}{(2\pi)^4 |\Sigma|^{-1}} \exp\left(-\frac{1}{2} (\tilde{\mathbf{y}} - \boldsymbol{\mu})^{\mathrm{T}} \Sigma^{-1} (\tilde{\mathbf{y}} - \boldsymbol{\mu}) \right) \frac{\tilde{d}^{40.4} (1-\tilde{d})^{12.8}}{\mathrm{B}(41.4, 13.8)}, \end{aligned} \tag{4.4}$$

where $\tilde{d}$ is the random variable representing the SDR in W3 and theta is the vector describing the parameters of interest, $\{\tilde{\mathbf{y}}, \tilde{d}\}$.

4.4.2 Information Acquisition and Likelihood Functions

The next step in the framework is to determine the information acquisition actions and their associated likelihood functions. For each watershed, three information acquisition actions are considered: gauging, sediment fingerprinting or a sediment source analysis (SSA). This suite of actions and their associated expected cost was selected to reflect the current research conducted in the Minnesota River Basin and was acquired during interviews on January 9, January 20, February 6, February 9, and May 6 of 2009 with Dr. Peter Wilcock, who is an NCED principal investigator and professor at Johns Hopkins University specializing in erosion and sedimentation research.

For a particular watershed, gauging is defined as the placement of a single gauge at the watershed outlet and is estimated to cost \$15,000/yr. Over the course of the year, streamflow and sediment samples are collected 20-25 times over the seven month gauging season (April – November) and used to compute an annual average sediment loading for the watershed (Guy 1969). Sediment fingerprinting involves using atmospherically deposited radionuclides as tracers for sediment sources (Walling and Woodward 1992) . Over the year, twelve samples are collected and analyzed for cesium-137 and lead-210 to estimate the proportion of field derived sediment. The cost to collect the samples is estimated as \$2000, and the cost of analyzing the samples to determine the relative field contribution is \$1500/sample. Thus, a year long sediment fingerprinting study costs an estimated \$20,000/yr.

The last research action under consideration is a sediment source analysis action, which produces estimates of the loadings from each source in each watershed. Similar to a sediment budget (Reid and Dunne 1996), the SSA is defined as a year long study, typically performed by a master's level graduate student, in which field work, aerial photograph analysis and literature review are combined to estimate the sediment contribution from each sediment source in the watershed. The cost of an SSA is estimated to be \$40,000/yr. In addition to performing each action separately in each watershed, combinations of two concurrent actions are also considered, resulting in a total of 46 actions, including a "no learning" action. Combining actions allows for interesting possibilities because the different actions provide very different information. Stream gauging quantifies the total sediment passing the gauge, with no distinction regarding sources. Fingerprinting provides an estimate of the proportion of field sediment from the watershed, with no information on the magnitude of the load. A SSA produces estimates of the sediment loading from each sediment source. The complete list of actions considered is presented in the Table 4-10. This limit of at most two concurrent actions was chosen to allow for a manageable number of information acquisition actions to illustrate the framework. Figure 4-3 shows a sampling of only five of the 46 possible actions, in order to keep the figure readable.

The probability of observing a particular observation is defined using a log-normal likelihood function, again because the observations from the research actions must be non-negative, and to facilitate computation of the posterior distributions. Each research action produces a different type of observation. Research actions that combine two individual research actions produce composite observations. For example, if both gauging and fingerprinting were performed in watershed W1 concurrently, the action would produce an estimate of the total sediment loading in W1 and the proportion of field sediment. Thus, depending on the action, the observation produced is either a scalar or a vector. The likelihood function for each scalar observation is written as

$$f_{\tilde{z}_a|\boldsymbol{\theta}}(\tilde{z}_a \mid \tilde{\boldsymbol{\theta}}) = \frac{1}{\tilde{z}_a \sigma_a^{lik} \sqrt{2\pi}} \exp\left[-\frac{1}{2} \left(\frac{\ln(\tilde{z}_a) - \mu_a^{lik}}{\sigma_a^{lik}} \right)^2 \right], \tag{4.5}$$

where

$A :=$ $\{a \mid a = 1, 2, ..., 45\}$ is the index set of research actions (see Table 4-10 for a description of each action and its corresponding index value)

$\tilde{z}_a :=$ observed value from research action a

$\mu_a^{lik} :=$ mean of natural log-transformed observation for research action a

$\sigma_a^{lik} :=$ standard deviation of natural log-transformed observations for research action a

For observations with more than one element, the likelihood function is determined by recognizing that taking the natural logarithm of a log-normally distributed random variable results in a normally distributed random variable. The likelihood function for each vector observation is written as

$$f_{\tilde{\mathbf{z}}'_a|\boldsymbol{\theta}}(\tilde{\mathbf{z}}'_a \mid \tilde{\boldsymbol{\theta}}) = \frac{1}{(2\pi)^4 \left|\Sigma_a^{lik}\right|^{-1}} \exp\left(-\frac{1}{2}\left(\tilde{\mathbf{z}}'_a - \boldsymbol{\mu}_a^{lik}\right)^{\mathrm{T}} \left(\Sigma_a^{lik}\right)^{-1} \left(\tilde{\mathbf{z}}'_a - \boldsymbol{\mu}_a^{lik}\right)\right), \tag{4.6}$$

where

$\tilde{\mathbf{z}}'_a :=$ vector of natural log-transformed observation for research action a, with elements $\ln(\tilde{z}_{a,b})$

$\boldsymbol{\mu}_a^{lik} :=$ vector of means of natural log-transformed observations for research action a

$\rho_{a,b,b'}^{lik} :=$ correlation coefficient for elements b and b' of natural log-transformed observation for research action a

$\Sigma_a^{lik} :=$ covariance matrix of natural log-transformed obervations, with elements $\Sigma_{a,b,b'}^{lik}$

$\Sigma_{a,b,b'}^{lik} :=$ $\rho_{a,b,b'}^{lik}\sigma_{a,b}^{lik}\sigma_{a,b'}^{lik}$, where $\sigma_{a,b}^{lik}$ and $\sigma_{a,b'}^{lik}$ are the standard deviations for observation elements b and b', respectively

For each likelihood function, the expected value was found as a function of the parameter values, and was assumed to be unbiased. Gauging produces an estimate of the total sediment exiting the watershed above where the gauge is placed, thus the expected value of gauging was found as the sum of the expected value of the parameters representing sediment loadings from the upstream watershed. For example, gauging in watershed W1 (*a* = 16) would have an expected value of $E(\tilde{z}_{16}) = E(\tilde{x}_{FW1} + \tilde{x}_{SW1}) = \tilde{x}_{FW1} + \tilde{x}_{SW1}$, where $\tilde{z}_{16}$ represents the observation resulting from gauging in W1. Fingerprinting produces an estimate of the proportion of field sediment. The expected value of fingerprinting in W1 (*a* = 1) is then $E(\tilde{z}_1) = E(\frac{\tilde{x}_{F1}}{\tilde{x}_{F1} + \tilde{x}_{S1}})$, with $\tilde{z}_1$ represents the observation from the research action. A second order Taylor series expansion was used to produce an estimate of $E\left(\tilde{z}_a\right)$ (Rice 1995). For example, the expected value of fingerprinting in W1 is

$$\begin{aligned} E(\tilde{z}_1) &= E\left(\frac{\tilde{x}_{F1}}{\tilde{x}_{F1} + \tilde{x}_{S1}}\right) \\ &\approx \frac{\tilde{x}_{F1}}{\tilde{x}_{F1} + \tilde{x}_{S1}} + \frac{1}{2}\left\{ f''_{\tilde{x}_{F1}\tilde{x}_{F1}} Var(\tilde{x}_{F1}) + f''_{\tilde{x}_{S1}\tilde{x}_{S1}} Var(\tilde{x}_{S1}) - 2 f''_{\tilde{x}_{F1}\tilde{x}_{S1}} Cov(\tilde{x}_{F1}, \tilde{x}_{S1}) \right\}, \end{aligned} \tag{4.7}$$

where $f''_{\tilde{x}_{F1}\tilde{x}_{F1}}$ is the partial second derivative of $\frac{\tilde{x}_{F1}}{\tilde{x}_{F1} + \tilde{x}_{S1}}$ with respect to $\tilde{x}_{F1}$, $f''_{\tilde{x}_{S1}\tilde{x}_{S1}}$ is the partial second derivative with respect to $\tilde{x}_{S1}$, and $f''_{\tilde{x}_{F1}\tilde{x}_{S1}}$ is the partial second derivative with respect to $\tilde{x}_{F1}$ and $\tilde{x}_{S1}$.

Finally, an SSA produces an estimate of each sediment source. For example, the expected value of a sediment source analysis in W1 (*a* = 40) is a two dimensional vector: $E(\tilde{z}_{40}) = [\tilde{x}_{FW1}, \tilde{x}_{SW1}]$.

The standard deviation for each likelihood function was determined through interviews with Dr. Patrick Belmont (personal communication, July 2, 2008) about fingerprinting methods, and with Dr. Peter Wilcock (personal communication, January 9, 2009) regarding gauging and sediment source analysis. Each research action is subject to several sources of error. For example, samples collected with each method are prone to both spatial and temporal variability. There are sample errors associated with sample collection,

Table 4-3: Actions and Corresponding Errors

Action	95% CI
Gauging	70% - 140% of mean
Fingerprinting	80% - 120% of mean
Sediment Source Analysis	
Fields	50% - 200% of mean
Streams	50% - 200% of mean
Ravines	50% - 200% of mean
Bluffs	50% - 200% of mean

Table 4-4: Correlations between actions

Research Actions	Correlations
Concurrent gauging in two watersheds	High (0.85)
Sediment Source Analysis in W3 (alone or with any other action)	
Ravine and Bluff observations	Low (0.3)
Sediment Source Analysis in W1 and W2	
Field observations	Med (0.5)
Stream observations	Low (0.3)
Sediment Source Analysis in W1 (or W2) and W3	
Field observations	Med (0.5)
Stream observations	Med (0.5)

as well as measurement errors associated with the calculation of sediment loadings for each method. For each research action, the expert was asked to provide a 95% confidence interval reflecting these sources of error. In addition to means and standard deviations of observations from research actions, the expert was asked if the errors were correlated. Correlations were elicited for all combinations of actions, as well as correlations in the accuracies of the individual components of a sediment source analysis. The parameters of observational errors elicited from the experts are summarized in Table 4-3 and Table 4-4.

4.4.3 Discrete Observations and Posterior Generation

Once the prior distribution and likelihood functions are parameterized, the next step involves determining the set of observations for each research action. The probability density for the outcomes $\tilde{\mathbf{z}}_a$ of each research action is defined as $f_{\tilde{z}_a}(\tilde{z}_a) = \int_{\tilde{\theta}} f_{\tilde{z}_a|\tilde{\theta}}(\tilde{z}_a \mid \tilde{\boldsymbol{\theta}}) f_{\tilde{\theta}}(\tilde{\boldsymbol{\theta}}) d\tilde{\boldsymbol{\theta}}$. The probability density function is continuous; however, because we use a decision tree methodology, we require a discrete set of observations. Therefore, the continuous distribution was discretized using the following procedure. For each research action, the range of possible $\tilde{\mathbf{z}}_a$ values was divided into discrete intervals. For the research actions for which $\tilde{\mathbf{z}}_a$ are scalar, the range of possible values was divided into 9 mutually exclusive intervals. For the remaining research actions, each of which have vector $\tilde{\mathbf{z}}_a$ outcome, the ranges of values for each dimension were divided into 3 intervals. For the actions with scalar outcomes (gauging and fingerprinting), the probability of each interval was estimated using Monte Carlo integration with antithetic sampling to reduce variance (Fishman 1996). For the research actions with two dimensional observations (gauging and fingerprinting simultaneously, or sediment source analysis in W1 or W2), the probability of each of the 9 joint probability intervals was estimated using the same Monte Carlo integration approach. For the research actions whose $\tilde{\mathbf{z}}_a$ have more than two dimensions, a subset of discrete intervals was selected using Latin hypercube sampling (McKay et al. 1979). The probability of each of these intervals was also estimated with Monte Carlo integration using antithetic sampling.

These estimated probabilities are used to determine nine discrete values $\zeta_{a,n}$, (n={1,2,…,9}), of $\tilde{\mathbf{z}}_a$ and their associated discrete probabilities, p($\zeta_{a,n}$). These discrete probabilities and observations are illustrated in Figure 4-3 as the outcomes of the chance nodes. We selected these discrete values in order to approximate the original distribution. To do this, a "moment matching" approach was taken. This was implemented by running an optimization model that minimized the sum of squared deviations between the discrete probabilities and the Monte Carlo estimates of the probabilities for their associated intervals, subject to the constraints that the means and covariances of the discrete distribution matched the means and covariances of the continuous distribution as estimated by the MC integration. The model formulation used is presented in the Appendix to this chapter.

Based on the prior distribution and likelihood functions, WinBUGS (Lunn et al. 2000) was used to simulate the posterior distribution, $f_{\boldsymbol{\theta}|\zeta_{a,n}}(\tilde{\boldsymbol{\theta}} \mid \zeta_{a,n})$, for each action and discrete observation. WinBUGS is a Microsoft Windows based software program that performs Bayesian inference using Gibbs sampling (Geman and Geman 1984), which is a Markov chain Monte Carlo (MCMC) algorithm. MCMC is essentially Monte Carlo integration using Markov chains. Monte Carlo integration draws random samples from distributions of interest and then computes sample averages as approximations to expectations of the distribution. MCMC draws samples with the use of a Markov chain, which is a sequence of random variables such that at each time $t \geq 0$, the following state, X_{t+1} is sampled from the transition probability distribution $P(X_{t+1}|X_t)$, which depends only on the current state, X_t, not the entire history of the chain of states prior to t. The Gibbs sampler, on which WinBUGS relies, is a special case of the Metropolis-Hastings algorithm, which is the first algorithm developed to perform MCMC (see Gilks et al. 1996 for a thorough discussion of MCMC, Metropolis-Hastings, and Gibbs Sampling).

For each research action and observation, two Markov chains are used to check convergence. While it is difficult to say conclusively that a chain has converged, it is possible to investigate if it has not. For each research action a and discrete observation $\zeta_{a,n}$, the estimated potential scale reduction (EPSR) was one or nearly one, suggesting convergence (Gelman and Rubin 1992).

The Bayesian inference performed with WinBUGS resulted in estimates of the expectation of the posterior distribution, $E(\tilde{\boldsymbol{\theta}} \mid \zeta_{a,n})$, for each action a and observation n. These expectations were then transformed back to the original parameters $\tilde{x}_{i,j}$ describing the long term annual average sediment loadings (in tons) from source i in watershed j, by taking the average of the exponential of the posterior samples. The procedure used to determine the discretized observations and their associated probabilities, was prone to errors, due to the use of approximation techniques (Monte Carlo integration). Therefore, it was necessary to adjust the results of the Bayesian inference to ensure that the following relationship was satisfied:

$$\sum_{n=1}^{9} p(\zeta_{a,n}) E(\tilde{\mathbf{x}} | \zeta_{a,n}) = E(\tilde{\mathbf{x}}), \tag{4.8}$$

where $\tilde{\mathbf{x}}$ is the vector of long term annual average sediment loadings. For each action in which the relationship did not hold, the following adjustment was made for each observation *n* and action *a*:

$$E'(\tilde{\mathbf{x}} \mid \zeta_{a,n}) = E(\tilde{\mathbf{x}} \mid \zeta_{a,n}) - \left(\sum_{n=1}^{9} p(\zeta_{a,n}) E(\tilde{\mathbf{x}} \mid \zeta_{a,n}) - E(\tilde{\mathbf{x}}) \right). \tag{4.9}$$

This adjustment was necessary to prevent negative values of information from being calculated and to prevent the inappropriate selection of research actions based on incorrect values of $E(\tilde{\mathbf{x}} \mid \zeta_{a,n})$.

4.4.4 Multiobjective Linear Program

For each action and observation, the corrected expected value of each posterior distribution, $E'(\tilde{\mathbf{x}} \mid \zeta_{a,n})$, was then used as an input to the multiobjective linear program used to select the optimal suite of management actions. The multiobjective linear program (MOLP) seeks to select the optimal amount of each BMP to install in order to minimize a weighted sum of expected annual cost and expected long term annual sediment loss in the three watershed model:

$$\min \quad \sum_{i=1}^{4} \sum_{j=1}^{3} \sum_{k=1}^{8} \sum_{m=1}^{7} \left(C_{i,k} - W * \frac{S_{i,j}^{m} * F_{i,k} * E'(\tilde{x}_{i,j} \mid \zeta_{a,n})}{A_{i,j}} \right) b_{i,j,k}^{m} \tag{4.10}$$

$$\text{subject to} \quad \frac{\sum_{k=1}^{8} b_{i,j,k}^{m}}{A_{i,j}} \le UP_{i,j}^{m} \qquad \forall i,j,m \tag{4.11}$$

$$\sum_{m} b_{i,j,k}^{m} \le UB_{i,j,k} \qquad \forall i,j,k \tag{4.12}$$

$$b_{i,j,k}^{m} \ge 0 \qquad \forall i,j,k,m \tag{4.13}$$

where

K := $\{k \mid k = 1, 2, \ldots, 8\}$ is the index set of BMP types.

M := $\{m \mid m = 1, 2, \ldots, 7\}$ is the index set of segments of the cumulative percent soil loss versus cumulative percent area (or length) curve used to calculate sediment reduced by BMPs

$b_{i,j,k}^{m}$:= decision variable describing the amount (in km^2 or km) of BMP type k addressing sediment source i in watershed j for segment m.

$C_{i,k}$:= cost ($/$\text{km}^2$ or $/km) of BMP type k addressing sediment source i

$F_{i,k}$:= fractional sediment reduction (dimensionless) by BMP type k addressing sediment source i

$S_{i,j}^{m}$:= slope (dimensionless) of the m^{th} segment of the soil loss curve for sediment source i in watershed j

$A_{i,j}$:= area (km^2) or length (km) contributing soil for source i in watershed j

$UP_{i,j}^{m}$:= range (dimensionless) of segment m of the soil loss curve for source i in watershed j

$UB_{i,j,k}$:= upper bound (km^2 or km) for BMP type k addressing sediment source i in watershed j

W := weight ($/ton) on the sediment reduction objective

The objective function (4.10) minimizes the sum of the expected costs of the BMPs, and maximizes the weighted expected sediment reduced by the BMPs. The sediment portion of the objective is equivalent to maximizing expected long term annual sediment reduction because the initial (unabated) loadings are inputs into the linear program and are considered constants, which cannot be optimized.

The sediment reduced (tons) by each BMP is calculated based on the assumption that each particular sediment source type has a distribution of loss within the areas making up that type, and that the areas with the highest relative contribution of sediment would be addressed first. To accomplish this, a curve representing the distribution of soil loss must be constructed for each source and watershed that relates cumulative % soil loss and cumulative % area (or % length for streambanks and bluffs). The tons of sediment reduced per unit of each BMP installed are then calculated as $\frac{S_{i,j}^{m} * F_{i,k} * E'(\tilde{x}_{i,j} \mid \zeta_{a,n})}{A_{i,j}}$. The first set of constraints (4.11) limits the amount of each decision variable corresponding to each segment of the soil loss curve to be within the range of that segment. For example, if the first segment of the soil loss curve addresses the worst 1% of the total area contributing soil loss from source i in watershed j, the constraint would limit the percent of area occupied by this BMP k for this segment to be less than 1%. The second set of constraints (4.12) limits the total amount of each type of BMP to be less than the upper bound on the available area (or length) for the BMP. The last set of constraints (4.13) enforces non-negativity of BMP amounts.

To determine the tradeoff between cost and sediment reduced, the model is run with a range of values of W to show the decisions under different relative valuations of soil loss. The next section describes the management actions under consideration and the parameter values used in the MOLP.

4.4.5 Management Practices and Parameter Values

A portfolio of possible best management practices and their associated costs and effectiveness was identified through literature review and interviews with experts involved in managing sediment in the Minnesota River Basin. The data sources for each BMP are summarized in Table 4-5. The BMPs under consideration address all four sources of sediment and were chosen to illustrate a sampling of the different kinds of BMPs currently in place or under consideration. The costs and effectiveness for each BMP is summarized in Table 4-6. As costs and reductions are subject to uncertainty and variation, the costs and reduction parameters used are assumed to represent expected costs and expected reductions.

Table 4-5: Data sources for each BMP

BMP	Cost Data	Effectiveness Data
Critical Area Planting	LARS and eLINK	RUSLE2
Conservation Tillage	EQUIP	RUSLE2
Stream Stabilization	Steve Becker and John Brach	Peter Wilcock
Stream Restoration	LARS and eLINK	Peter Wilcock
Land Retirement	MN Land Economics Database	Peter Wilcock
Drainage Pipe	Global Pipe Installation Cost Estimator	Peter Wilcock
Toe Protection	Steve Becker and John Brach	Peter Wilcock
Complete Stabilization	John Brach	John Brach

Table 4-6: Expected Costs and Expected Fractional Reductions for each BMP type

Sediment Source (i)	BMP Type	Annualized Cost ($C_{i,k}$)	Fractional Reduction ($F_{i,k}$)
Fields	Critical Area Planting (k = 1)	\$20,510/km^2	0.65
Fields	Conservation Tillage (k = 2)	\$7,414/km^2	0.85
Streambanks	Streambank Stabilization (k = 3)	\$22,966/km	1.00
Streambanks	Streambank Restoration (k = 4)	\$36,417/km	1.00
Ravines	Land Retirement (k = 5)	\$118,858/km^2	0.70
Ravines	Drainage Pipe (k = 6)	\$24,606/km	0.90
Bluffs	Toe Protection (k = 7)	\$22,966/km	0.61
Bluffs	Complete Stabilization (k = 8)	\$71,850/km	1.00

The field BMPs under consideration are critical area planting (CAP) and conservation tillage (CT). Critical area planting addresses areas of high erosion and involves establishing permanent vegetation, such as perennial grasses, perennial legumes, trees, shrubs, vines or mixture on these sites as defined by the Natural Resource Conservation Service (NRCS), an agency under the United State Department of Agriculture (USDA) (USDA NRCS 2008). Less drastic is conservation tillage, which allows continued production of row crops such as soybeans and corn, which dominate in this region. Conservation tillage is an agricultural practice that manages the "the amount, orientation, and distribution of crop and other plant residue on the soil surface year round while limiting soil-disturbing activities to only those necessary to place nutrients, condition residue, and plant crops" (USDA NRCS 2008).

The cost of critical area planting was found from the Minnesota Government Annual Reporting System (LARS) (Mohring 2004) and eLINK (Minnesota Board of Water and Soil Resources 2009) datasets for the Le Sueur River watershed, which contain all existing BMPs in the watershed[1]. The average cost of all critical area planting BMPs was calculated and annualized by assuming a 5% discount factor. The cost of conservation tillage was determined from the 2008 MN Environmental Quality Incentives Program (EQIP) (Minnesota Natural Resource Conservation Service (MN NRCS) 2007). For both field BMPs, the fractional sediment reduction was determined using the revised universal soil loss equation, RUSLE2 (Foster et al. 2003), which predicts soil loss from cropland as the product of factors representing climate, soil erodibility, slope length, slope steepness, cover management, and support practices. Management practices cause changes in both

[1] Pearl Zheng provided extensive information on costs and reductions by compiling data from expert interviews, literature review, and RUSLE2 computations.

cover management and support practices. Letting C and P represent those two factors, respectively, the effect of the placement of a BMP can be determined by the ratio $(C)/(C^oP^o)$, where C' and P' are the factor values with the BMP implemented while C^o and P^o are the values representing the cover management and support practices currently in place. C and P depend on soil type. The dominant soil type in highly erosive areas with more than 50 tons/acre/yr is Stroden Complex, while Beauford clay, Marna silty clay loam, and Canisteo-glenco complex are the dominant soil types basin-wide.

The BMPs addressing streambank erosion include streambank stabilization (SS) and streambank restoration (SR). Streambank stabilization includes stream barbs and rock riprap. Stream barbs are used to redirect streamflow away from an eroding bank and are defined by NRCS as “low rock sills projecting out from a streambank and across the stream’s thalweg” (USDA NRCS 1996). Rock riprap involves installing a blanket of graded rocks on a streambank in order to protect the slope by reducing erosion (USDA NRCS 1996). Streambank restoration involves shoreline protection using treatments such as redirecting stream flows, reshaping slopes, and vegetation of streambanks (USDA NRCS 2008). The cost of the streambank stabilization action combines the cost of both stream barbs and rock riprap. The costs for these practices were determined through expert elicitation with John Brach, former NRCS Area Engineer in Minnesota and current NRCS State Conservation Engineer in Montana (personal communication, March 26, 2008), and Steve Becker, NRCS State Conservation Engineer in St. Paul, MN (personal communication, March 27, 2008). The costs provided by the experts were annualized using a discount factor of 5% over the project lifespan of 40 years. For streambank restoration, costs were determined from LARS and eLINK by calculating the average annualized costs based on existing streambank restoration projects in the Le Sueur River watershed. The fractional sediment reduction for each management action was determined through interviews with Dr. Peter Wilcock, who has expertise in stream restoration (personal communication, February 7, 2009).

To reduce erosion from ravines, two BMPs are under consideration: land retirement (LT) and tile drainage pipes (DP). Land retired consists of fields near the edges of ravines that are planted with perennial vegetation, which filters out sediment and slows the velocity of runoff. In contrast, tile drainage pipes are laid along the bottom of the ravine to direct the flow from the ravine to the stream channel, thus reducing erosion along the bottom of the ravine. The cost of the first ravine BMP, land retirement, was found by consulting the Minnesota Land economics database (Flowtite Technology AS 2005). The average 2007 land purchase price for the Le Sueur River watershed was annualized using a 5% discount factor over a 10 year time period. For tile drainage pipe, it was assumed that a 12-inch PVC pipe would be used. Based on current retail prices, the cost of the pipe was assumed to be \$21/ft. The Global Pipe Installation Cost Estimator software from Flowtite was used (Flowtite Technology AS 2005) to estimate installation costs, and the maintenance costs were assumed to be 3% of the material and installation cost. The cost was annualized over a 10 year time period using a 5% discount factor. The fractional reductions for each management practice were determined through expert elicitation (P. Wilcock, personal communication, February 7, 2009).

Lastly, the two BMPs under consideration for addressing soil loss from bluffs are toe protection (TP) measures and complete stabilization (CS) of the bluff. Toe protection includes the placement of stream barbs and rock riprap, as described above. Complete stabilization of the bluff involves grading the slope and installing a retaining wall. For complete stabilization of bluffs, the cost was determined from consulting with John Brach (personal communication, March 26, 2008). The cost was then annualized using a 5% discount factor over a 50 year lifespan. The fractional reduction for complete stabilization of bluffs was determined though expert elicitation (P. Wilcock, personal communication, February 7, 2009).

The cost of toe protection was assumed to be identical to the cost of streambank stabilization, as the types of structures are implemented in both cases. For the fractional sediment reduction associated with toe protection, a more complex calculation was necessary because the reduction is not constant over time. It was assumed that toe protection results in a time profile of sediment loss over time that decreases according to a negative exponential function $E * \exp(-kt)$ where E is the base rate of sediment loss over time, and k is the exponential decay rate. Thus, toe protection does not immediately result in reduction in sedimentation, but the rate of soil loss decreases over time. The rationale for this is that with no protection, the bluff with erode over time at the base rate E. With toe protection, the bottom of the bluff stops eroding, but the top of the bluff continues to erode until a more stable slope is established. The fractional reduction in Table 4-6 equals the levelized value, R, over an infinite lifetime, which is the constant reduction rate whose present worth (at an annual discount rate of r/year) is the same as the present worth (PW) of the assumed actual time profile of reductions, based on the negative exponential relationship. The present value of sediment loss with no abatement is defined as $\int_{t=0}^{\infty} E \exp(-rt)dt = \frac{E}{r}$, and the present value of sediment loss with toe protection is defined as $\int_{t=0}^{\infty} E\exp(-kt)\exp(-rt)dt = \frac{E}{r+k}$. The levelized fractional reduction, *R*, resulting from toe protection is then *k*(*r*+*k*). If we assume an annual discount rate, say *r* = 5%, we can calculate *R* from the following calculation. Based on expert judgment (P. Wilcock, personal communication, January 29, 2009), we assumed that over the first 20 years after BMP installation, the average soil loss from the bluff would be halved; which allows us to solve for k in $\int_{t=0}^{20} E\exp(-kt) = \frac{1}{2} * 20 * E$, by using the average reduction over the first 20 years after the BMP installation.

As described in section 4.4.4, the tons of sediment reduced per amount of BMP installed are based upon the fractional reductions (based on the values of Table 4-6) and the uncontrolled sediment loss $\tilde{\mathbf{x}}$. We assume that each particular source type has a distribution of loss within the areas making up that type, and that the areas with the highest relative contribution of sediment would be addressed first. To accomplish this, a curve representing the distribution was constructed for each source and watershed that related cumulative % soil loss and cumulative % area (or % length for streambanks and bluffs). For field soil loss, this curve was constructed using GIS data provided in June 2007 by Rick Moore, GIS specialist at the Water Resources Center (WRC) at Minnesota State University Mankato, which classified the Maple River Watershed by area into ten soil loss classes based on calculations from RUSLE2. The resulting soil loss curves for the upper watersheds and lower watershed are piece-wise linear with seven segments each.

Expert guidance indicated that sediment supply from streambank erosion would be evenly distribution along approximately one half of the stream length in both the upper and lower watersheds (P. Belmont, personal communication, February 2, 2009; P. Wilcock, personal communication, February 3, 2009). This model uses a linear relationship between stream length and soil loss, resulting in a curve with a single segment.

Expert judgment also informed the cumulative soil loss curves for ravines and bluffs in watershed W3. The ravine soil loss curve is made up of four segments, and the bluff soil loss curve contains three. The soil loss curves are presented in Figure 4-4. The x axis is the proportion of the total area or total length, $A_{i,j}$ and the y axis is the proportion of the total uncontrolled soil loss $\tilde{\mathbf{x}}$. The slopes and segment lengths for each soil loss curve are required parameters for the MOLP. These parameter values are presented in Table 4-7.

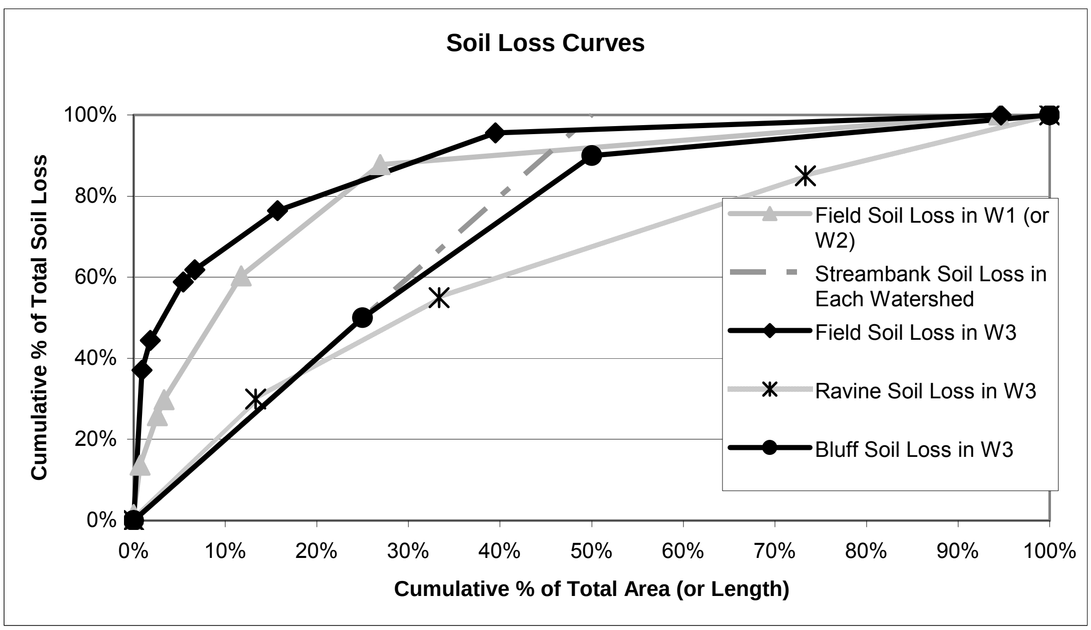

Figure 4-4: Soil Loss Curves for each Sediment Source

To calculate the sediment reduced by each BMP installed, a separate decision variable $b_{i,j,k}^{m}$ was created for each segment m of the cumulative soil loss curve for each sediment source type i (field, streambank, ravine or bluff), watershed j, and BMP type k. Certain

BMPs apply to only highly erosive areas represented by the steepest segments of the soil loss curve. In particular, it is assumed that critical area planting is applied to only 2.5% of the total contributing area, which corresponds to the first three segments of the soil loss curves. Conservation tillage only applies only to the remaining contributing area, or the remaining four segments of the soil loss curve. Meanwhile, complete stabilization of bluffs is only considered for the worst 25% (first segment) of contributing bluff length, while toe protection addresses the remaining 75% of bluffs (two segments). The remaining BMPs can be applied to all contributing areas.

Table 4-7: Soil Loss Curve Slopes and Segment Lengths

Sediment Source (*i*)	Watershed (*j*)	BMP Type (*k*)	Segment (m)	Segment Slope $\left(S_{i,j}^{m}\right)$	Segment Length $\left(UP_{i,j}^{m}\right)$
Fields	W1 or W2	Critical Area Planting (1)	1	90.4286	0.0002
Fields	W1 or W2	Critical Area Planting (1)	2	18.0857	0.0068
Fields	W1 or W2	Critical Area Planting (1)	3	6.4581	0.0189
Fields	W1 or W2	Conservation Tillage (2)	4	5.4257	0.0074
Fields	W1 or W2	Conservation Tillage (2)	5	3.6171	0.0843
Fields	W1 or W2	Conservation Tillage (2)	6	1.8086	0.1518
Fields	W1 or W2	Conservation Tillage (2)	7	0.1809	0.6753
Fields	W3	Critical Area Planting (1)	1	40.2599	0.0092
Fields	W3	Critical Area Planting (1)	2	8.0520	0.0091
Fields	W3	Critical Area Planting (1)	3	4.0260	0.0357
Fields	W3	Conservation Tillage (2)	4	2.4156	0.0127
Fields	W3	Conservation Tillage (2)	5	1.6104	0.0902
Fields	W3	Conservation Tillage (2)	6	0.8052	0.2380
Fields	W3	Conservation Tillage (2)	7	0.0805	0.5519
Streambanks	W1 or W2	Streambank Stabilization (3)	1	2.0000	0.5000
Streambanks	W1 or W2	Streambank Restoration (4)	1	2.0000	0.5000
Streambanks	W3	Streambank Stabilization (3)	1	2.0000	0.5000
Streambanks	W3	Streambank Restoration (4)	1	2.0000	0.5000
Ravines	W3	Land Retirement (5) and Drain Pipe (6)	1	2.2500	0.1333
Ravines	W3	Land Retirement (5) and Drain Pipe (6)	2	1.2500	0.2000
Ravines	W3	Land Retirement (5) and Drain Pipe (6)	3	0.7500	0.4000
Ravines	W3	Land Retirement (5) and Drain Pipe (6)	4	0.5625	0.2667
Bluffs	W3	Toe Protection (7) and Stabilization (8)	1	1.6000	0.2500
Bluffs	W3	Toe Protection (7) and Stabilization (8)	2	0.2000	0.2500
Bluffs	W3	Toe Protection (7) and Stabilization (8)	3	2.0000	0.5000

The remaining information required for the MOLP are the total contributing area or length of each source in each watershed, and the upper bound on each BMP. This information is summarized in Table 4-8 and Table 4-9. The total field areas and total stream length for each watershed were determined through GIS analysis of the Maple River watershed using data provided by in June 2007 by Rick Moore at the Water Resources Center (WRC). The proportion of the total field area and stream length that contribute sediment in each watershed was found from the soil loss curves. Two values exist for ravines. This is because land retirement targets land areas surrounding the ravines, while drainage pipes are use along the length of ravines. Patrick Belmont (personal communication, January, 23, 2009) informed us that there are about thirty ravines in the lower Maple River watershed, with an average length of 2.5 km. Thus, a total of 75 km of ravine length contribute sediment. For each ravine, we assumed that land retirement could be applied to 50 meter wide areas along the full length of the ravine. Thus, the area contributing sediment was calculated as 3.75 km^2. The contributing length of bluffs was found though expert elicitation. Dr. Patrick Belmont (personal communication, January 23, 2009) stated that about 30% of the total 40 km of streams in watershed W3 were adjacent to bluffs and that all of these bluffs contribute sediment. Thus, the contributing length of bluffs is 12 km.

Lastly, the upper bounds for each BMP were determined. It was assumed that critical area planting would be done in 50 meter wide strips along in agricultural fields along waterways. From discussions with Drs. Patrick Belmont and Peter Wilcock on January 23, 2009, the total length of waterways, including drainage ditches and small streams that are along agricultural fields was assumed to be 250 km for each upper watershed, and 40 km for the lower watershed. It was further assumed that critical area planting would apply to only 20% of these areas. Conservation tillage was estimated similarly, with the assumption that the width of conservation tillage would be 500 m on each side of the waterway. Conservation tillage was considered appropriate on lands along all 250 km of waterways in the upper watershed and 40 km in the lower. The upper bounds for the remaining measure were found by assuming that each BMP could be used to treat all contributing areas or lengths.

Table 4-8: Area (or Length) Contributing Sediment ($A_{i,j}$)

Sediment Source (*i*)	Watershed *j* = W1	*j* = W2	*j* = W3
Fields	387 km^2	387 km^2	64 km^2
Streambanks	80 km	80 km	40 km
Ravines	NA	NA	3.75 km^2/75 km
Bluffs	NA	NA	12 km

Table 4-9: BMP Upper Bounds

Sediment Source (*i*)	Watershed (*j*)	BMP Type	BMP Upper Bound ($UB_{i,j,k}$)
Fields	W1 or W2	Critical Area Planting (*k* = 1)	5 (km^2)
Fields	W1 or W2	Conservation Tillage (*k* = 2)	250 (km^2)
Fields	W3	Critical Area Planting (*k* = 1)	0.8 (km^2)
Fields	W3	Conservation Tillage (*k* = 2)	8 (km^2)
Streambanks	W1 or W2	Streambank Stabilization (*k* = 3)	40 (km)
Streambanks	W1 or W2	Streambank Restoration (*k* = 4)	40 (km)
Ravines	W3	Streambank Stabilization (*k* = 3)	20 (km)
Ravines	W3	Streambank Restoration (*k* = 4)	20 (km)
Bluffs	W3	Land Retirement (*k* = 5)	3.75 (km^2)
Bluffs	W3	Drainage Pipe (*k* = 6)	75 (km)
Bluffs	W3	Toe Protection (*k* = 7)	9 (km)
Bluffs	W3	Complete Stabilization (*k* = 8)	3 (km)

The parameters presented here were then used in the MOLP to determine the optimal set of BMPs for each observation from each research action. Tradeoffs between costs and sediment reduction were determined by solving the MOLP for each of 12 values of W, the weight on the sediment reduction objective. The results of the MOLP were then used to compute the expected value of imperfect information for each research action, which is explained in the next subsection.

4.4.6 Expected Value of Imperfect Information

The results of the linear program were used to compute the expected value of imperfect, or sample, information for each sediment objective weight for each of the 45 research actions. This was done using the following formula

$$EVII(a,W)=\sum_{n=1}^{9}\left(p(\zeta_{a,n})*Obj(\zeta_{a,n},W)\right)-Obj(a_o,W), \tag{4.14}$$

where $Obj(\zeta_{a,n},W)$ is the sum of cost and weighted sediment loss based on the optimal management actions determined from the multiobjective LP with sediment objective weight W and expected sediment loadings, $E'(\tilde{\mathbf{x}}\,|\,\zeta_{a,n})$, resulting from observation n of research action a. Each optimal objective is weighted by the probability, $p(\zeta_{a,n,})$ of the corresponding observation. The optimal sum of cost and weighted sediment loss for the no research action, $Obj(a_o,W)$, is then computed based on the optimal management actions determined from the multiobjective LP based on the expected sediment loadings from the prior distribution. The expected value of imperfect information is found by subtracting the optimal objective based on no learning from the optimal expected objective based on implementing a learning, or research action. The next section presents the results of applying the methodology to this case study, including the calculated values of EVII for each research action.

4.5 Results

This section describes the results of the framework's application to the case study of the Maple River Basin. First, the discrete observations and probabilities are presented. Next, the posterior distributions for each observation are discussed, followed by the results of the multiobjective linear program and the value of information analysis.

4.5.1 Discrete Observations and Probabilities

Research actions produce information on a continuous scale. In order to use a decision tree framework to investigate the impacts of performing research, a discrete set of observations was developed to represent the observed outcomes of research. The process for selecting this discrete set, and the associated probability of each observation occurring, was presented in section 4.4.3. The probabilities, $p(\zeta_{a,n,})$, for each action, along with the action's description, are presented for each observation (Obs.) in Table 4-10. Examples of the resulting observations, $\zeta_{a,n}$, are presented in Table 4-11. The research actions are identified by their ID, which is described Table 4-10. For actions involving observations of more than one type, the column labeled "observation element" indicates to which element the observation applies. The full list of observations is presented in Appendix II. For each observation, posterior distributions are generated according to the procedure presented in section 4.4.3. The probabilities are used to evaluate the value of information contained in each action.

Table 4-10: Probabilities, $p(\zeta_{a,m})$, for each Research Action Observation

Action ID (a)	Description	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	Obs 6	Obs 7	Obs 8	Obs 9
1	Fingerprinting in W1	0.065	0.149	0.139	0.240	0.144	0.061	0.150	0.044	0.008
2	Fingerprinting & SSA in W1	0.021	0.160	0.073	0.344	0.008	0.086	0.012	0.093	0.203
3	Fingerprinting in W1 & SSA in W2	0.021	0.160	0.073	0.344	0.008	0.086	0.012	0.093	0.203
4	Fingerprinting in W1 & SSA in W3	0.013	0.086	0.060	0.017	0.113	0.140	0.001	0.094	0.476
5	Fingerprinting in W1 & W2	0.016	0.090	0.049	0.038	0.241	0.166	0.024	0.195	0.181
6	Fingerprinting in W1 & W3	0.097	0.224	0.303	0.109	0.108	0.131	0.017	0.005	0.006
7	Fingerprinting in W2	0.065	0.149	0.139	0.240	0.144	0.061	0.150	0.044	0.008
8	Fingerprinting in W2 & SSA in W1	0.021	0.160	0.073	0.344	0.008	0.086	0.012	0.093	0.203
9	Fingerprinting & SSA in W2	0.021	0.160	0.073	0.344	0.008	0.086	0.012	0.093	0.203
10	Fingerprinting in W2 & SSA in W3	0.013	0.086	0.060	0.017	0.113	0.140	0.001	0.094	0.476
11	Fingerprinting in W2 & W3	0.097	0.224	0.303	0.109	0.108	0.131	0.017	0.005	0.006
12	Fingerprinting in W3	0.115	0.202	0.273	0.123	0.099	0.072	0.079	0.028	0.010
13	Fingerprinting in W3& SSA in W1	0.093	0.035	0.038	0.500	0.025	0.028	0.038	0.145	0.099
14	Fingerprinting in W3 & SSA in W2	0.093	0.035	0.038	0.500	0.025	0.028	0.038	0.145	0.099
15	Fingerprinting and SSA in W3	0.012	0.001	0.022	0.013	0.742	0.031	0.041	0.100	0.038
16	Gauging in W1	0.099	0.123	0.106	0.171	0.204	0.141	0.080	0.040	0.035
17	Gauging & Fingerprinting in W1	0.009	0.031	0.064	0.167	0.183	0.142	0.189	0.150	0.065
18	Gauging in W1 & Finger-printing in W2	0.009	0.031	0.064	0.167	0.183	0.142	0.189	0.150	0.065
19	Gauging in W1 & Finger-printing in W3	0.031	0.085	0.204	0.097	0.123	0.205	0.141	0.084	0.030
20	Gauging & SSA in W1	0.480	0.017	0.289	0.040	0.008	0.060	0.026	0.016	0.065
21	Gauging in W1 & SSA in W2	0.480	0.017	0.289	0.040	0.008	0.060	0.026	0.016	0.065
22	Gauging in W1 & SSA in W3	0.208	0.500	0.108	0.043	0.000	0.056	0.018	0.023	0.043
23	Gauging in W1 & W2	0.289	0.076	0.002	0.107	0.174	0.041	0.002	0.115	0.196
24	Gauging in W1 & W3	0.347	0.245	0.051	0.013	0.080	0.083	0.012	0.067	0.101
25	Gauging in W2	0.099	0.123	0.106	0.171	0.204	0.141	0.080	0.040	0.035
26	Gauging in W2 & Finger-printing in W1	0.009	0.031	0.064	0.167	0.183	0.142	0.189	0.150	0.065
27	Gauging & Fingerprinting in W2	0.009	0.031	0.064	0.167	0.183	0.142	0.189	0.150	0.065
28	Gauging in W2 & Finger-printing in W3	0.031	0.085	0.204	0.097	0.123	0.205	0.141	0.084	0.030
29	Gauging in W2 & SSA in W1	0.480	0.017	0.289	0.040	0.008	0.060	0.026	0.016	0.065
30	Gauging & SSA in W2	0.480	0.017	0.289	0.040	0.008	0.060	0.026	0.016	0.065
31	Gauging in W2 & SSA in W3	0.208	0.500	0.108	0.043	0.000	0.056	0.018	0.023	0.043
32	Gauging in W2 & W3	0.347	0.245	0.051	0.013	0.080	0.083	0.012	0.067	0.101
33	Gauging in W3	0.072	0.207	0.287	0.127	0.102	0.109	0.058	0.025	0.013
34	Gauging in W3 & Finger-printing in W1	0.037	0.035	0.097	0.178	0.117	0.137	0.158	0.119	0.122

Action ID (a)	Description	Obs 1	Obs 2	Obs 3	Obs 4	Obs 5	Obs 6	Obs 7	Obs 8	Obs 9
35	Gauging in W3 & Finger-printing in W2	0.037	0.035	0.097	0.178	0.117	0.137	0.158	0.119	0.122
36	Gauging & Fingerprinting in W3	0.082	0.055	0.341	0.130	0.045	0.221	0.098	0.015	0.012
37	Gauging in W3 & SSA in W1	0.493	0.002	0.011	0.234	0.015	0.013	0.023	0.000	0.208
38	Gauging in W3 & SSA in W2	0.493	0.002	0.011	0.234	0.015	0.013	0.023	0.000	0.208
39	Gauging in W3 & SSA in W3	0.610	0.001	0.001	0.023	0.116	0.076	0.086	0.001	0.087
40	SSA in W1	0.208	0.160	0.089	0.039	0.042	0.030	0.148	0.163	0.120
41	SSA in W1 & W2	0.550	0.012	0.002	0.143	0.006	0.001	0.013	0.258	0.016
42	SSA in W1 & W3	0.593	0.006	0.044	0.049	0.058	0.038	0.193	0.001	0.017
43	SSA in W2	0.208	0.160	0.089	0.039	0.042	0.030	0.148	0.163	0.120
44	SSA in W2 & W3	0.593	0.006	0.044	0.049	0.058	0.038	0.193	0.001	0.017
45	SSA in W3	0.461	0.038	0.174	0.068	0.026	0.023	0.027	0.076	0.108

Table 4-11: Example Observations for Several Research Actions

Action ID (a)	Observation Element	Observations ($\zeta_{a,n}$)								
		$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$
10	Fingerprinting W2	0.740	0.869	1.129	1.129	0.740	0.869	0.740	0.869	1.129
10	Fields W3	1660	7233	8400	1660	8400	7233	1660	8400	7233
10	Streams W3	2146	6534	12230	6534	12230	2146	12230	6534	2146
10	Ravines W3	14895	27738	36117	36117	14895	27738	14895	36117	27738
10	Bluffs W3	18695	29296	53572	29296	18695	53572	53572	18695	29296
11	Fingerprinting W2	0.629	0.925	0.989	0.629	0.925	0.989	0.629	0.925	0.989
11	Fingerprinting W3	0.197	0.197	0.197	0.233	0.233	0.233	0.368	0.368	0.368
21	Gauging W1	7385	9908	14259	14259	7385	9908	7385	14259	9908
21	Fields W2	5629	10612	14030	10612	14030	5629	10612	5629	14030
21	Streams W2	146	1200	3429	146	1200	3429	3429	1200	146
29	Gauging W2	7385	9908	14259	14259	7385	9908	7385	14259	9908
29	Fields W1	5629	10612	14030	10612	14030	5629	10612	5629	14030
29	Streams W1	146	1200	3429	146	1200	3429	3429	1200	146
30	GaugingW2	7385	9908	14259	14259	7385	9908	7385	14259	9908
30	Fields W2	5629	10612	14030	10612	14030	5629	10612	5629	14030
32	Gauging W2	7689	9715	12813	7689	9715	12813	7689	9715	12813
32	Gauging W3	61601	61601	61601	93750	93750	93750	120257	120257	120257
36	Gauging W3	58888	80739	120729	58888	80739	120729	58888	80739	120729
36	FingerprintingW3	0.197	0.197	0.197	0.242	0.242	0.242	0.303	0.303	0.303
43	Fields W2	5702	9671	13276	5702	9671	13276	5702	9671	13276
43	Streams W2	289	289	289	881	881	881	2174	2174	2174
45	Fields W3	3846	7144	17429	3846	17429	7144	17429	7144	3846
45	Streams W3	3860	5950	13243	3860	5950	13243	5950	3860	13243
45	Ravines W3	15014	23799	46822	33601	15014	46822	15014	46822	23799
45	Bluffs W3	17150	23799	49523	49523	23799	17150	49523	17150	23799

4.5.2 Posterior Distributions

To illustrate the results of the Bayesian inference, the marginal posterior distributions for a single observation from two research actions are presented in Figure 4-5 and Figure 4-6 as examples. Figure 4-5 displays the marginal posterior distributions for observation $\zeta_{21,1}$. Action 21 involves gauging in watershed W1 and an SSA in W2. Figure 4-6 shows the posterior marginal distributions for observation $\zeta_{11,7}$. Action 11 is defined as fingerprinting in watersheds W2 and W3 concurrently. The dashed curves represent the marginal prior distributions, and the solid curves represent the marginal posteriors distributions. The x-axis is the value of the log of the parameter of interest, $\ln(\tilde{x}_{i,j})$, and the y-axis is the value of the marginal posterior probability density function, *f*, defined in terms of $\ln(\tilde{x}_{i,j})$.

The posterior distributions shown in Figure 4-5 result from observation 1 of action 21, as indicated in Table 4-11, observation 1 has a low value for the gauging observation in W1, and a low value for both the field and streambank observations in W2. Based on the prior information, the expected W1 gauging observation is $E(\tilde{x}_{FW1} + \tilde{x}_{SW1})$ = 10,000 tons/yr, but the actual observation is instead 7385 tons/yr. For the SSA inW2, the expected values from the prior distribution are 9000 tons/yr for field sediment, and 1000 tons/yr for streambank sediment; however, the observation produces an estimate of 5629 tons/yr from fields and 147 tons/yr from streambanks. As a result of the low gauging observation in W1, the posterior distributions describing loadings from fields and streambanks in W1 have shifted left. In addition, the loadings from fields and streambanks in W2 have also shifted left, with a large shift in streambank loading. This large shift is due to the very low observation of streambanks loadings. Note that while no information is being directly revealed about the loadings in watershed W3, there are also shifts in the posterior distributions for these loadings. These shifts are due to the fact that the loadings in W3 are correlated with the loadings in W1 and W2. So, for example, if streambank loadings are low in W1 and W2, then it would be expected that the streambanks loadings would also be lower in W3 since the streambank loadings between the upper and lower watersheds are moderately correlated.

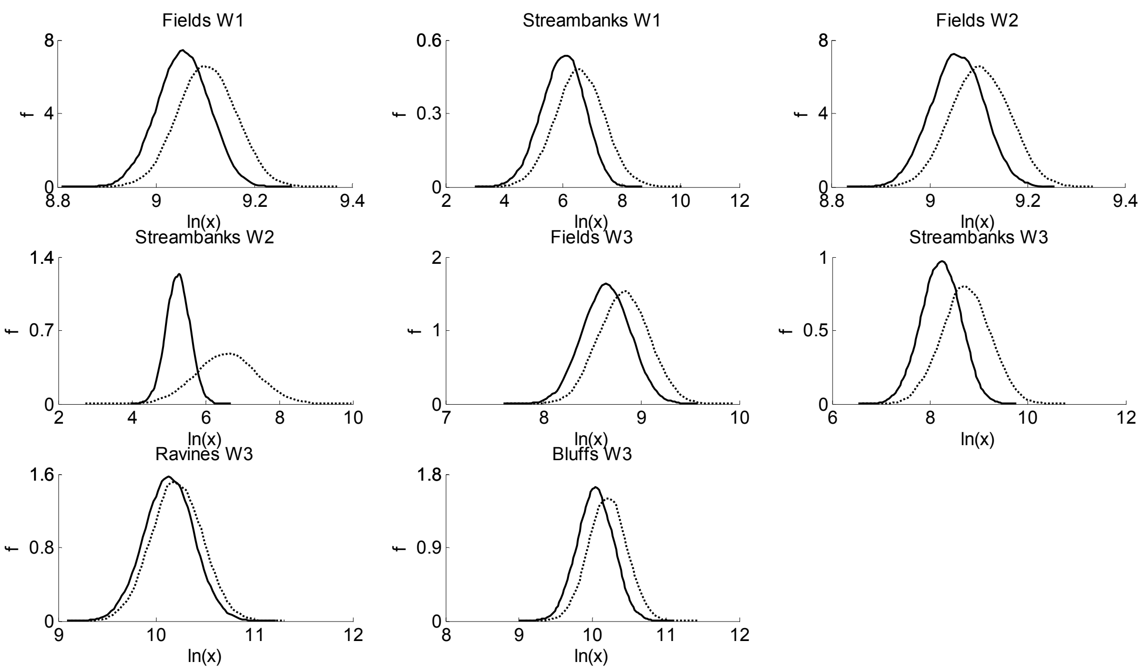

Figure 4-5: Marginal Prior Distributions (dashed lines) and Marginal Posterior Distributions (solid lines) of $\ln\left(\tilde{x}_{i,j}\right)$ resulting from observation $\zeta_{21,1}$ of Gauging in W1 and a Sediment Source Analysis in W2

Figure 4-6 shows the prior and posterior marginal distributions resulting from performing fingerprinting in both W2 and W3 concurrently (a = 11), and obtaining observation $\zeta_{11,7}$. In this case, little change is observed in the loadings in W1. There is a large shift to the

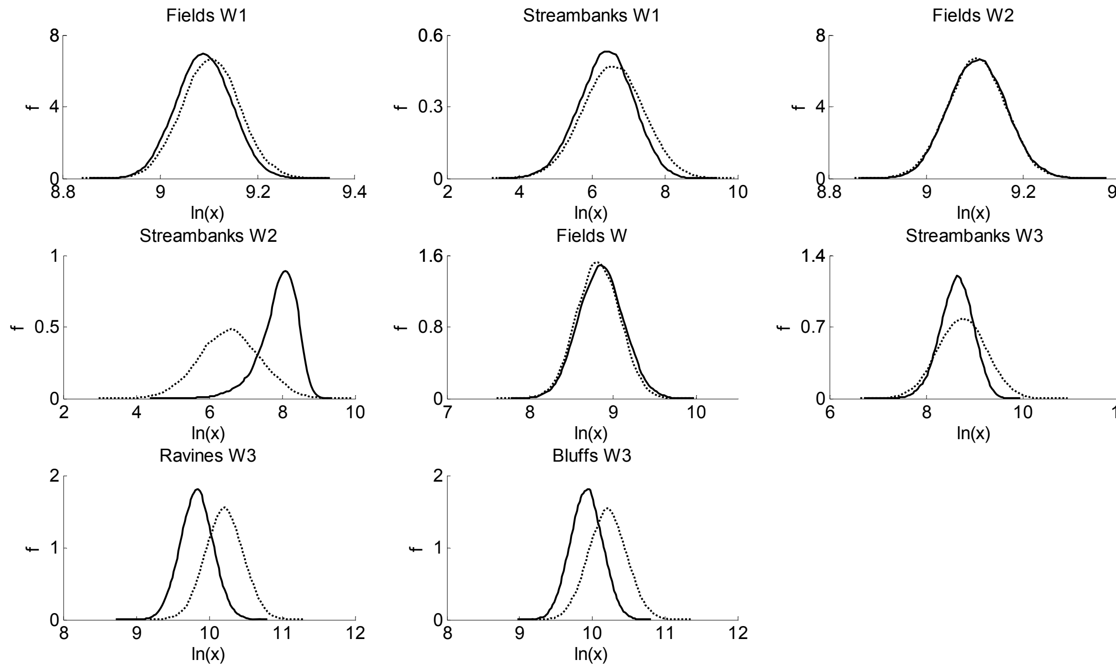

Figure 4-6: Marginal Prior Distributions (dashed lines) and Marginal Posterior Distributions (solid lines) of $\ln\left(\tilde{x}_{i,j}\right)$ resulting from observation $\zeta_{11,7}$ of Fingerprinting in W2 and W3 Concurrently.

right in the streambank loadings in W2, reflecting the fact that observation 7 has a low fingerprinting result in W2. Since fingerprinting is determined as the ratio of field contributions to the total loading, the low observation caused an increase in the denominator, which translated to an increase in streambank loadings. Observation 7 resulted in a high fingerprinting observation in W3 (i.e., a high proportion of field contributions in W3), which corresponds with a decrease in the non-field sediment loadings. In general, the posterior distributions have lower variance than the prior distribution, indicating the improved understanding of the system resulting from undertaking a learning action. The full set of expected posterior loadings for each research action and observation are presented in the appendix. While the shifts in the distributions from priors to posteriors are interesting, the impacts that these shifts have on the choice of management actions is what determines whether the research action yields valuable information. The next section discusses these impacts and the results of the multiobjective linear programs.

4.5.3 Multiobjective Linear Program Solutions

For each action and each observation, the multiobjective linear program detailed in section 4.4.4 was run using 12 values for the weight on the sediment objective, as shown in Table 4-12. The value of the sediment objective weight is equivalent to the marginal cost of sediment reduction in \$/ton. Marginal costs for sediment reduction have been reported in the literature from \$10/ton up to \$126/ton (Yuan et al. 2002; Khanna et al. 2003; Moore et al. 1992; Yang et al. 2003). A much larger range (\$1/ton up to \$5000/ton) is used here in order to generate a tradeoff curve to investigate the tradeoffs between expected cost and expected sediment reduction for each research action and observation. Large marginal costs for sediment reduction represent decision maker preferences in which sediment reduction is paramount.

Figure 4-7 shows an example tradeoff curve for the no research action, a = 0, and action a =*21* consisting of gauging in W1 and a sediment source analysis in W2. For the research action, the outcomes displayed in the figure are found by weighting the optimal expected cost and expected sediment remaining from each observation by the probability of that observation, $\sum_{n=1}^{9}\left(p(\zeta_{a,n})*Obj(\zeta_{a,n},W)\right)$, and do not include the cost of implementing the research action. The 12 sediment objective weights are also shown in the figure. The expected total sediment loading without abatement is 90,000 tons/yr. Thus, the management actions selected by each research action will reduce this amount of sediment remaining in the watershed. Information does not necessarily increase or decrease the

Table 4-12: Optimal Research Actions, BMPs and Resulting Expected Sediment Reductions (Averaged over Observation Outcomes)

Weight	Optimal Research Action (a)	Tons Reduced: Actions in Watershed W1			Actions in Watershed W2			Actions in Watershed W3						
		CAP	CT	SS	CAP	CT	SS	CAP	CT	SS	LT	DP	TP	CS
1	0 (No Action)	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	1687	0	0	0	0	0	0
20	0	85	0	0	85	0	0	1687	0	0	10780	0	6832	14000
50	21 or 29	85	0	0	85	0	0	1808	644	4634	19600	0	6832	14000
50	0	85	0	0	85	0	0	1808	1047	0	19600	0	6832	14000
100	0	799	307	0	799	307	0	1808	1153	7000	19600	0	8540	14000
200	36	799	2640	0	799	3150	0	1808	1153	7000	10508	11690	8540	14000
200	0	799	2640	0	799	2640	0	1808	1153	7000	8820	13860	8540	14000
400	6 or 11	1026	4739	0	1026	4739	754	1808	1153	7000	3038	21293	8540	14000
400	0	1026	4739	0	1026	4739	0	1808	1153	7000	2940	21420	8540	14000
500	6 or 11	1026	4739	0	1026	4739	754	1808	1153	7000	422	24657	8540	14000
500	0	1026	4739	0	1026	4739	0	1808	1153	7000	0	25200	8540	14000
600	21 or 29	1026	4740	653	1026	4740	943	1808	1153	7000	0	25200	8540	14000
600	0	1026	4739	0	1026	4739	0	1808	1153	7000	0	25200	8540	14000
1000	21 or 29	1026	4740	711	1026	4740	992	1808	1153	7000	0	25200	8540	14000
1000	0	1026	4739	1000	1026	4739	1000	1808	1153	7000	0	25200	8540	14000
5000	20 or 30	1026	5296	1000	1026	5296	993	1808	1153	7000	0	25200	8540	14000
5000	0	1026	5296	1000	1026	5296	1000	1808	1153	7000	0	25200	8540	14000
Max Expected Reduction		**1026**	**5296**	**1000**	**1026**	**5296**	**1000**	**1808**	**1153**	**7000**	**19600**	**25200**	**8540**	**14000**

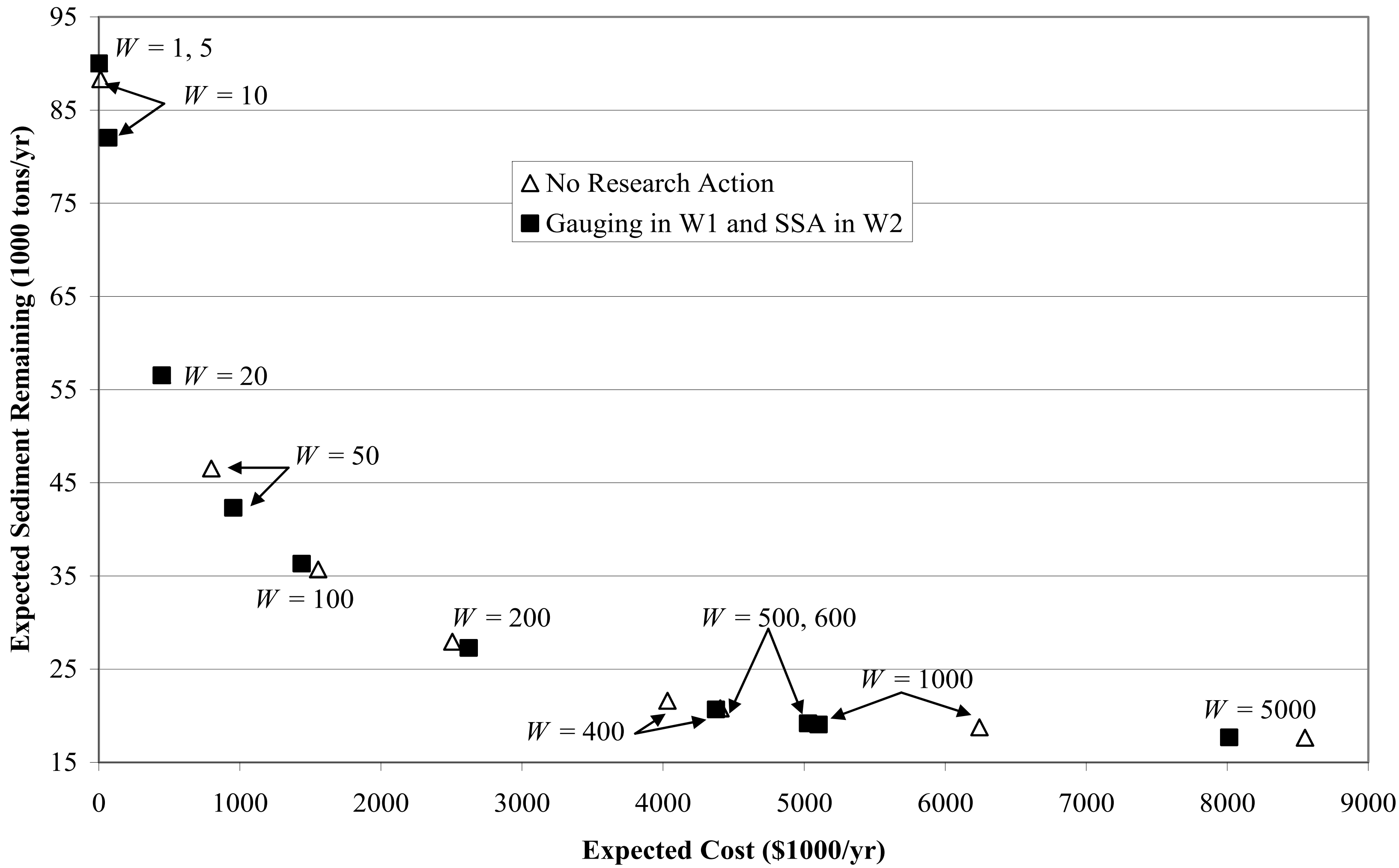

Figure 4-7: Tradeoff Curves for No Research Action and Gauging in W1 and Sediment Source Analysis in W2 (excludes cost of action χ_a)

individual cost or sediment reduction objectives; however, the probability weighted sum of the MOLP objective will not be higher with the action, as compared to the no research action because EVII is necessarily nonnegative. The optimal objective function for each research action is presented in the Appendix. The optimal decision variable values are available on CD from the author on request.

Table 4-12 summarizes the optimal research actions for each sediment objective weight found by determining the research action that minimizes

$$\min_{\{a\}} \sum_{n=1}^{9} \left(\sum_{i=1}^{4} \sum_{j=1}^{3} \sum_{k=1}^{8} \sum_{m=1}^{7} \left(C_{i,k} - W * \frac{S_{i,j}^{m} * F_{i,k} * E'(\tilde{x}_{i,j} \mid \zeta_{a,n})}{A_{i,j}} \right) b_{i,j,k}^{m} \right) p(\zeta_{a,n}) + \chi_a , \quad (4.15)$$

where χ_a is the expected cost (\$/yr) of the research action, as described in section 4.4.2. The cost of the no research action, χ_0, is zero. Therefore, a research action will only be optimal if the cost of the research action and probability weighted optimal objective function value is less than the optimal objective function value under the no research action. When it is optimal to perform a research action, the no research action is displayed for comparison. The values in Table 4-12 are the probability weighted tons of sediment reduced by each of the optimal management actions, which are found by weighting the tons reduced for each observation by the probability of that observation. The values obtained following individual observation outcomes will vary, of course, because observations affect the posterior distribution of loadings. For each BMP in each watershed j, the probability weighted tons reduced is defined as follows

Critical Area Planting (CAP) = $$\sum_{n=1}^{9} \left(\sum_{m=1}^{3} \frac{S_{F,j}^{m} F_{F,1} E'(\tilde{x}_{F,j} \mid \zeta_{a,n}) b_{F,j,1}^{m}}{A_{F,j}} \right) p(\zeta_{a,n}) \quad (4.16)$$

Conservation Tillage (CT) = $$\sum_{n=1}^{9}\left(\sum_{m=4}^{7}\frac{S_{F,j}^{m}F_{F,2}E'(\tilde{x}_{F,j}\,|\,\zeta_{a,n})b_{F,j,2}^{m}}{A_{F,j}}\right)p(\zeta_{a,n}) \quad (4.17)$$

Streambank Stabilization (SS) = $$\sum_{n=1}^{9}\left(\frac{S_{S,j}^{1}F_{S,3}E'(\tilde{x}_{S,j}\,|\,\zeta_{a,n})b_{S,j,3}^{1}}{A_{S,j}}\right)p(\zeta_{a,n}) \quad (4.18)$$

Streambank Restoration (SR) = $$\sum_{n=1}^{9}\left(\frac{S_{S,j}^{1}F_{S,4}E'(\tilde{x}_{S,j}\,|\,\zeta_{a,n})b_{S,j,4}^{1}}{A_{S,j}}\right)p(\zeta_{a,n}) \quad (4.19)$$

Land Retirement (LT) = $$\sum_{n=1}^{9}\left(\sum_{m=1}^{4}\frac{S_{R,j}^{m}F_{R,5}E'(\tilde{x}_{R,j}\,|\,\zeta_{a,n})b_{R,j,5}^{m}}{A_{R,j}}\right)p(\zeta_{a,n}) \quad (4.20)$$

Drainage Pipe (DP) = $$\sum_{n=1}^{9}\left(\sum_{m=1}^{4}\frac{S_{R,j}^{m}F_{R,6}E'(\tilde{x}_{R,j}\,|\,\zeta_{a,n})b_{R,j,6}^{m}}{A_{R,j}}\right)p(\zeta_{a,n}) \quad (4.21)$$

Toe Protection (TP) = $$\sum_{n=1}^{9}\left(\sum_{m=1}^{2}\frac{S_{B,j}^{m}F_{B,7}E'(\tilde{x}_{B,j}\,|\,\zeta_{a,n})b_{B,j,7}^{m}}{A_{B,j}}\right)p(\zeta_{a,n}) \quad (4.22)$$

Complete Stabilization (CS) = $$\sum_{n=1}^{9}\left(\frac{S_{B,j}^{1}F_{B,8}E'(\tilde{x}_{B,j}\,|\,\zeta_{a,n})b_{B,j,8}^{1}}{A_{B,j}}\right)p(\zeta_{a,n}) \quad (4.23)$$

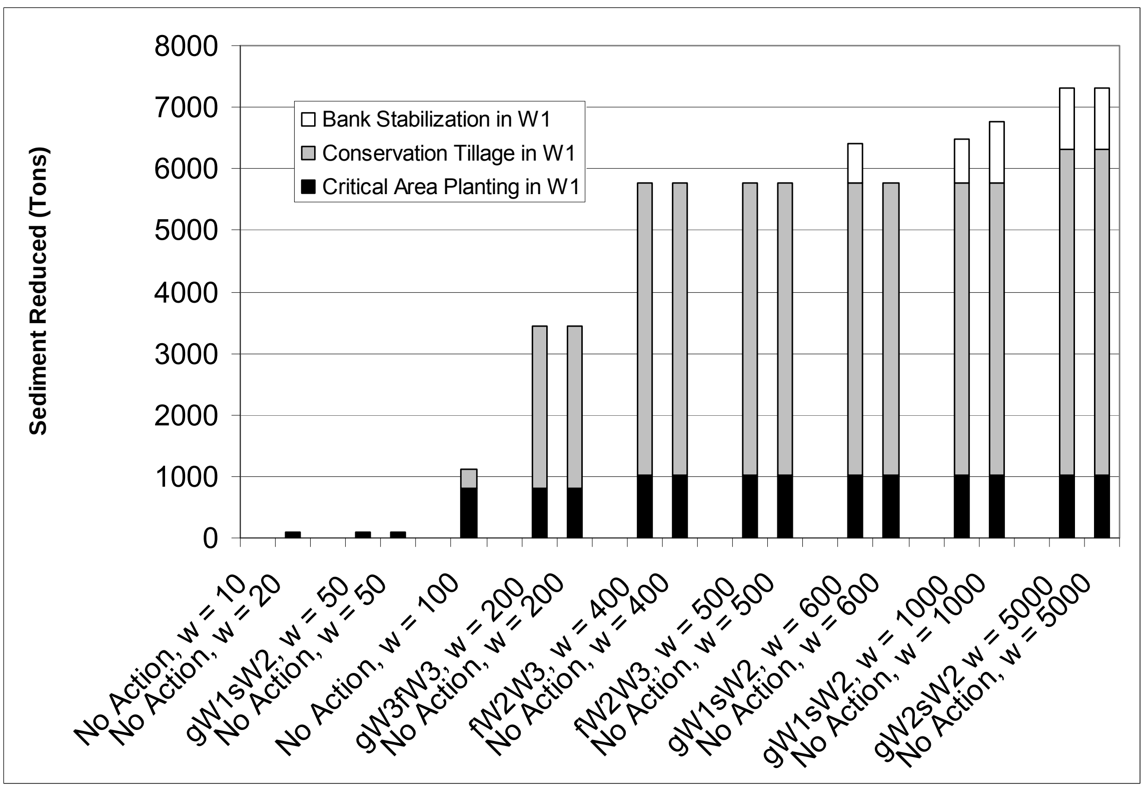

Figure 4-8: Optimal BMPs in Watershed W1 (Expected Unabated Sediment Loss = 90,000 tons)

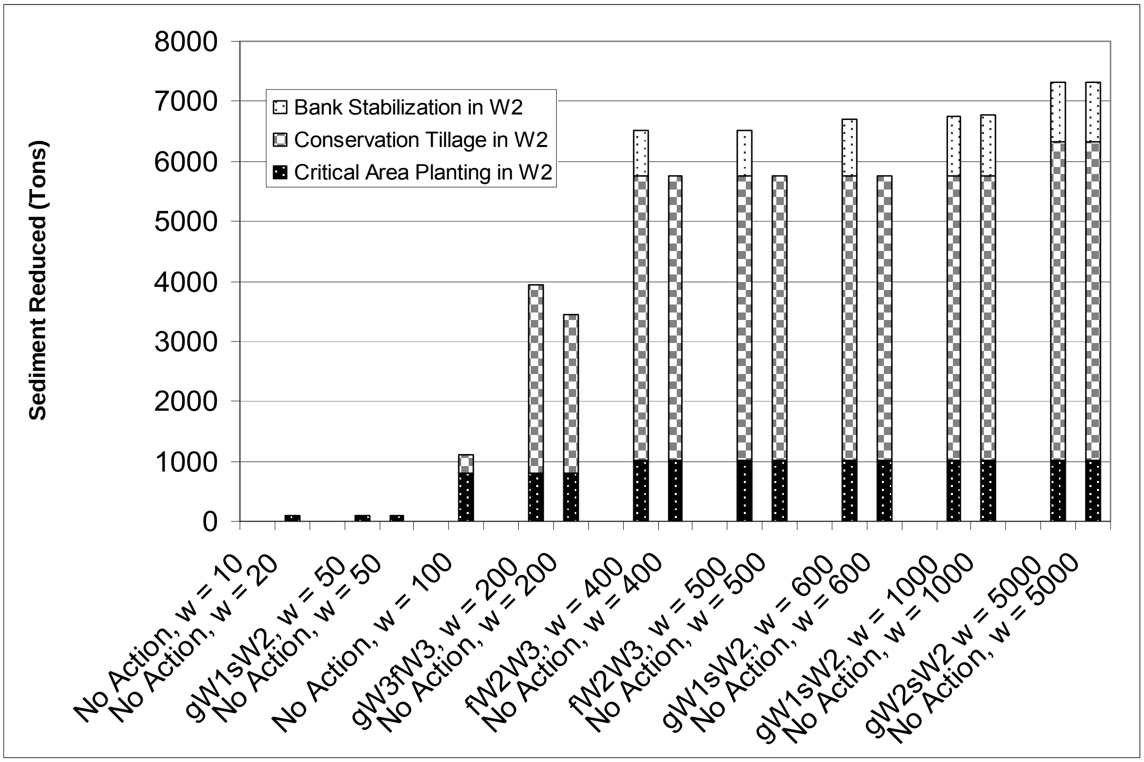

Figure 4-9: Optimal BMPs in Watershed W2 (Expected Unabated Sediment Loss = 90,000 tons)

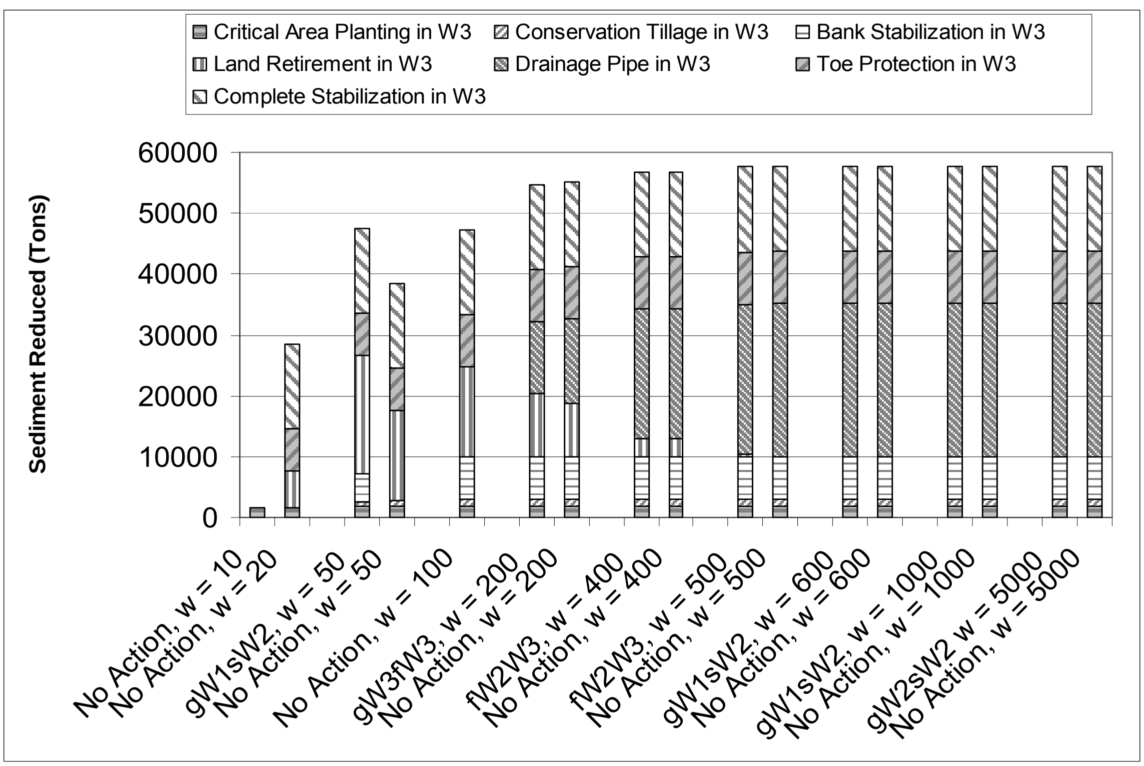

Figure 4-10: Optimal BMPs in Watershed W3 (Expected Unabated Sediment Loss = 90,000 tons)

The last line of Table 4-12 displays the maximum expected tons reduced to illustrate how much of the total achievable reduction is accomplished with each BMP.

Figure 4-8, Figure 4-9, and Figure 4-10 graphically compare the probability weighted tons reduced for each BMP under each of the optimal research actions and compares these values to the tons reduced for each BMP under the no research action for each sediment objective weight.

Table 4-13: Implied cost effectiveness assuming expected prior loadings

Cost Effectiveness ($/ton)	Actions in Watershed W1			Actions in Watershed W2			Actions in Watershed W3						
	CAP	CT	SS	CAP	CT	SS	CAP	CT	SS	LT	DP	TP	CS
Low	15	69		15	69		7	33		10	33	10	
High	210	2074		210	2074		72	990		40	130	81	
Average	100	1071	919	100	1071	919	38	293	66	25	80	45	15

Table 4-13 shows the implied cost effectiveness for each action based on the expected prior loadings. Since reductions are a function of the soil loss curve, the low, high and average reductions are listed. For streambank stabilization and complete stabilization, there is only one segment of the soil loss curve, and therefore only an average effectiveness value. (The posterior cost effectiveness numbers for particular observation values will in general differ from these values because posterior expected total loadings from various sources can be higher or lower than the prior expectations.)

This table shows that many actions will not be taken unless the penalty for sediment loss is above $100, but that some are worth doing even for penalties of $10/ton or less. Thus, it is not surprising that for sediment weights of $1/ton and $5/ton, the optimal management action is to do nothing. None of the management actions is cost effective at these sediment weights. When the weight is increased to $10/ton, it is optimal to employ critical area planting in W3 and to not perform any research action. According Table 4-13, the effectiveness of critical area planting in W3 is $7/ton when applied to the areas in the watershed contributing the most sediment. As the sediment objective weight is further increased to $50/ton, critical area planting in both W1 and W2 enter the solution along with land retirement, toe protection and complete stabilization, which goes immediately to its upper bound. At this sediment reduction weight, it is optimal to perform research in the form of either gauging in W1 and a sediment source analysis in W2 (a = 21), or gauging in W2 and a sediment source analysis in W1 (a = 29). Since watershed W1 and W2 are symmetric, these two actions produce nearly identical objective function values, with minor differences resulting from sampling error in the generation of the posterior distributions.

The optimal management actions resulting from these research actions differ from the optimal management actions under the no research action in one fundamental way: streambank stabilization is optimal under some outcomes when the research action is performed. In particular, there are four research action observations of gauging in W1 and SSA in W2 that lead to streambank stabilization in W3 – observations 3, 6, 7, and 8, as shown in Table 4-11. These four observations all lead to large expected posterior loadings for streambanks in W3 (see appendix for details) compared to the expected prior loadings for streambanks in W3 (7000 tons/yr). Observations 3, 6, and 7 each have large observed values for streambanks loadings in W2 resulting from the SSA component of the action. Since streambank loadings between W2 and W3 are moderately correlated, these large observations of streambank loadings in W2 result in large posterior loadings of streambanks in W3. Observation 8 also results in large streambank loadings in W3.

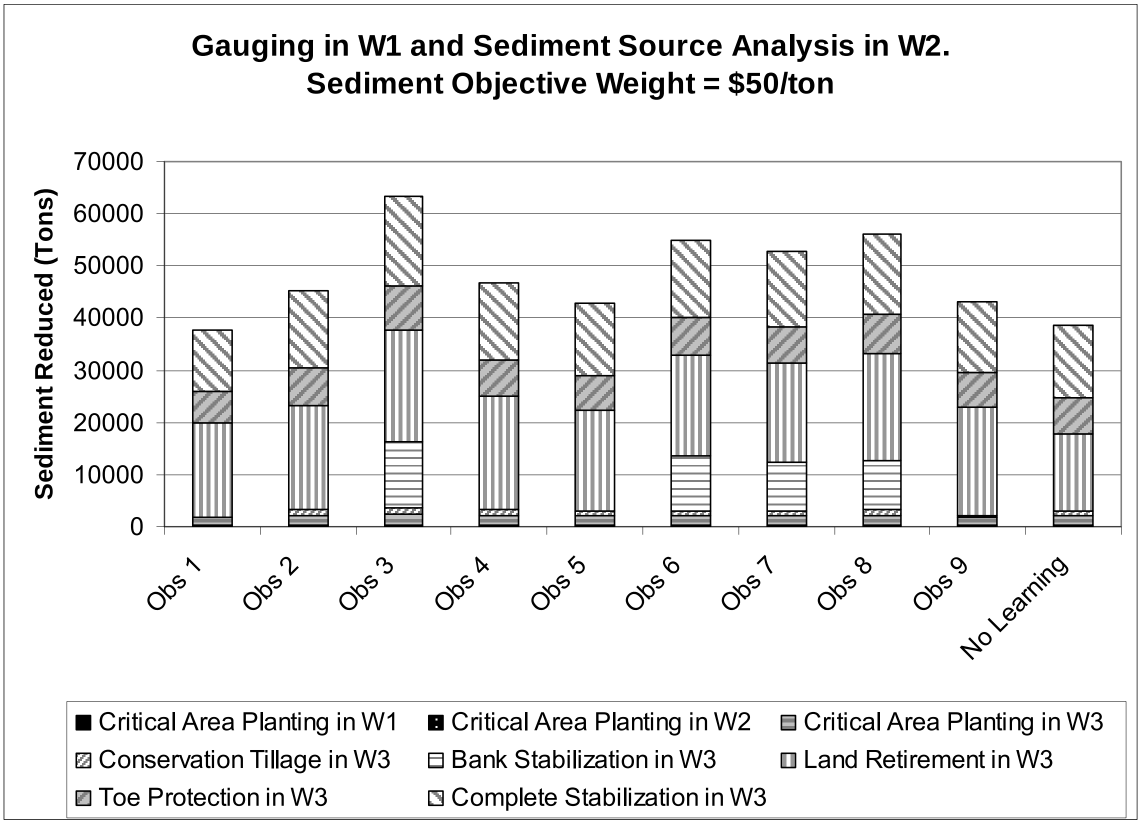

Figure 4-11: Optimal BMPs for Each Observation of Gauging in W1 and a Sediment Source Analysis in W2

Observation 8 has a large observed value for gauging in W1, a moderate observed value for streambanks in W2, and a low observed value for fields in W2. Since fields in W1 and W2 are strongly correlated, the low observed field loadings in W2 pushes down the field loading in W1; however, the large gauging observation counteracts the low field observation, resulting in a moderate expected posterior field loading. The large gauging observation in conjunction with the moderate observed streambank loading in W2 leads to a moderately large streambank loading in W3, again due to correlations between streambank loadings in the upper and lower watersheds. Figure 4-11 illustrates the different management actions selected for each outcome, as well as the management action selected under the no research action.

Table 4-14: Expected Value of Imperfect Information for Research Actions

Action	Cost ($/yr)	EVII ($1000/yr) for each weight											
		1	5	10	20	50	100	200	400	500	600	1000	5000
1 or 7	10	0	0	0.53	0	5.50	0.35	1.42	13.46	31.86	72.44	211.55	0
2 or 9	50	0	0	1.15	0	2.60	0.23	3.01	1.71	13.04	39.98	122.96	0
3 or 8	50	0	0	1.32	0	7.97	0.08	2.61	10.51	47.19	95.49	259.17	0
6 or 11	50	0	0	4.25	1.38	27.18	2.07	7.66	13.46	46.66	105.20	291.15	0
5	20	0	0	0.47	0.00	4.98	0.09	1.49	12.87	29.90	62.30	184.52	0
6 or 11	20	0	0	4.47	2.62	17.96	4.22	12.79	100.73	178.49	248.32	495.27	10.12
12	10	0	0	4.11	1.47	0.49	2.56	10.69	2.92	3.41	0.84	39.16	0
13 or 14	50	0	0	7.63	0.23	7.69	5.01	17.69	9.53	0	5.23	128.28	0
15	50	0	0	1.89	1.81	5.40	8.56	5.74	3.50	2.73	3.71	57.54	0
16	15	0	0	1.66	0.00	6.44	0.26	5.58	34.79	52.59	71.93	120.67	0
17 or 27	25	0	0	2.45	0.03	13.85	0.21	5.28	47.00	80.80	116.00	228.87	0
18 or 26	25	0	0	2.67	0.00	18.84	0.21	4.59	62.40	106.46	170.41	352.83	0
19 or 34	25	0	0	7.14	4.31	7.16	11.84	16.07	12.32	10.96	14.99	236.56	0
20 or 30	55	0	0	5.45	0.00	38.24	19.79	12.33	34.83	127.17	221.47	624.68	497.34
21 or 29	55	0	0	6.71	0.00	55.43	54.44	17.12	32.47	174.07	333.67	843.77	487.34
22 or 31	55	0	0.12	9.23	13.22	33.61	42.12	28.48	74.37	122.27	168.68	316.77	0
23	30	0	0	2.66	0.00	1.74	0.27	6.62	26.85	71.15	121.45	318.80	0
24 or 32	30	0	0.001	11.06	4.28	5.23	12.45	28.39	47.65	46.59	52.85	177.94	0
33	15	0	0.02	5.11	0.88	3.63	3.33	12.83	4.71	3.44	7.40	22.59	0
34 or 35	25	0	0	7.94	3.17	22.93	5.79	19.01	86.55	144.27	191.92	396.55	4.93
36	25	0	0	11.26	10.25	6.97	18.50	26.31	31.28	21.75	4.14	158.95	0
37 or 38	55	0	0	7.18	1.56	9.72	7.00	18.94	39.72	47.05	63.30	71.74	0
39	55	0	0	6.14	0.01	10.38	6.68	16.85	8.33	0	10.54	105.62	0
40 or 43	40	0	0	1.80	0	3.18	0.31	3.35	0	11.00	92.30	377.53	0
41	80	0	0	1.74	0	12.78	0.13	8.37	43.69	95.39	146.74	221.48	0
42 or 44	80	0	0.08	11.34	2.45	32.41	15.29	35.16	30.91	96.19	177.63	400.71	0
45	40	0	0.17	13.21	5.94	40.19	19.13	39.02	41.62	43.66	90.76	368.98	0

While it is optimal to perform a research action when the sediment reduction weight is $50/ton, when the weight is further increased to $100/ton, it turns out that it is optimal to do no research. However, as the weight on the sediment objective is increased even further, it is always optimal to perform a research action. This is due to the fact that at a weight of $100/ton, each research action that produces a different set of optimal management actions as compared to the no research action would cost more to implement than the value of the information it would yield (i.e., the improvement in the weighted value of cost and sediment). For example, consider action 21, gauging in W1 and SSA in W2. As described above, when the sediment objective weight is $50/ton, there are four observations that lead to streambank stabilization in W3. Under the no research action, streambank stabilization in W3 is not part of the optimal solution. When the weight is increased to $100/ton, streambank stabilization is performed under research action 21 for all but three observations, 1, 4, and 9; these three observations have very low posterior streambank loadings in W3. Under the optimal solution of the no research action, streambank stabilization in W3 is implemented. The value of performing research action 21 lies in dictating under what conditions to perform streambank stabilization in W3; however, the value of this information is slightly less than the cost of performing the research action, and therefore is it optimal to not perform any learning and implement streambank stabilization in W3.

As the weight on sediment is further increased, the value produced from research outweighs the cost for at least one research action, making it worthwhile to invest in research. Table 4-14 shows which research actions (highlighted in black) have a value of information that exceeds the cost of the action for each value of the sediment objective.

To further investigate the differences between the “no research” action and the optimal research action Figure 4-12, Figure 4-13, and Figure 4-14 show, for each watershed, the optimal BMPs for three actions: no research action; gauging in W1 and a sediment source analysis in W2; and fingerprinting in W2 and W3 concurrently. Each line represents each BMP in the optimal solution. The y-axis shows the rescaled value of the BMP as compared to its maximum possible value. For example, a value of 0.5 indicates that the BMP is in the solution at 50% of its maximum possible value. When a research action is undertaken, the expected level of BMP implementation is shown, weighted by the probability of each outcome. The x-axis shows the sediment objective weight.

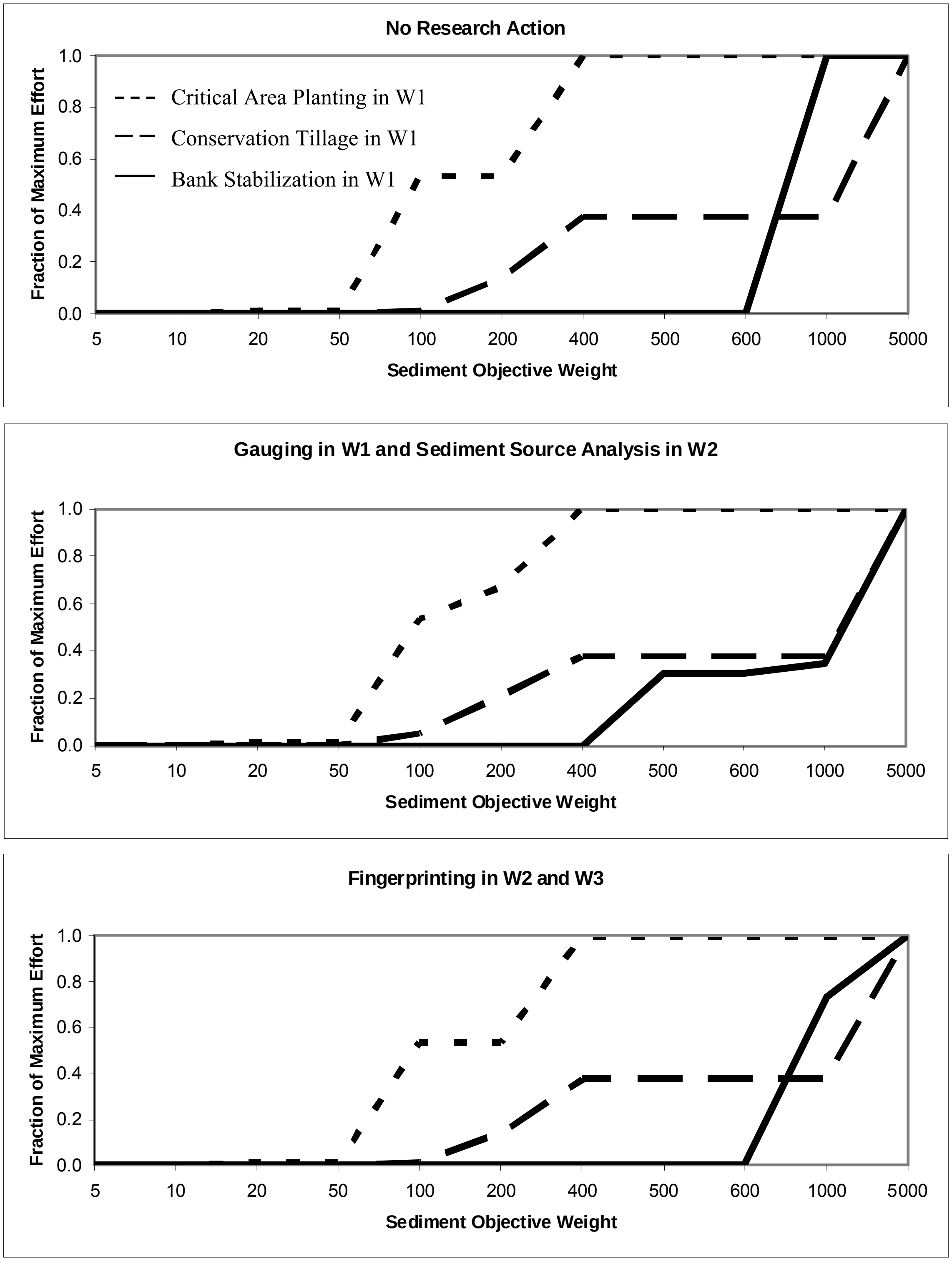

Figure 4-12: Expected Level of BMP Implementation in Watershed W1, Averaged over Research Outcomes

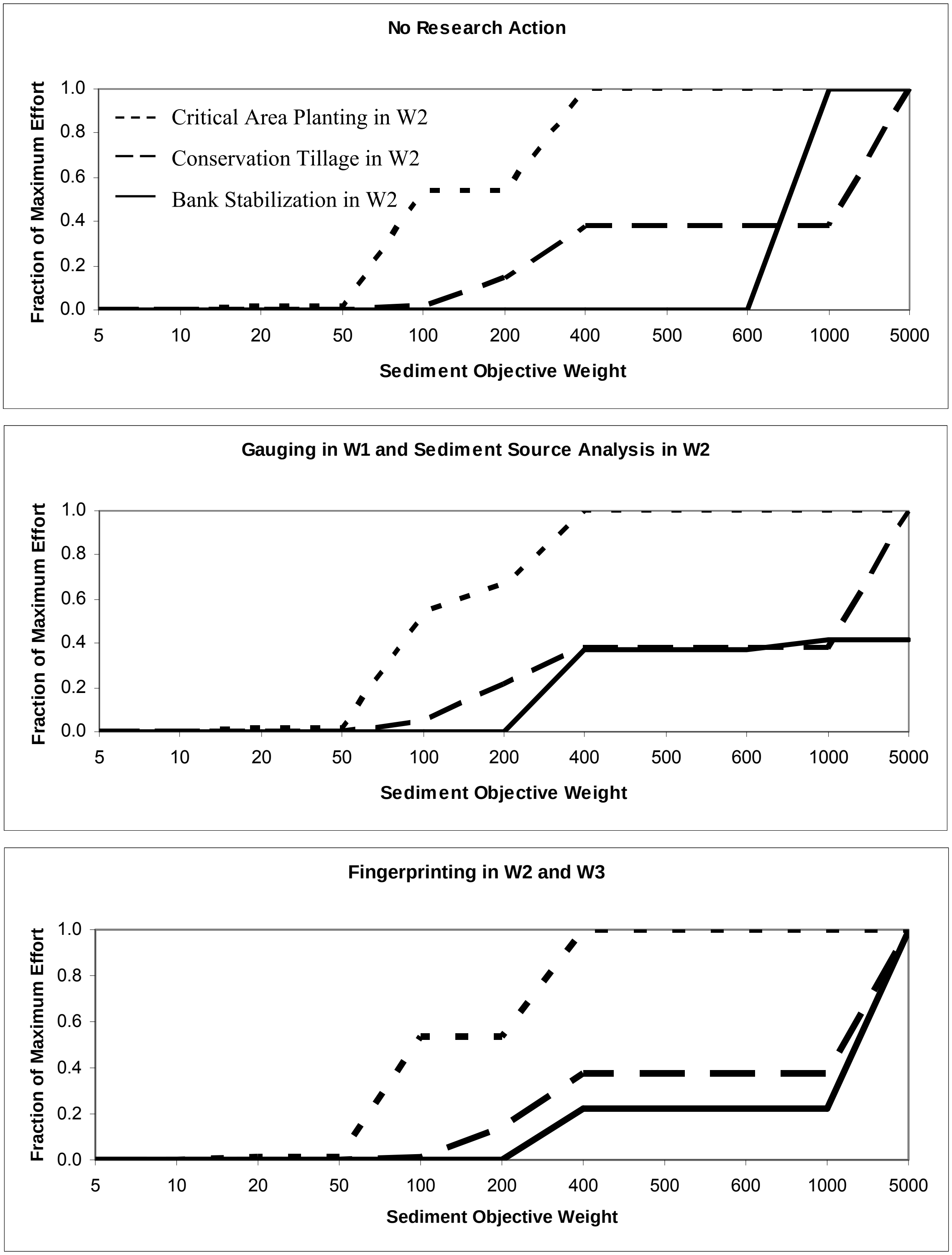

Figure 4-13: Expected Level of BMP Implementation in Watershed W2, Averaged over Research Outcomes

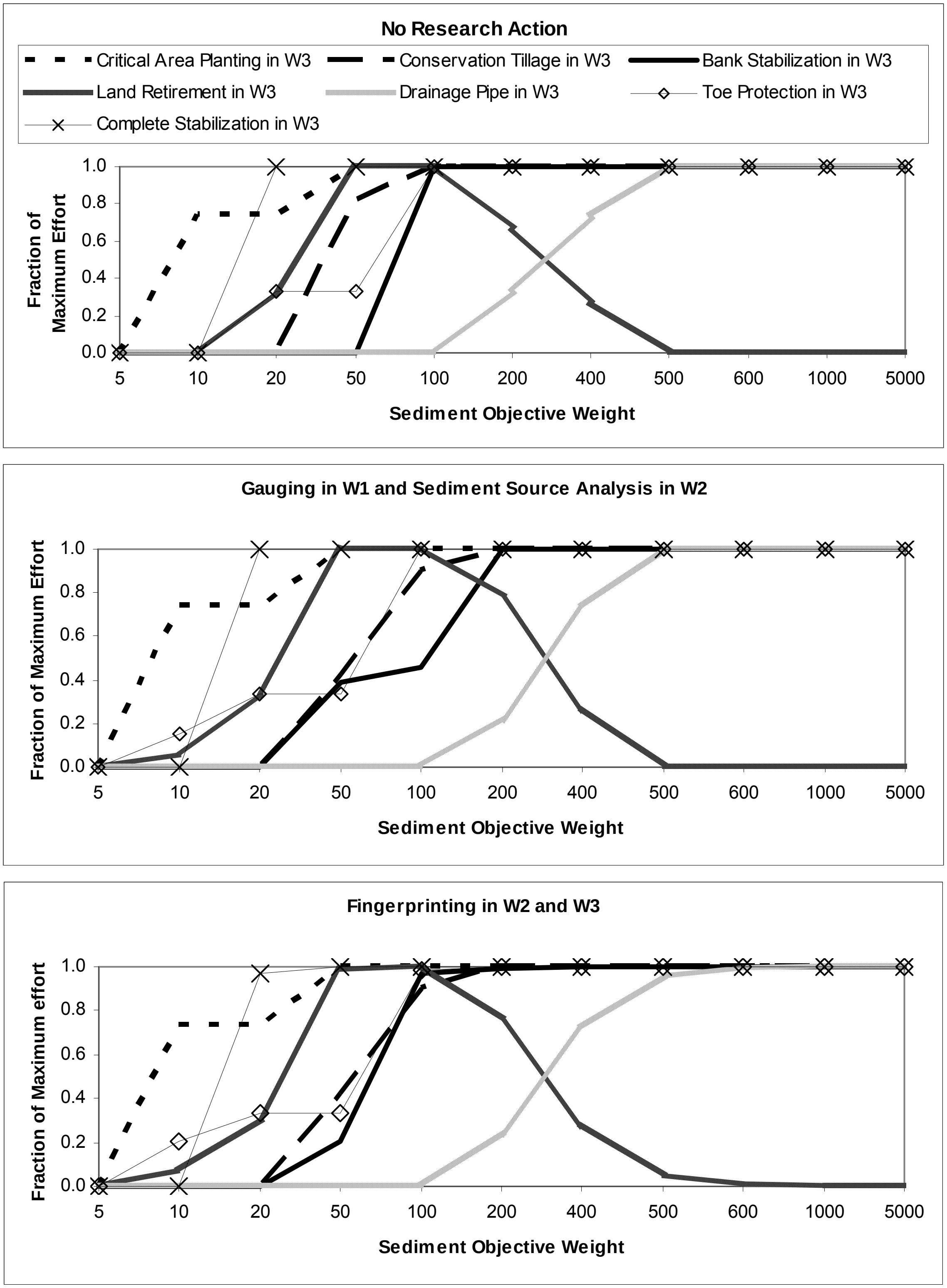

Figure 4-14: Expected Level of BMP Implementation in Watershed W3, Averaged over Research Outcomes

In watershed W1, the difference between the no research action and the other two research actions indicates the value of streambank stabilization. For the no research action, once streambank stabilization enters the solution, it goes immediately to its upper bound, while the transition is more gradual for the other two actions. This is due to the fact that for sediment reduction weights less than \$5000/ton, streambank stabilization is only chosen for a subset of the observed research action outcomes. For example, for fingerprinting in W2 and W3 (a = 11), streambank stabilization in W1 only enters the solution for observations 1 – 4 at a sediment weight of \$1000/ton (because those observations indicate relatively high levels of streambank sediment), and then enters the solution for all observations at a weight of \$5000/ton.

Similarly, differences exist in the value of streambank stabilization in W2. For the no research action, streambank stabilization again transitions immediately to its upper bound, while for gauging in W1 and sediment source analysis in W2, streambank stabilization never exceeds 50% of its upper bound. This is again due to the differences between the outcomes of the research action. Note too that the management actions selected in W2 differ from W1. This is due to the fact that the research actions have different impacts in each watershed. For action 21, gauging in W1 produces estimates of the total loadings in W1, while SSA in W2 produces estimates of streambank and field loadings. Thus, the different observations translate to different posterior loadings and therefore different management actions between the two watersheds.

Differences can also be seen in watershed W3 between the research actions and no action. The differences are particularly evident for streambank stabilization and toe protection. This is again due to the different outcomes resulting from each research action, as discussed above. In general, there are two management actions most affected by observation outcomes. First, when posterior streambank loadings are high and the weight on the sediment objective is also adequately large, streambank stabilization enters the solution. As mentioned above, four observations from actions 21 and 29 produce large expected posterior streambank loadings in W3; however, streambank stabilization only enters the solution when the weight on the sediment reduction objective reaches $50/ton. Similarly, there are two observations that produce large expected posterior streambank loadings in W1; however, streambank stabilization in W1 only enters the solution when the sediment reduction weight is $400/ton or greater.

The other management actions most influenced by observations are land retirement and drainage pipe. For low sediment weights, the preferred action is land retirement because it is cheaper to implement, but it is also less efficient, with a fractional reduction of 0.7. As the weight on sediment reduction is increased, it becomes cost effective to install pipe, since it has a fractional reduction of 0.9. As Table 4-12 indicates, the transition from land retirement to drainage pipe begins to happen at a sediment reduction weight of $200/ton. Consider the optimal action for this sediment weight, gauging and fingerprinting in W3 (a = 36). When the expected posterior ravine loadings are high, as in observations 3 and 6, the tons reduced by drainage pipe exceed the tons reduced by land retirement. When the expected posterior ravine loading is low, as is the case in observation 7, the total ravine tons reduced goes down, with a particularly large decrease in tons reduced by land retirement. The impact of the individual observations on the management actions selected is illustrated in Figure 4-15.

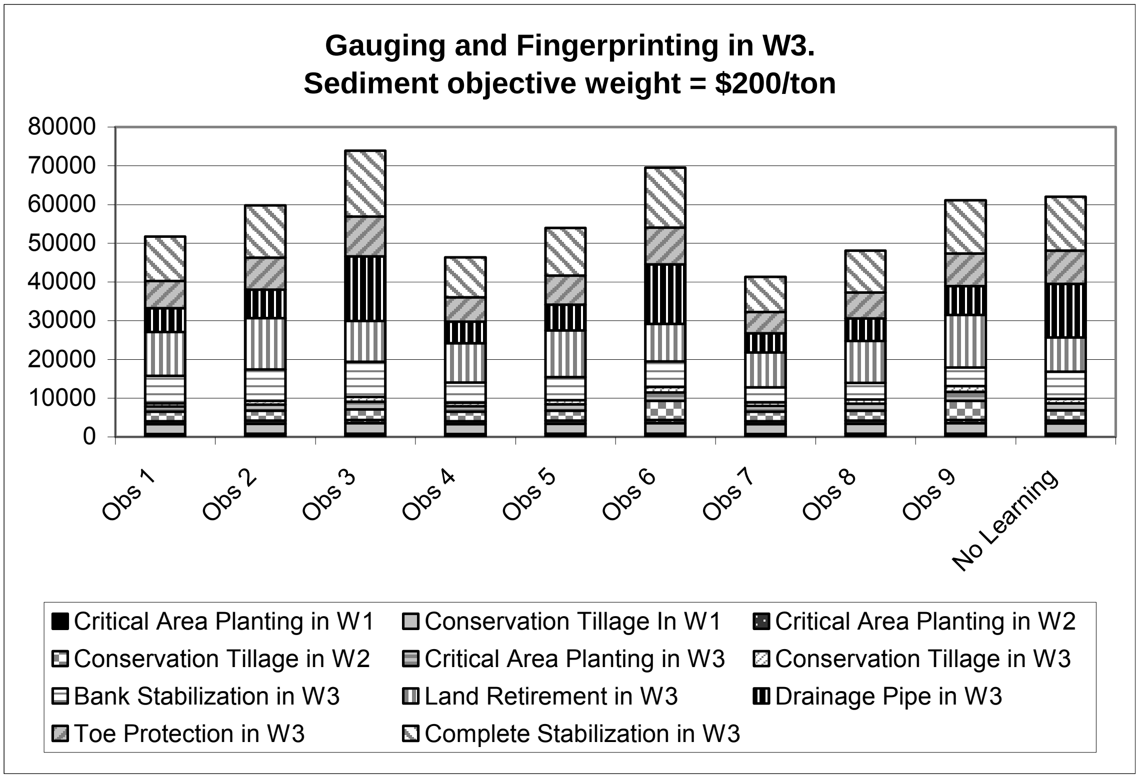

Figure 4-15: Optimal BMPs for Each Observation of Gauging and Fingerprinting in W3 Concurrently

Note that streambank restoration is never chosen under any research action or sediment reduction weight. This is due to the fact that streambank stabilization can be applied to the same streambanks as restoration, but is cheaper and just as effective. Therefore, streambank restoration will never be selected if the only objectives are to minimize cost and minimize sediment remaining; however, streambank restoration has other benefits, such as aesthetics. These additional benefits can be incorporated into the objective function of the multiobjective linear program, which could result in a different set of optimal research and management actions.

4.5.4 Value of Imperfect Information

Lastly, the expected value of imperfect information (EVII) for each research action and each weight is investigated. Table 4-14 shows the results of this analysis. The black boxes indicate that the EVII exceeds the cost of the action. Since watershed W1 and W2 are symmetric, several actions produce nearly identical EVII, with minor differences resulting from sampling error in the generation of the posterior distributions. To simplify the results, the actions that produce nearly equal EVII are represented by one set of values in Table 4-14.

For low values of the sediment objective weight ($1/ton and $5/ton), very few research actions have non-zero values for EVII because no sediment control actions have costs of less than $5/ton of sediment removed (see Table 4-13). As the sediment objective weight increases to $10/ton, all actions have increased values of EVII; however, when the weight is further increased to $20/ton, the value of information drops for all but two actions. This initial increase and decrease is due to fact that at first, learning provides value because certain observations result in the selection of different types of management actions compared to the "no research" action. For example, research action 24, gauging in W1 and W3, has an EVII of about $11,000/yr for a sediment weight of $10/ton. Under the no research action, it is optimal to only employ critical area planting in W3 (see Table 4-12), whereas for research action 24, it is optimal to employ critical area planting for all observations, as well as land retirement for observations 4,5, 7,8 and 9, toe protection for observations 4-9, and complete stabilization in W3 for observation 7. These observations (4,5,7,8 and 9) correspond to large expected posterior ravine loadings, and observations 4-9 correspond to large expected posterior bluff loadings, with observation 7 having the largest. Thus, at a weight of \$10/ton, there are different types of BMPs that are optimal under research action 24 compared with the no research action. When the sediment weight is increased to \$20/ton, the EVII for action 24 drops to \$4280/yr. At this sediment weight, critical area planting in all three watersheds, along with land retirement and toe protection are optimal management actions under the no research action. Under research action 24, these same management actions are optimal, yet the amount of each action differs across observations. In general, the value of information of a research action is larger when the research action selects different *types* of management actions instead of different *amounts* of the same management actions as compared with the no research action.

Table 4-14 shows an important trend. For composite actions performed in the same watershed, such as gauging and fingerprinting in W1 (a = 17), the EVII tends to be lower than if the same actions were performed in different watersheds, such as gauging in W1 and fingerprinting in W2 (a = 18). This trend is due to the fact that performing actions in two different watersheds provides more information than performing two actions in the same watershed. For example, gauging and fingerprinting in W1 provides improved understanding of only watershed W1, whereas gauging in watershed W1 and fingerprinting in W2 provides improved understanding of both watersheds.

There are several actions that have large EVIIs for many sediment weights. For example, there are 15 actions whose EVIIs exceed the cost of the action for sediment reduction weights between and including \$400/ton to \$1000/ton (e.g., action *1*, fingerprinting in W1). In general, the EVII rises and falls as sediment weights increase due to two main factors. The first is the result described above – research actions that cause different types of management actions to be selected have larger EVII compared to research actions that cause different amounts of the same management actions to be selected as compared with the no research action. The second factor is the existence of management actions that, under certain sediment weights, are adopted under some observations but not under others. As the weight increases, the actions are chosen under all observations, resulting in a drop in the EVII.

For example, at a sediment weight of $10/ton, action 13, fingerprinting in W3 and SSA in W1, has an EVII of $7600/yr. At this weight, all observations except 3, 4, and 8 result in the selection of land retirement and toe protection in the optimal solution. When the weight increases to $20/ton, land retirement and toe protection are selected under all observations and the EVII drops to under $1000/yr. When the sediment weight is larger, the observations do not provide enough additional information to cause significant changes in the management actions.

While there are different optimal actions for different sediment objective weights, several research actions are robust in that the value of information for the action is high for a range of sediment weights. In particular, actions 45 (SSA in W3), 21 (gauging in W1 and SSA in W2), and 29 (gauging in W2 and SSA in W1) each have an EVII that exceeds the cost of the action for 5 sediment weights, and actions 21 and 29 are optimal for sediment weights of $50/ton, $600/ton, and $1000/ton. It is not clear why these actions tend to perform well compared to other actions; however, the observed streambank loadings do seem to play a role. For example, when action 21 is optimal, the difference between the optimal management actions selected under this action as compared to the optimal management actions selected under the no research action tend to be due to differences in streambank stabilization. For example, at a sediment weight of $600/ton, action 21 selects streambank stabilization in watershed W1 and W2 for certain observations, while under the no research action, streambank stabilization is not selected in watersheds W1 and W2. Interestingly, the observations for action 21 do not produce posterior streambank loadings with higher variability as compared to other actions. For example, the posterior streambank loadings in W3 are more variable under action 11, fingerprinting in W1 and W3. Similarly, posterior streambanks loadings in W1 and W2 are more variable under research action 23, gauging in W1 and W2, as compared with action 21.

In general, the results indicate that a complex combination of factors determines which actions do well for each sediment weight. For example, there is no clear indication of where it is best to perform gauging. For certain sediment weights, such as $10/ton, it is more valuable to gauge in W3, as compared to W1 or W2; however, at a weight of $50/ton, the opposite is true. The lack of clean stories in the data indicates the complexity of factors influencing the outcomes. In particular, the observations and their associated probabilities impact both the posterior distributions as well as the expected sediment reduction of each set of management actions. As mentioned above, at one sediment weight, the observations can lead to a large EVII; however, at another sediment weight, the observations have less of an impact. Similarly, it is not clear how the magnitudes and variability of individual observations impact the outcome of management actions. This result supports the need for a systems approach to uncover the best combination of research and management actions.

While the results indicate that a complex combination of factors influence the optimal research and management actions, the framework and results are still useful for managing sediment reduction. First, if a manager can identify his or her preference for sediment reduction, the framework dictates which combination of research and management actions will optimally achieve the maximum sediment reduction at the least cost in an expected value sense. For example, if the manager feels that sediment reduction is ten times as important as cost minimization, the results indicate that it is best to manage the system based on the prior understanding of the sediment loadings, and to forego research. At this sediment reduction weight, it is best to employ only critical area planting in watershed W3. If the manager's preference for sediment reduction is 50 times that of cost minimization, the framework reveals that it would be best to invest in research to learn more about the location and magnitude of sediment loadings before managing the system. This investment in learning will allow better targeting of management actions, resulting in a lower overall cost. If the manager's preferences are not well defined, the framework can be used to investigate the impact of different preference weights.

4.6 Conclusions

This chapter developed a framework to determine the optimal combination of research and management actions to efficiently reduce sediment and turbidity. A complex case study representing the Maple River watershed was used to illustrate the framework. The methodology combined Bayesian inference and multiobjective linear programming to determine when it is best to act based on our current understanding of the physical system, and when it is cost effective to invest in research.

In general, the results demonstrate that the value placed on the sediment reduction objective has a strong impact not only on the extent of BMP implementation, but also on whether or not research actions are worthwhile. For low sediment objective weights, it is best to act based on the current belief about sediment loadings and implement low cost BMPs. As more value is placed on reducing sediment loading, research actions become worthwhile for determining which types of BMPs most effectively reduce sediment given the posterior loadings. However, when the value on sediment reduction reaches \$5000/ton, the value produced by nearly all research actions does not exceed the cost of performing the research. This reflects the fact that at very high sediment reduction preference weights, it is optimal to implement the highest level of management, except when research reveals that the sediment loadings are significantly lower than expected under the prior information.

Several of the research actions were robust in that they were cost effective for a range of sediment weights suggesting that these actions are good candidates when the value of sediment reduction is uncertain. However, the complexity of the problem prevents clear trends from being readily identified, indicating that a variety of interacting factors influence the results. Thus, in order to effectively reduce sediment, the manager must determine his or her values in terms of sediment reduction, or use the framework to investigate the impacts of different sediment reduction values.

While the framework presented here explicitly addresses uncertainty in the understanding of the physical system and the accuracy of various research actions, other major sources of uncertainty are disregarded or only addressed implicitly. In particular, the cost and percentage effectiveness of the sediment reduction management actions were assumed to be known. Further work should investigate the impact of the uncertainty on these parameters on the choices of management actions; for instance, if managers are risk averse, they may prefer BMPs that have less uncertainty, even if their expected cost per ton of removal is greater. Or there may exist management or research actions that could provide valuable information on the relative costs and effectiveness of different BMPs. In addition, sensitivity analysis can investigate the impacts of choosing different forms for the prior distributions or likelihood functions, as well as the consequences of disagreeing expert opinions. For example, the impact of different expected prior loadings can be investigated by rerunning the framework with new prior distributions and examining the research or management actions identified as optimal.

In general, the framework can help managers systematically sort through the potentially large combination of research and management actions. By investigating the potential outcomes of research using Bayesian inference, the framework allows managers to evaluate the benefits of research without having to wait for it to be implemented. Sensitivity analysis can also be used to inform the manager of which parameters in the model most influence the choice of actions.

The framework presented here is a useful tool for aiding in management decisions for sediment reduction. While the methodology relies on a simplified view of a complicated system, it is useful for illustrating the consequences of conducting research on the choice of management actions; however, the framework does have limitations. For example, the three watershed model used to illustrate the framework was a greatly simplified model of Maple River watershed. Despite the simplification, the results depended on a complex combination of factors. Thus, it is necessary to carefully consider the problem being addressed with this framework, and to include only as much detail as is necessary. By carefully considering the problem at hand, the framework presented in this chapter can successfully be used to illustrate the consequences of conducting research on the choice of management actions.

Chapter 5
Conclusions

The research presented in this dissertation developed three novel approaches for optimally managing environmental management problems utilizing approaches from the fields of environmental systems analysis and decision analysis: designing nature reserves for species protection, addressing behavior biases affecting preferences in the context of environmental externalities in electricity generation planning, and reducing sediment to impaired water bodies.

The first study successfully developed two novel integer programming models for determining nature reserve sites with irreplaceability value of 1.0, meaning that the site appears in all alternate optimal sets. The first model efficiently identified irreplaceable sites in the context of the Species Set Covering Problem, and the second model identified irreplaceable sites in the context of the Maximal Covering Species Problem. Using data on terrestrial invertebrates in the state Oregon, the research revealed that the designation of a site as irreplaceable is not an intrinsic quality of the site, but rather, the irreplaceability is context dependent, depending on the total number of sites available for selection.

While the irreplaceable value of a nature reserve site is context dependent, the implications of identifying irreplaceable sites can still be extremely important to decision makers. Understanding which sites are required to protect a maximum number of species at a given resource level allows the decision maker flexibility in the reserve design. The set of irreplaceable sites can be protected first, while the combination of additional sites that best complements the irreplaceable sites can be addressed in the future, possibly by considering a variety of criteria for these additional sites. The knowledge gained from identifying which sites are irreplaceable allows for improved decision making and environmental management aimed at protecting vulnerable species.

While the models presented in Chapter 2 successfully identified irreplaceable sites, further work can be done to address the shortcomings of the model. For example, reserve design should concurrently consider all objectives, such as the spatial composition of the sites. If constraints on connectivity of sites were incorporated into the model, it is likely that the set of irreplaceable sites identified would be different. By considering all reserve objectives simultaneously, the contribution of each site can be described in a way that is more relevant for species management. The model could also be expanded by considering species data other than presence-absence data. The presence of a species in a particular site says nothing about its ability to survive or thrive in that site. If instead, the probability of survival of each species in each site was considered, the set of irreplaceable sites that protect the maximum number of species with a user-prescribed reliability could be determined.

While the first study identifies alternate optimal solutions to single objective problems, the second study addresses alternate solutions to problems with multiple conflicting objectives, using approaches from the field of decision analysis. When more than one alternative solution exists for a multiobjective problem, preference weights can be used to determine the preferred alternative; however, certain techniques aimed at eliciting decision makers' preferences are prone to biases. The second research study in this dissertation quantified and mitigated biases that appear when using value trees to aid in preference weight elicitation. This research considered the problem of selecting among electricity generation planning alternatives, while considering environmental externalities.

The framework developed in Chapter 3 successfully determined a debiased set of preference weights that better reflected the preferences of the decision makers in the absence of value tree induced-bias, and investigated the impact of making decision using biased weights. This research can be useful for debiasing preference weights when time is limited. In these cases, it is often not possible to investigate the source of the bias and perform additional tasks to identify preference weights that resonate with the decision makers.

While this research successfully demonstrated the usefulness of the model for debiasing weights, the small sample size used in the research prevented generalizations about the method from being made. To improve upon this work, larger datasets should be used to investigate the performance of the model. It would also be informative to present the debiased weights to the decision makers to ask whether the debiased weights better represent their preferences. This would provide the most powerful evidence that the weights produced by the model are better representations of the decision maker's beliefs.

The third research study addressed water quality impairments in the Minnesota River basin using a multiobjective, stochastic approach. Chapter 4 presented the successful development and application of a framework that combines expert elicitation, Bayesian inference and multiobjective linear programming to identify the optimal set of management actions and research actions that minimize both the expected cost and expected sediment loadings to waterways in the Minnesota River basin. The multiobjective stochastic framework can be expanded in several ways. First, other sources of uncertainty can be included in the framework, such as uncertainty in the costs and sediment reduction effectiveness of the management actions. In addition, the framework can be expanded to consider sediment reduction at a finer spatial scale. Other areas for future work include the incorporation of other ancillary objectives, such as the aesthetics of stream restoration, and expanding the geographic scope to that consistent with the TMDL decision making – for instance, the full Minnesota River Basin.

The shared goal of the methods developed in this dissertation is to help managers improve environmental management decisions. The common thread is the use of environmental systems analysis to produce optimal management decisions. This dissertation has illustrated the range of applications in which systems analysis and decision analysis can be used to improve choices in environmental management.

Each method has applicability beyond the particular case used in its development. The models developed to determine irreplaceable nature reserve sites can be used to determine the decision variables that appear in all alternate optimal solutions. For many combinatorial problems, alternate optimal solutions exist. When this is the case, it is often useful to identify these alternate solutions to provide the decision maker with flexibility in his or her decision. For example, the problem of determining the optimal combination of obnoxious facilities, such as waste disposal sites, that impact the least number of people can be explored with the models in Chapter 2. The set of locations that must be included in any optimal facility design can be determined. This set identifies the facilities that appear in all alternate optimal solutions. Knowing this set allows the decision maker to implement the design beginning with the irreplaceable facilities.

Similarly, the method for debiasing weights in Chapter 3 can be applied to variety of environmental management problem. In particular, the methodology can be used to identify and mitigate weighting biases in any problem in which two sets of preference weights are elicited with two different value trees. For example, further objectives can be included in the sediment reduction framework in Chapter 4, and the relative importance of each objective can be elicited with the use of two value trees. If the weights suffer from value tree-induced weighting biases, debiased weights can be determined, and then used to select the optimal management actions to reduce sediment loadings.

The framework developed in Chapter 4 can also help decision makers to assess important tradeoffs between information acquisition and abatement effort in other contexts. These types of tradeoffs are extremely common in environmental management problems. Decision makers must often choose between learning more about the problem (and thus possibly delaying the solution of the problem), and acting with limited knowledge (and therefore risking expending resources on ineffective solutions). While the methodology of Chapter 4 is applied to sediment reduction, the same concepts can be extended to other environmental management problems. For example, nature reserve design can be examined with this framework. If uncertainty exists about the success of protecting different nature reserves, the decision maker must choose between learning more about how the protection measure will perform, and protecting species based on the current level of understanding. While it may be costly in the short term to investigate the performance of protection measures, the long term benefits of improved understanding may incur a lower overall cost.

Overall, this dissertation has developed three widely applicable approaches that can improve environmental management. The methods incorporate techniques from environmental systems analysis and decision analysis to provide decision makers with tools to aid in managing complex, real-world environmental problems. As is the case with any model, simplifications were required to represent complex systems in manageable ways. Despite these simplifying assumptions, the novel approaches developed in this dissertation can lead to improved choices regarding the management of environmental problems.

Appendices

Appendix I: Chapter 3 Data and Results

Table A.I- 1: Weights Elicited with the Nonhierarchical (NH) Value Tree and Hierarchical (H) Value Tree

Subject	Tree	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}
S1	NH	0.122	0.099	0.153	0.107	0.076	0.061	0.069	0.061	0.061	0.046	0.031	0.015	0.099
	H	0.150	0.000	0.300	0.050	0.080	0.013	0.040	0.023	0.023	0.011	0.000	0.010	0.300
S2	NH	0.000	0.000	0.500	0.300	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.200
	H	0.030	0.030	0.360	0.180	0.007	0.011	0.018	0.001	0.001	0.006	0.008	0.050	0.300
S3	NH	0.150	0.070	0.200	0.100	0.030	0.020	0.010	0.100	0.040	0.050	0.100	0.080	0.050
	H	0.050	0.025	0.400	0.025	0.008	0.006	0.006	0.024	0.008	0.008	0.040	0.200	0.200
S4	NH	0.015	0.008	0.153	0.137	0.107	0.092	0.092	0.061	0.076	0.046	0.031	0.061	0.122
	H	0.040	0.000	0.200	0.160	0.081	0.063	0.036	0.024	0.060	0.012	0.024	0.000	0.300
S5	NH	0.060	0.070	0.300	0.210	0.050	0.030	0.030	0.040	0.030	0.030	0.020	0.070	0.060
	H	0.000	0.000	0.450	0.150	0.060	0.020	0.020	0.030	0.020	0.040	0.010	0.100	0.100
S6	NH	0.048	0.190	0.333	0.143	0.048	0.019	0.019	0.019	0.019	0.010	0.048	0.010	0.095
	H	0.040	0.104	0.640	0.016	0.050	0.014	0.007	0.002	0.001	0.001	0.014	0.010	0.100
S7	NH	0.090	0.100	0.130	0.110	0.080	0.070	0.070	0.020	0.040	0.040	0.030	0.110	0.110
	H	0.060	0.120	0.240	0.180	0.023	0.018	0.010	0.025	0.015	0.008	0.003	0.150	0.150
S8	NH	0.137	0.157	0.196	0.176	0.098	0.020	0.010	0.010	0.010	0.020	0.098	0.029	0.039
	H	0.100	0.100	0.175	0.125	0.048	0.024	0.008	0.018	0.024	0.042	0.036	0.100	0.200
S9	NH	0.082	0.148	0.164	0.082	0.016	0.041	0.016	0.074	0.066	0.074	0.098	0.090	0.049
	H	0.060	0.180	0.240	0.120	0.008	0.016	0.016	0.018	0.006	0.018	0.018	0.150	0.150
S10	NH	0.101	0.131	0.202	0.182	0.091	0.040	0.030	0.020	0.010	0.051	0.061	0.040	0.040
	H	0.050	0.100	0.150	0.200	0.091	0.014	0.035	0.003	0.003	0.012	0.042	0.050	0.250
S11	NH	0.078	0.059	0.196	0.137	0.049	0.049	0.118	0.098	0.118	0.059	0.020	0.010	0.010
	H	0.040	0.040	0.240	0.080	0.008	0.056	0.011	0.068	0.113	0.034	0.011	0.100	0.200

Table A.I- 2: Rankings of Alternatives for Each Subject Using Nonhierarchical (NH), Hierarchical (H), and Model weights

Subject	Method	Ref	A	B	C	D	E	F	G	H	I	J	K	L	M	N
S1	NH	6	3	1	14	15	12	8	5	2	9	7	4	13	10	11
	H	3	2	9	12	14	10	8	6	1	7	5	4	13	11	15
	Model	4	3	8	13	15	12	7	5	1	6	9	2	10	11	14
S2	NH	1	4	7	6	11	5	8	10	3	12	13	14	15	2	9
	H	1	2	7	6	12	3	10	9	4	15	8	11	14	5	13
	Model	1	3	7	6	10	5	8	9	2	13	12	14	15	4	11
S3	NH	2	3	1	10	14	13	9	8	6	15	4	5	12	7	11
	H	1	2	3	9	13	8	11	10	7	15	4	5	12	6	14
	Model	1	3	2	8	11	13	10	9	7	15	4	5	12	6	14
S4	NH	8	6	7	14	13	15	10	3	4	1	5	2	11	12	9
	H	5	2	12	15	13	10	9	3	1	6	7	4	14	8	11
	Model	6	5	11	15	13	12	9	4	1	2	7	3	14	10	8
S5	NH	3	2	1	12	15	10	8	5	4	13	9	6	14	7	11
	H	4	3	1	12	15	13	9	8	2	5	10	6	14	7	11
	Model	4	3	1	12	15	13	10	6	2	8	9	5	14	7	11
S6	NH	4	3	1	9	15	5	7	6	2	14	12	11	13	8	10
	H	5	6	2	8	15	4	7	3	1	12	14	11	13	9	10
	Model	6	5	2	8	15	4	7	3	1	13	14	11	12	9	10
S7	NH	5	4	1	14	15	8	10	7	6	9	3	2	11	12	13
	H	2	3	1	10	15	5	11	9	6	14	4	8	13	7	12
	Model	2	3	1	10	15	4	11	9	6	14	5	8	13	7	12
S8	NH	3	2	1	11	15	4	7	6	5	14	9	8	13	10	12
	H	3	2	1	10	15	5	11	9	6	14	4	7	12	8	13
	Model	3	2	1	10	15	4	8	6	5	14	9	11	13	7	12
S9	NH	2	3	1	12	14	10	9	8	5	15	4	6	13	7	11
	H	2	3	1	8	15	4	11	10	6	14	5	9	12	7	13
	Model	2	3	1	7	15	4	9	8	5	14	10	11	13	6	12
S10	NH	5	2	1	10	15	6	8	4	3	14	9	7	13	11	12
	H	3	2	5	10	15	4	9	6	1	14	8	7	12	11	13
	Model	6	3	2	9	15	5	7	4	1	14	13	8	12	10	11
S11	NH	7	6	1	15	12	13	9	4	5	2	8	3	14	11	10
	H	1	3	2	14	12	13	9	8	5	10	4	6	15	7	11
	Model	2	3	1	14	12	13	11	8	4	7	6	9	15	5	10

Appendix II: Chapter 4 Probability and Observation Calculation and Results

II.1 Probability and Observation Calculation Procedure

To estimate discrete probabilities and observations of $\tilde{\mathbf{z}}_a$, the following procedure is used.

Step 1) If the observation has 1 element, for example gauging in W1, the range of possible values of $\tilde{\mathbf{z}}_a$ is divided into 9 ranges. If the observation has two elements, for example fingerprinting in W2 and W3, the range of each element is divided into 3 ranges, low, medium, and high. For observations with more than 2 elements, 9 Latin hypercube samples are determined. For example, fingerprinting & SSA in W3 produces observations of fingerprinting in W3 and fields, streambanks, ravines, and bluffs in W3. The range of possible values for each element is subdivided into three ranges: low (l), medium (m), and high (h). Thus, there are 3^5 or 243 combinations of the three ranges for the five elements. Nine sample ranges from the set of 243 are determined with Latin hypercube sampling, resulting in the following, as an example.

	Sample								
Element	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Fingerprinting in W3	l	m	h	h	l	m	l	m	h
Fields W3	l	m	h	l	h	m	l	h	m
Streambanks W3	l	m	h	m	h	l	h	m	l
Ravines W3	l	m	h	h	l	m	l	h	m
Bluffs W3	l	m	h	m	l	h	h	l	m

Step 2) Estimate the (joint) probability for each sample using Monte Carlo integration with antithetic sampling.

Step 3) From Monte Carlo results, estimate mean and covariance of $\tilde{\mathbf{z}}_a$.

Step 4) Determine the discrete probabilities and observations such that the means and covariances of the discrete distribution match the estimated means and covariances of the continuous distribution:

$$\min \sum_{n=1}^{9} \left(p(\tilde{\mathbf{z}}'_{a,n}) - p(\boldsymbol{\zeta}_{a,n}) \right)^2 \quad \text{(A.II.1)}$$

$$\text{subject to} \qquad \eta_{a,n,e} = \sum_{n=1}^{9} p(\zeta_{a,n}) \zeta_{a,n,e} \quad \forall e \in \mathrm{E} \quad \text{(A.II.2)}$$

$$\phi_{a,n,e,e'} = \sum_{n=1}^{9} \left(p(\zeta_{a,n})(\zeta_{a,n,e} - \eta_{a,n,e}) p(\zeta_{a,n})(\zeta_{a,n,e'} - \eta_{a,n,e'}) \right) \forall e, e' \in \mathrm{E} \quad \text{(A.II.3)}$$

$$\sum_{n=1}^{9} p(\zeta_{a,n}) = 1 \quad \text{(A.II.4)}$$

$$0 \leq p(\zeta_{a,n}) \leq 1 \quad \forall n = 1, 2, \ldots, 9 \quad \text{(A.II.5)}$$

$$\zeta_{a,n.e} \geq 0 \quad \forall e \in \mathrm{E},\ n = 1, 2, \ldots, 9 \quad \text{(A.II.6)}$$

$$\zeta_{a,n,e} = \zeta_{a,n',e} \quad \forall \text{ Latin Hypercube relationships,} \quad \text{(A.II.7)}$$

where

$p(\boldsymbol{\zeta}_{a,n}) :=$ discrete probability for observation n of action a (decision variable)

$\zeta_{a,n,e} :=$ discrete observation element e for observation n of action a (decision variable)

$p(\tilde{\mathbf{z}}'_{a,n}) :=$ estimated probability for observation n of action a

$\eta_{a,n,e} :=$ estimated mean for observation element e of observation n of action a

$\phi_{a,n,e,e'} :=$ estimated covariance between observation elements e and e' of observation n of action a

The objective function (A.II.1) minimizes the sum of squared deviations between the estimated probabilities and the calculated discrete probabilities. The first constraint (A.II.2) requires that the estimated means equal the discrete means. The second constraint (A.II.3) requires that the estimated covariances equal the discrete covariances. Constraint (A.II.4) requires that the calculated discrete probabilities sum to 1, while the fourth constraint (A.II.5) makes sure that the probabilities are between zero and 1. The fifth constraint (A.II.6) ensures that the discrete probabilities are non-negative, and the last constraint (A.II.7) requires that only three discrete observations are produced for each element, in accordance with the Latin Hypercube sample. Referring to the example Latin hypercube samples in Step 1 above, the Latin Hypercube constraints for element 1 would be

$$\begin{aligned}\zeta_{a,1,1} &= \zeta_{a,5,1} = \zeta_{a,7,1} \\ \zeta_{a,2,1} &= \zeta_{a,6,1} = \zeta_{a,8,1} \\ \zeta_{a,3,1} &= \zeta_{a,4,1} = \zeta_{a,9,1}\end{aligned}.$$

II.2 Results

Table A.II- 1: Observations for Each Research Action

Action ID (a)	Observation Element	Observations $\zeta_{a,n}$ $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
1	Fingerprinting W1	0.6694	0.7341	0.8	0.8525	0.9408	0.9877	1.0951	1.2	1.4985
2	Fingerprinting W1	0.6726	0.8416	1.0161	1.0161	0.6726	0.8416	0.6726	1.0161	0.8416
2	Fields W1	7650	7686	15097	7686	15097	7650	7686	7650	15097
2	Streambanks W1	499	854	2186	499	854	2186	2186	854	499
3	Fingerprinting W1	0.6726	0.8416	1.0161	1.0161	0.6726	0.8416	0.6726	1.0161	0.8416
3	Fields W2	7650	7686	15097	7686	15097	7650	7686	7650	15097
3	Streambanks W2	499	854	2186	499	854	2186	2186	854	499
4	Fingerprinting W1	0.7402	0.8687	1.1291	1.1291	0.7402	0.8687	0.7402	0.8687	1.1291
4	Fields W3	1660	7233	8400	1660	8400	7233	1660	8400	7233
4	Streambanks W3	2146	6534	12230	6534	12230	2146	12230	6534	2146
4	Ravines W3	14895	27738	36117	36117	14895	27738	14895	36117	27738
4	Bluffs W3	18695	29296	53572	29296	18695	53572	53572	18695	29296

Action ID (a)	Observation Element	Observations $\zeta_{a,n}$ $n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$
5	Fingerprinting W1	0.646	0.8577	0.9992	0.646	0.8577	0.9992	0.646	0.8577	0.9992
5	Fingerprinting W2	0.6875	0.6875	0.6875	0.8223	0.8223	0.8223	1.0402	1.0402	1.0402
6	Fingerprinting W1	0.6288	0.925	0.9886	0.6288	0.925	0.9886	0.6288	0.925	0.9886
6	Fingerprinting W3	0.1966	0.1966	0.1966	0.2334	0.2334	0.2334	0.3677	0.3677	0.3677
7	Fingerprinting W2	0.6694	0.7341	0.8	0.8525	0.9408	0.9877	1.0951	1.2	1.4985
8	Fingerprinting W2	0.6726	0.8416	1.0161	1.0161	0.6726	0.8416	0.6726	1.0161	0.8416
8	Fields W1	7650	7686	15097	7686	15097	7650	7686	7650	15097
8	Streambanks W1	499	854	2186	499	854	2186	2186	854	499
9	Fingerprinting W2	0.6726	0.8416	1.0161	1.0161	0.6726	0.8416	0.6726	1.0161	0.8416
9	Fields W2	7650	7686	15097	7686	15097	7650	7686	7650	15097
9	Streambanks W2	499	854	2186	499	854	2186	2186	854	499
10	Fingerprinting W2	0.7402	0.8687	1.1291	1.1291	0.7402	0.8687	0.7402	0.8687	1.1291
10	Fields W3	1660	7233	8400	1660	8400	7233	1660	8400	7233
10	Streambanks W3	2146	6534	12230	6534	12230	2146	12230	6534	2146
10	Ravines W3	14895	27738	36117	36117	14895	27738	14895	36117	27738
10	Bluffs W3	18695	29296	53572	29296	18695	53572	53572	18695	29296
11	Fingerprinting W1	0.6288	0.925	0.9886	0.6288	0.925	0.9886	0.6288	0.925	0.9886
11	Fingerprinting W3	0.1966	0.1966	0.1966	0.2334	0.2334	0.2334	0.3677	0.3677	0.3677
12	Fingerprinting W3	0.1958	0.2107	0.2252	0.2414	0.2523	0.2638	0.2824	0.307	0.3989
13	Fingerprinting W3	0.2173	0.235	0.3335	0.3335	0.2173	0.235	0.2173	0.3335	0.235
13	Fields W1	7650	7811	15050	7811	15050	7650	7811	7650	15050
13	Streambanks W1	365	1197	1274	365	1197	1274	1274	1197	365
14	Fingerprinting W3	0.2173	0.235	0.3335	0.3335	0.2173	0.235	0.2173	0.3335	0.235
14	Fields W2	7650	7811	15050	7811	15050	7650	7811	7650	15050
14	Streambanks W2	365	1197	1274	365	1197	1274	1274	1197	365
15	Fingerprinting W3	0.1964	0.22	0.3318	0.3318	0.1964	0.22	0.1964	0.22	0.3318
15	Fields W3	2824	8012	10784	2824	10784	8012	2824	10784	8012
15	Streambanks W3	2907	8400	12835	8400	12835	2907	12835	8400	2907
15	Ravines W3	19961	29276	43220	43220	19961	29276	19961	43220	29276
15	Bluffs W3	19713	23799	60000	23799	19713	60000	60000	19713	23799
16	Gauging W1	7377	8235	8625	8912	9500	10500	11500	13182	18731
17	Gauging W1	7945	9273	13328	7945	9273	13328	7945	9273	13328
17	Fingerprinting W1	0.6278	0.6278	0.6278	0.8741	0.8741	0.8741	0.9897	0.9897	0.9897
18	Gauging W1	7945	9273	13328	7945	9273	13328	7945	9273	13328
18	Fingerprinting W2	0.6278	0.6278	0.6278	0.8741	0.8741	0.8741	0.9897	0.9897	0.9897
19	Gauging W1	7425	10105	13266	7425	10105	13266	7425	10105	13266
19	Fingerprinting W3	0.2032	0.2032	0.2032	0.2363	0.2363	0.2363	0.317	0.317	0.317
20	Gauging W1	7385	9908	14259	14259	7385	9908	7385	14259	9908
20	Fields W1	5629	10612	14030	10612	14030	5629	10612	5629	14030
20	Streambanks W1	146	1200	3429	146	1200	3429	3429	1200	146
21	Gauging W1	7385	9908	14259	14259	7385	9908	7385	14259	9908
21	Fields W2	5629	10612	14030	10612	14030	5629	10612	5629	14030
21	Streambanks W2	146	1200	3429	146	1200	3429	3429	1200	146
22	Gauging W1	6756	9000	14256	14256	6756	9000	6756	9000	14256
22	Fields W3	3145	6159	17718	3145	17718	6159	3145	17718	6159
22	Streambanks W3	2468	6922	15442	6922	15442	2468	15442	6922	2468
22	Ravines W3	9955	27925	35174	35174	9955	27925	9955	35174	27925
22	Bluffs W3	10682	24991	50165	24991	10682	50165	50165	10682	24991
23	Gauging W1	7641	9888	12904	7641	9888	12904	7641	9888	12904
23	Gauging W2	7722	7722	7722	9636	9636	9636	13239	13239	13239
24	Gauging W1	7689	9715	12813	7689	9715	12813	7689	9715	12813
24	Gauging W2	61601	61601	61601	93750	93750	93750	120257	120257	120257
25	Gauging W2	7377	8235	8625	8912	9500	10500	11500	13182	18731
26	Gauging W2	7945	9273	13328	7945	9273	13328	7945	9273	13328
26	Fingerprinting W1	0.6278	0.6278	0.6278	0.8741	0.8741	0.8741	0.9897	0.9897	0.9897
27	Gauging W2	7945	9273	13328	7945	9273	13328	7945	9273	13328
27	Fingerprinting W2	0.6278	0.6278	0.6278	0.8741	0.8741	0.8741	0.9897	0.9897	0.9897
28	Gauging W2	7425	10105	13266	7425	10105	13266	7425	10105	13266
28	Fingerprinting W3	0.2032	0.2032	0.2032	0.2363	0.2363	0.2363	0.317	0.317	0.317
29	Gauging W2	7385	9908	14259	14259	7385	9908	7385	14259	9908
29	Fields W1	5629	10612	14030	10612	14030	5629	10612	5629	14030
29	Streambanks W1	146	1200	3429	146	1200	3429	3429	1200	146
30	Gauging W2	7385	9908	14259	14259	7385	9908	7385	14259	9908
30	Fields W2	5629	10612	14030	10612	14030	5629	10612	5629	14030
30	Streambanks W2	146	1200	3429	146	1200	3429	3429	1200	146

Action ID (a)	Observation Element	Observations $\zeta_{a,n}$ $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
31	Gauging W2	6756	9000	14256	14256	6756	9000	6756	9000	14256
31	Fields W3	3145	6159	17718	3145	17718	6159	3145	17718	6159
31	Streambanks W3	2468	6922	15442	6922	15442	2468	15442	6922	2468
31	Ravines W3	9955	27925	35174	35174	9955	27925	9955	35174	27925
31	Bluffs W3	10682	24991	50165	24991	10682	50165	50165	10682	24991
32	Gauging W2	7689	9715	12813	7689	9715	12813	7689	9715	12813
32	Gauging W3	61601	61601	61601	93750	93750	93750	120257	120257	120257
33	Gauging W3	59331	67980	73080	80000	85000	90000	100811	115388	198045
34	Gauging W3	62805	90238	110474	62805	90238	110474	62805	90238	110474
34	Fingerprinting W1	0.6315	0.6315	0.6315	0.8324	0.8324	0.8324	1.0392	1.0392	1.0392
35	Gauging W3	62805	90238	110474	62805	90238	110474	62805	90238	110474
35	Fingerprinting W2	0.6315	0.6315	0.6315	0.8324	0.8324	0.8324	1.0392	1.0392	1.0392
36	Gauging W3	58888	80739	120729	58888	80739	120729	58888	80739	120729
36	Fingerprinting W3	0.1973	0.1973	0.1973	0.2421	0.2421	0.2421	0.3027	0.3027	0.3027
37	Gauging W3	68057	78750	112930	112930	68057	78750	68057	112930	78750
37	Fields W1	6108	7650	14894	7650	14894	6108	7650	6108	14894
37	Streambanks W1	193	857	3735	193	857	3735	3735	857	193
38	Gauging W3	68057	78750	112930	112930	68057	78750	68057	112930	78750
38	Fields W2	6108	7650	14894	7650	14894	6108	7650	6108	14894
38	Streambanks W2	193	857	3735	193	857	3735	3735	857	193
39	Gauging W3	59404	88691	93751	93751	59404	88691	59404	88691	93751
39	Fields W3	3102	5981	9860	3102	9860	5981	3102	9860	5981
39	Streambanks W3	3696	5950	10492	5950	10492	3696	10492	5950	3696
39	Ravines W3	16136	25980	58000	58000	16136	25980	16136	58000	25980
39	Bluffs W3	14475	23799	33601	23799	14475	33601	33601	14475	23799
40	Fields W1	5702	9671	13276	5702	9671	13276	5702	9671	13276
40	Streambanks W1	289	289	289	881	881	881	2174	2174	2174
41	Fields W1	5824	9496	22745	5824	22745	9496	22745	9496	5824
41	Streambanks W1	503	876	7000	503	876	7000	876	503	7000
41	Fields W2	6171	7650	14648	8232	6171	14648	6171	14648	9853
41	Streambanks W2	261	1200	2393	2393	1200	261	2393	261	1200
42	Fields W1	5587	9452	18573	5587	9452	18573	9452	18573	5587
42	Streambanks W1	244	850	2605	2605	850	244	2605	244	850
42	Fields W3	3593	5950	14167	5950	14167	3593	14167	3593	5950
42	Streambanks W3	3628	5950	16018	16018	5950	3628	3628	16018	5950
42	Ravines W3	17257	26084	55677	26084	55677	17257	26084	55677	17257
42	Bluffs W3	16216	29383	49540	16216	49540	29383	29383	16216	49540
43	Fields W2	5702	9671	13276	5702	9671	13276	5702	9671	13276
43	Streambanks W2	289	289	289	881	881	881	2174	2174	2174
44	Fields W2	5587	9452	18573	5587	9452	18573	9452	18573	5587
44	Streambanks W2	244	850	2605	2605	850	244	2605	244	850
44	Fields W3	3593	5950	14167	5950	14167	3593	14167	3593	5950
44	Streambanks W3	3628	5950	16018	16018	5950	3628	3628	16018	5950
44	Ravines W3	17257	26084	55677	26084	55677	17257	26084	55677	17257
44	Bluffs W3	16216	29383	49540	16216	49540	29383	29383	16216	49540
45	Fields W3	3846	7144	17429	3846	17429	7144	17429	7144	3846
45	Streambanks W3	3860	5950	13243	3860	5950	13243	5950	3860	13243
45	Ravines W3	15014	23799	46822	33601	15014	46822	15014	46822	23799
45	Bluffs W3	17150	23799	49523	49523	23799	17150	49523	17150	23799

Table A.II- 2: Expected Loadings from the Posterior for each Research Action and Observation

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n})$							
			Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
7	Fingerprinting in W2	1	9117	1229	9109	2813	7405	10497	27564	29733
		2	9057	1133	9052	1770	7201	8916	27717	28971
		3	9027	1062	9028	1162	7098	7671	27949	28418
		4	9002	1000	9000	865	6996	6890	28022	27995
		5	8985	928	8989	583	6950	6028	28283	27700
		6	8963	919	8968	497	6877	5759	28127	27372
		7	8920	864	8924	365	6732	5291	28041	26852
		8	8924	851	8930	293	6747	4985	28333	26878
		9	8905	807	8911	186	6667	4458	28415	26500
8	Fingerprinting in W2 & SSA in W1	1	9011	685	9011	2893	7119	9311	27104	28666
		2	8987	1005	8993	1164	7014	7716	27459	28103
		3	9200	2112	9183	781	7519	8253	29020	30055
		4	8893	656	8903	669	6702	5593	27766	26736
		5	9222	1038	9193	3041	7611	10493	28343	30712
		6	9078	2127	9084	1257	7342	10037	27470	29458
		7	9149	2193	9144	3133	7585	13886	27032	30658
		8	8946	989	8955	704	6881	6495	27780	27441
		9	9095	671	9072	1135	7145	6537	28916	28630
9	Fingerprinting & SSA in W2	1	8884	939	8867	852	6612	6768	26955	26821
		2	8974	1019	8962	1084	6899	7459	27468	27819
		3	9254	1198	9284	1684	7907	8571	29597	30829
		4	8921	914	8918	669	6739	6085	27678	27072
		5	9076	1086	9083	1362	7273	8051	28175	28940
		6	9080	1185	9073	2023	7262	9515	27621	29217
		7	9041	1214	9020	2747	7139	10760	26866	29046
		8	8977	988	8972	944	6922	7026	27713	27818
		9	9044	975	9058	740	7140	6311	28853	28290
10	Fingerprinting in W2 & SSA in W3	1	8358	543	8105	980	3965	5416	17436	18616
		2	9001	1220	9000	1248	6976	8773	28693	28589
		3	9171	2029	9192	992	7686	11369	35014	36090
		4	8548	912	8278	684	4323	8458	25878	23321
		5	8776	1831	8831	2511	6534	12661	23940	25390
		6	9216	733	9188	869	7619	5246	28130	32692
		7	8545	1522	8266	2247	4314	13424	20187	27847
		8	9001	1120	9035	1187	7183	8748	30923	25855
		9	9001	701	9004	609	6989	4931	27700	26949
11	Fingerprinting in W2 & W3	1	9107	1378	9091	3566	7354	12571	27094	29950
		2	9032	1030	9007	418	6988	6890	30247	29732
		3	9022	982	8991	278	6917	6384	30552	29553
		4	9048	1119	9090	3340	7408	9449	23896	26480
		5	8931	858	8960	348	6875	5129	26239	25701
		6	8897	813	8923	219	6763	4750	26211	25435
		7	8782	647	8965	2607	7125	4373	16444	17982
		8	8578	538	8741	187	6301	2247	17220	16663
		9	8566	516	8727	97	6236	2038	17253	16512
12	Fingerprinting in W3	1	9101	1199	9046	1205	7105	9124	31777	31864
		2	9073	1104	9043	1093	7112	7994	30189	30308
		3	9017	1016	9008	1026	7024	7214	28617	28609
		4	9001	955	9013	946	7038	6459	27337	27291
		5	8941	914	8966	906	6914	6093	26105	26057
		6	8908	860	8945	857	6861	5634	25249	25185
		7	8876	829	8935	840	6825	5205	23945	23824
		8	8823	767	8909	757	6769	4551	22473	22309
		9	8594	606	8748	600	6305	3188	17827	17560

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n}))$ Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
13	Fingerprinting in W3 & SSA in W1	1	9108	777	9012	1163	6979	8361	34192	33382
		2	9199	1545	9144	1243	7450	9832	31927	32985
		3	9154	1555	9188	1023	7571	7063	26053	27032
		4	8863	756	8902	879	6725	5784	25466	25148
		5	9388	1573	9283	1390	7777	10553	34760	35825
		6	9201	1615	9149	1263	7474	9993	31847	33036
		7	9242	1636	9156	1332	7443	10784	33379	34625
		8	8971	1481	9018	957	7120	7083	25005	25969
		9	9253	776	9162	1132	7356	7548	33886	33025
14	Fingerprinting in W3 & SSA in W2	1	9136	1154	9030	777	7023	8275	34386	33540
		2	9198	1249	9132	1550	7401	9857	31764	32924
		3	9136	1027	9215	1555	7729	7107	26219	27135
		4	8861	874	8892	755	6683	5780	25410	25113
		5	9384	1390	9317	1574	7943	10538	35050	35976
		6	9203	1271	9138	1614	7417	9998	31745	32931
		7	9224	1341	9127	1635	7360	10862	33175	34452
		8	8991	971	9033	1483	7154	7091	25118	26029
		9	9223	1130	9172	776	7445	7586	33845	33003
15	Fingerprinting & SSA in W3	1	8684	432	8389	437	4614	4614	23859	23982
		2	9018	1183	8994	1207	6875	8540	28424	27033
		3	9274	1711	9540	1726	9441	7965	26979	28495
		4	8625	828	8551	822	5147	6403	20681	17837
		5	8981	995	8985	997	6961	6981	28067	28255
		6	9314	636	9242	625	7697	4128	28804	34948
		7	8816	1527	8580	1537	5138	11371	23216	31134
		8	9174	1031	9206	1016	7770	7790	33141	27127
		9	8921	569	9027	568	7131	3328	21999	20870
16	Gauging in W1	1	8780	612	8810	876	6514	6169	26524	26035
		2	8877	696	8893	929	6720	6425	27198	26865
		3	8906	738	8916	952	6765	6558	27382	27113
		4	8942	774	8947	947	6855	6592	27628	27389
		5	9008	848	9007	989	7002	6781	28110	27985
		6	9104	993	9090	1049	7217	7144	28846	28884
		7	9158	1181	9136	1086	7341	7551	29027	29413
		8	9283	1625	9250	1172	7674	8351	29791	30720
		9	9475	4816	9433	1486	8329	12548	30176	33339
17	Gauging & Fingerprinting in W1	1	8802	2208	8851	1115	6769	10229	25254	27127
		2	8921	2677	8962	1184	7080	10809	26034	28239
		3	9190	3935	9199	1375	7729	12309	27899	30877
		4	8834	654	8854	920	6627	6456	26854	26551
		5	8979	772	8981	975	6940	6689	27882	27734
		6	9300	1220	9260	1145	7670	7657	30139	30689
		7	8828	474	8845	859	6562	5725	27065	26234
		8	8967	544	8963	928	6843	5948	28081	27395
		9	9296	798	9248	1061	7588	6531	30594	30358
18	Gauging in W1 & Fingerprinting in W2	1	8948	801	8962	3559	7027	10664	26192	28320
		2	9085	987	9078	3667	7332	11239	27189	29624
		3	9384	2002	9346	3916	8103	13582	29007	32668
		4	8830	659	8852	742	6618	6121	26891	26416
		5	8962	820	8965	791	6881	6528	27786	27558
		6	9273	1651	9242	908	7650	8028	29812	30574
		7	8802	638	8823	484	6496	5400	26971	25954
		8	8944	785	8949	518	6817	5726	28047	27219
		9	9252	1571	9223	593	7557	6982	30066	30110

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n})$							
			Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
19	Gauging in W1 & Fingerprinting in W3	1	8823	513	8802	1003	6490	7584	30453	29588
		2	9092	860	9037	1128	7074	8261	31954	31654
		3	9282	1651	9211	1277	7550	9656	32369	33195
		4	8739	468	8759	869	6367	6172	27072	26279
		5	8982	771	8970	975	6883	6701	28266	27929
		6	9221	1437	9206	1094	7590	7609	29367	29950
		7	8554	378	8655	674	6088	4144	21391	20713
		8	8803	615	8882	757	6668	4451	22516	22176
		9	9044	1141	9111	845	7298	5001	23403	23654
20	Gauging & SSA in W1	1	8661	0	8694	738	6141	4336	26320	24569
		2	9172	1168	9161	1105	7440	7943	28739	29714
		3	9462	2559	9421	1414	8231	11225	30002	32922
		4	9357	111	9279	874	7477	4081	32391	29904
		5	8995	1074	9010	1037	7060	7798	27325	28215
		6	9022	2469	9049	1225	7273	10874	26678	29109
		7	8959	2172	8990	1162	7102	10269	26272	28436
		8	9301	1371	9272	1175	7718	8305	29646	30863
		9	9113	37	9077	824	7001	4151	30246	27967
21	Gauging in W1 & SSA in W2	1	8605	347	8613	0	5862	3653	25991	23782
		2	9150	788	9157	1192	7412	7423	28677	29430
		3	9568	2156	9558	2418	8709	12518	30719	34333
		4	9255	1469	9221	119	7401	4754	31005	29448
		5	8954	491	9003	1191	7026	6796	27384	27725
		6	9125	881	9107	2835	7323	10445	27371	29696
		7	9008	553	9050	2874	7233	9343	26887	28737
		8	9309	1929	9257	1225	7661	9436	29115	31049
		9	9019	585	9030	54	6907	3795	29586	27223
22	Gauging in W1 & SSA in W3	1	8284	375	8239	446	4686	3607	17953	17958
		2	9040	839	9043	1001	6997	7335	29370	28268
		3	9814	3058	9973	2363	10958	13445	39136	43270
		4	9234	1533	9023	830	6352	8260	29370	27921
		5	8618	1074	8929	2317	7380	10532	24523	23840
		6	9285	529	9261	563	7670	3939	29155	33129
		7	8503	1005	8441	2020	5194	11529	21292	29636
		8	9201	863	9440	1048	8993	7034	34341	25748
		9	9450	854	9365	530	7703	4107	29105	28943
23	Gauging in W1 & W2	1	8789	705	8784	540	6368	5751	26636	25840
		2	8969	1690	8838	420	6384	7200	26983	27163
		3	9068	5236	8951	518	6814	11200	26566	28921
		4	8840	604	8944	1368	6994	6981	26993	27021
		5	9026	926	9000	643	6924	6349	28409	27924
		6	9191	2736	9085	587	7127	8917	28391	29639
		7	8955	715	9044	5598	7386	11699	26349	28921
		8	9114	803	9193	2625	7707	9070	28402	29800
		9	9281	1165	9281	1113	7795	7650	30302	30693
24	Gauging in W1 & W3	1	8813	620	8833	888	6501	6164	26121	25762
		2	8801	991	8752	804	6004	5857	22302	22672
		3	8825	3124	8740	808	5876	7448	19496	21311
		4	9242	582	9354	1313	8692	8627	39312	37928
		5	9212	784	9250	1167	7927	7746	33211	32784
		6	9206	1576	9165	1074	7289	7893	27740	28639
		7	9526	567	9687	1639	10323	10450	50409	47758
		8	9484	734	9575	1470	9407	9343	42534	41257
		9	9458	1232	9464	1340	8526	8919	34955	35300

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n})$							
			Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
25	Gauging in W2	1	8812	886	8785	613	6372	6190	26573	26076
		2	8895	924	8879	697	6626	6391	27194	26830
		3	8923	952	8914	743	6739	6555	27462	27204
		4	8959	968	8955	781	6862	6642	27792	27542
		5	8989	978	8989	844	6944	6773	27919	27814
		6	9080	1030	9093	998	7266	7137	28731	28779
		7	9158	1100	9180	1185	7542	7560	29309	29671
		8	9239	1164	9270	1611	7828	8303	29630	30561
		9	9433	1482	9473	4779	8588	12487	30146	33327
26	Gauging in W2 & Fingerprinting in W1	1	8966	3556	8955	801	6965	10646	26266	28419
		2	9061	3639	9063	983	7281	11200	26902	29384
		3	9341	3906	9379	1993	8330	13547	28935	32603
		4	8849	745	8829	663	6482	6136	26847	26427
		5	8959	787	8957	820	6848	6513	27692	27460
		6	9243	905	9274	1654	7844	8015	29836	30627
		7	8834	492	8813	642	6406	5438	27126	26049
		8	8943	518	8938	783	6766	5731	28004	27166
		9	9234	594	9266	1559	7772	6954	30229	30249
27	Gauging and Fingerprinting in W2	1	8849	1107	8796	2212	6478	10214	25244	27073
		2	8962	1200	8921	2678	6841	10921	26102	28293
		3	9187	1360	9183	3927	7659	12240	27716	30692
		4	8857	920	8837	646	6511	6396	26860	26529
		5	8968	971	8963	777	6880	6728	27735	27642
		6	9266	1136	9303	1222	7924	7622	30198	30738
		7	8851	881	8835	474	6490	5786	27156	26342
		8	8968	917	8972	547	6883	5921	28145	27435
		9	9245	1061	9292	797	7854	6551	30595	30330
28	Gauging W2 & Fingerprinting W3	1	8851	999	8753	509	6238	7546	30193	29280
		2	9078	1126	9017	875	6992	8364	31664	31477
		3	9262	1292	9232	1669	7688	9756	32524	33309
		4	8766	865	8720	456	6176	6098	26972	26137
		5	9025	976	9015	758	7013	6609	28699	28325
		6	9189	1084	9212	1441	7642	7624	29242	29871
		7	8590	673	8626	368	5918	4070	21273	20614
		8	8798	751	8881	610	6649	4435	22479	22109
		9	9005	836	9129	1101	7435	4980	23500	23703
29	Gauging in W2 & SSA in W1	1	8605	347	8613	0	5862	3653	25991	23782
		2	9150	788	9157	1192	7412	7423	28677	29430
		3	9568	2156	9558	2418	8709	12518	30719	34333
		4	9255	1469	9221	119	7401	4754	31005	29448
		5	8954	491	9003	1191	7026	6796	27384	27725
		6	9125	881	9107	2835	7323	10445	27371	29696
		7	9008	553	9050	2874	7233	9343	26887	28737
		8	9309	1929	9257	1225	7661	9436	29115	31049
		9	9019	585	9030	54	6907	3795	29586	27223
30	Gauging & SSA in W2	1	8696	730	8666	0	5982	4314	26370	24595
		2	9145	1132	9158	1171	7451	8052	28607	29687
		3	9424	1418	9462	2558	8460	11245	30023	32956
		4	9254	872	9329	111	7824	4093	32021	29596
		5	8996	1052	8983	1070	6927	7825	27213	28082
		6	9048	1239	9023	2469	7099	10881	26650	29042
		7	8987	1172	8954	2175	6893	10286	26221	28425
		8	9262	1212	9291	1374	7849	8404	29585	30871
		9	9073	828	9109	38	7165	4155	30119	27907

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n})$ Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
31	Gauging in W2 & SSA in W3	1	8319	454	8232	386	4633	3629	18012	18036
		2	9039	995	9037	849	6967	7344	29361	28254
		3	9789	2366	9991	2956	11089	13319	39213	43247
		4	9185	827	9059	1612	6536	8349	29173	27875
		5	8652	2331	8891	1067	7172	10520	24556	23901
		6	9284	565	9253	530	7630	3943	29139	33133
		7	8546	2032	8423	1022	5092	11525	21359	29801
		8	9200	1063	9424	850	8904	7041	34321	25727
		9	9384	525	9400	863	7924	4118	28949	28784
32	Gauging in W2 & W3	1	8834	883	8815	612	6419	6128	26115	25781
		2	8759	805	8768	989	6116	5880	22192	22511
		3	8772	805	8787	3189	6101	7485	19404	21238
		4	9328	1304	9285	584	8238	8682	39591	38096
		5	9244	1158	9243	783	7860	7735	33370	32901
		6	9165	1091	9196	1577	7483	7952	27593	28529
		7	9622	1688	9562	577	9503	10842	50695	48053
		8	9545	1471	9531	727	9100	9343	42703	41478
		9	9465	1340	9494	1233	8669	8877	35095	35457
33	Gauging in W3	1	8724	827	8714	835	6083	5959	23615	23587
		2	8864	902	8859	895	6518	6394	25646	25611
		3	8942	940	8939	951	6767	6670	26814	26798
		4	9033	1021	9032	1006	7076	7066	28307	28331
		5	9082	1047	9088	1059	7280	7306	29268	29321
		6	9141	1101	9146	1087	7487	7606	30433	30367
		7	9245	1189	9257	1180	7892	8158	32394	32452
		8	9383	1291	9403	1322	8446	8866	35201	35271
		9	9961	2005	9997	2007	11022	12725	48882	49415
34	Gauging in W3 & Fingerprinting in W1	1	8751	2970	8744	898	6153	8823	21188	23206
		2	9084	3394	9094	1236	7349	11106	26565	29113
		3	9266	3629	9284	1472	8063	12726	30107	32877
		4	8714	623	8701	795	5979	5706	22956	22939
		5	9073	798	9076	1051	7215	7102	28840	28903
		6	9262	900	9275	1199	7964	7981	32713	32837
		7	8681	172	8669	722	5870	4473	23532	22665
		8	9049	244	9050	914	7106	5458	29904	28606
		9	9255	304	9262	1061	7872	6192	33960	32642
35	Gauging in W3 & Fingerprinting in W2	1	8765	921	8737	2964	6126	8894	21270	23261
		2	9103	1232	9092	3404	7306	11093	26641	29160
		3	9285	1474	9284	3625	8031	12679	30257	33025
		4	8709	798	8700	617	5981	5691	22952	22923
		5	9076	1026	9074	799	7197	7031	28823	28879
		6	9261	1205	9275	903	7968	8036	32646	32837
		7	8672	713	8663	169	5861	4483	23553	22550
		8	9037	922	9045	258	7111	5528	29681	28584
		9	9263	1065	9278	301	7933	6160	34077	32707
36	Gauging & Fingerprinting in W3	1	8571	802	8461	796	5135	7105	22969	22899
		2	8862	1010	8785	985	6116	8056	27102	27138
		3	9256	1258	9234	1270	7748	9002	33657	33852
		4	8558	677	8521	671	5351	5228	20727	20613
		5	8857	844	8868	842	6478	5888	24668	24575
		6	9288	1092	9366	1090	8366	6586	31024	30961
		7	8536	597	8580	592	5588	3848	18364	18121
		8	8841	748	8943	733	6816	4287	21922	21715
		9	9261	958	9452	948	8898	4793	27701	27478

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n})$							
			Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
37	Gauging in W3 & SSA in W1	1	8777	849	8781	899	6334	6471	24745	24950
		2	9050	1508	9048	1124	7152	9093	26334	28261
		3	9590	3935	9578	1713	8931	14297	31903	36362
		4	9341	860	9361	1146	8252	7307	35342	34167
		5	9049	1497	9012	1052	6872	8416	24647	26580
		6	9096	3734	9098	1315	7343	12549	24519	28561
		7	8994	3666	8982	1171	6910	11695	22554	26427
		8	9365	1549	9391	1397	8421	10430	33008	34803
		9	9102	854	9073	993	7059	6532	28299	28062
38	Gauging in W3 & SSA in W2	1	8803	903	8788	850	6323	6473	24928	25008
		2	9053	1153	9048	1509	7143	9107	26243	28267
		3	9558	1721	9588	3934	9037	14293	31708	36323
		4	9324	1144	9336	861	8166	7321	34995	34092
		5	9008	1074	9014	1500	6933	8470	24372	26426
		6	9093	1308	9075	3742	7257	12574	24429	28540
		7	9008	1190	8992	3663	6933	11601	22533	26512
		8	9353	1424	9351	1545	8273	10513	32832	34640
		9	9065	983	9086	854	7174	6515	28300	28015
39	Gauging in W3 & SSA in W3	1	8862	835	8824	832	6415	6100	26002	25643
		2	9405	1194	9461	1188	8452	8181	34373	33873
		3	9711	1564	9834	1549	9975	11047	42742	38411
		4	9432	941	9330	950	7730	8522	39331	33012
		5	9016	1595	9202	1611	7895	9801	28253	27724
		6	9510	976	9548	987	8714	6401	34086	36071
		7	8889	1451	8843	1436	6460	10257	26290	30004
		8	9580	1060	9726	1060	9601	8048	41789	31804
		9	9488	955	9536	954	8700	6456	34840	34434
40	SSA in W1	1	8800	215	8814	807	6451	4895	27216	25808
		2	8926	217	8921	836	6694	4829	28292	26777
		3	9003	218	8986	854	6844	4789	28956	27376
		4	8916	791	8931	981	6842	7037	27189	27367
		5	9044	796	9040	1016	7099	6944	28264	28394
		6	9122	800	9105	1038	7257	6888	28927	29029
		7	9012	1876	9027	1148	7175	9395	27167	28699
		8	9141	1889	9137	1189	7444	9273	28240	29777
		9	9220	1897	9203	1214	7611	9200	28904	30441
41	SSA in W1 & W2	1	8901	896	8893	671	6679	6238	27419	26932
		2	9192	1239	9162	1564	7482	9481	28306	30210
		3	9678	5820	9629	2514	9019	17078	30377	36119
		4	9121	880	9135	2557	7487	9857	27742	29848
		5	9331	1245	9216	1571	7495	9342	29218	31065
		6	9310	5947	9332	663	8074	10543	29189	31766
		7	9392	1236	9277	2560	7710	10866	29198	31959
		8	9080	903	9101	675	7248	6067	29263	28514
		9	9311	5791	9340	1528	8230	15185	27931	32576
42	SSA in W1 & W3	1	8657	352	8606	853	5503	5731	24441	23303
		2	9170	957	9110	1115	6962	8103	29546	30559
		3	9828	2502	9833	2101	9792	15156	42819	42653
		4	9008	2399	9060	1766	7114	14619	29978	27766
		5	9785	987	9924	1281	10733	8211	42631	41559
		6	8852	355	8574	869	4920	5837	24196	26265
		7	9635	2426	9791	907	10177	6804	31294	34670
		8	8849	351	8571	1588	4913	12652	35346	24894
		9	9152	989	9186	1353	7511	7973	27820	35067

Action ID (a)	Action	Obs. (n)	Expected Loadings From Posterior $E(\tilde{x}_{i,j}\zeta_{a,n})$							
			Field W1	Stream W1	Field W2	Stream W2	Field W3	Stream W3	Ravine W3	Bluff W3
43	SSA in W2	1	8814	807	8800	213	6373	4895	27216	25808
		2	8921	836	8926	216	6722	4829	28292	26777
		3	8986	854	9003	217	6940	4789	28956	27376
		4	8931	981	8916	790	6756	7037	27189	27367
		5	9040	1016	9044	796	7125	6944	28264	28394
		6	9105	1038	9122	799	7356	6888	28927	29029
		7	9028	1148	9012	1877	7083	9395	27167	28699
		8	9137	1190	9141	1890	7469	9273	28240	29777
		9	9203	1215	9220	1897	7711	9200	28904	30441
44	SSA in W2 & W3	1	8662	853	8603	351	5489	5731	24444	23305
		2	9154	1113	9117	957	7003	8106	29493	30509
		3	9787	2092	9853	2502	9946	15169	42671	42519
		4	9024	1774	9054	2402	7070	14610	30034	27812
		5	9787	1284	9924	988	10731	8209	42678	41601
		6	8787	859	8600	354	5046	5847	24008	26080
		7	9638	908	9792	2430	10175	6802	31328	34706
		8	8784	1570	8598	350	5039	12675	35074	24718
		9	9167	1358	9180	990	7463	7970	27865	35125
45	SSA in W3	1	8672	620	8609	620	5510	4819	21806	22258
		2	9058	932	9090	933	7172	6532	28080	27846
		3	9714	1962	9895	1962	10783	11499	41606	41919
		4	9159	627	9036	628	6687	5139	29174	31709
		5	9226	1276	9453	1277	8971	6345	27710	30623
		6	9121	1318	9146	1318	7351	11231	34748	27241
		7	9426	1450	9634	1451	9685	6508	28863	37935
		8	9167	562	9187	562	7484	4966	32907	25792
		9	8830	1391	8748	1392	5873	11198	26765	26532

Table A.II- 3: Optimal Expected Cost and Expected Sediment Reduced for Each Research Action, Observation, and Weight on the Sediment Reduction Objective.

Action ID (a)	Weight (W)	Data (1000/yr)	Observation *n* = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9
1 or 7	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	80.99	80.99	12.09	12.09	12.09	12.09	12.09	71.52	71.52
		Reduction (tons/yr)	9.02	8.82	1.70	1.69	1.66	1.65	1.63	7.60	7.56
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	34.62	34.15	33.51	33.59	33.18	33.03	32.76	32.81	32.40
	50	Cost ($/yr)	1257.26	797.94	797.94	797.94	755.14	755.14	755.14	755.14	755.14
		Reduction (tons/yr)	55.28	44.08	43.41	43.62	42.35	42.20	41.94	42.07	41.64
	100	Cost ($/yr)	1555.10	1555.10	1555.10	1555.10	1544.62	1544.62	1544.62	1544.62	1085.30
		Reduction (tons/yr)	59.27	56.86	54.84	54.38	52.96	52.52	51.79	51.55	46.09
	200	Cost ($/yr)	2225.54	2225.54	2225.54	2505.48	2505.48	2505.48	2505.48	2505.48	2505.48
		Reduction (tons/yr)	65.63	63.22	61.17	62.14	60.81	60.37	59.62	59.42	58.40
	400	Cost ($/yr)	4950.29	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65
		Reduction (tons/yr)	76.18	70.95	68.88	68.48	67.15	66.69	65.94	65.75	64.70
	500	Cost ($/yr)	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	77.01	71.78	69.71	69.33	67.99	67.53	66.78	66.60	65.56
	600	Cost ($/yr)	5323.55	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	77.01	73.56	69.71	69.33	67.99	67.53	66.78	66.60	65.56
	1000	Cost ($/yr)	6242.19	6242.19	6242.19	5323.55	5323.55	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	78.27	74.69	71.91	70.32	68.92	67.53	66.78	66.60	65.56
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	79.39	75.82	73.02	72.30	70.61	70.05	69.15	68.85	67.65
2 or 9	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	140.42	12.09	80.99	80.99	80.99	12.09	71.52
		Reduction (tons/yr)	1.63	1.67	15.64	1.63	8.83	8.93	8.88	1.68	7.74
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	32.27	33.02	36.46	32.60	34.38	34.48	34.02	33.26	33.95
	50	Cost ($/yr)	755.14	797.94	797.94	755.14	797.94	1257.26	1257.26	797.94	797.94
		Reduction (tons/yr)	41.09	42.80	47.10	41.61	44.48	53.94	54.48	43.18	44.19
	100	Cost ($/yr)	1544.62	1555.10	1555.10	1544.62	1555.10	1555.10	1555.10	1555.10	1555.10
		Reduction (tons/yr)	52.38	54.08	59.86	52.21	56.49	57.90	58.42	54.08	54.34
	200	Cost ($/yr)	2225.54	2225.54	2505.48	2225.54	2505.48	2225.54	2225.54	2225.54	2505.48
		Reduction (tons/yr)	58.71	60.37	67.93	58.60	64.31	64.28	64.73	60.40	62.19
	400	Cost ($/yr)	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4950.29	4031.65	4031.65
		Reduction (tons/yr)	66.28	68.01	74.51	66.25	70.70	72.03	75.12	68.10	68.60
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	5323.55	5323.55	4404.91	4404.91
		Reduction (tons/yr)	67.10	68.84	75.40	67.09	71.54	74.89	75.93	68.94	69.46
	600	Cost ($/yr)	4404.91	4404.91	5323.55	4404.91	4404.91	5323.55	5323.55	4404.91	4404.91
		Reduction (tons/yr)	67.10	68.84	77.08	67.09	71.54	74.89	75.93	68.94	69.46
	1000	Cost ($/yr)	5323.55	6242.19	6242.19	5323.55	6242.19	6242.19	6242.19	6242.19	5323.55
		Reduction (tons/yr)	68.06	70.93	78.30	68.01	74.01	76.07	77.16	70.88	70.42
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	70.01	72.04	79.45	69.78	75.13	77.19	78.28	71.99	72.28

Action ID (a)	Weight (W)	Data (1000/yr)	Observation $n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$
3 or 8	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	80.99	12.09	140.42	12.09	140.42	80.99	80.99	12.09	140.42
		Reduction (tons/yr)	8.69	1.68	15.32	1.60	15.34	8.96	9.30	1.64	14.81
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	33.61	33.29	35.68	32.33	35.84	34.45	35.19	32.91	34.37
	50	Cost ($/yr)	1257.26	797.94	797.94	755.14	1257.26	1257.26	1257.26	755.14	797.94
		Reduction (tons/yr)	52.69	43.11	46.14	41.36	56.64	54.40	58.85	41.95	44.69
	100	Cost ($/yr)	1555.10	1555.10	1555.10	1544.62	1555.10	1555.10	1555.10	1544.62	1555.10
		Reduction (tons/yr)	56.59	54.67	58.48	51.41	60.71	58.37	62.91	52.99	55.12
	200	Cost ($/yr)	2225.54	2225.54	2505.48	2225.54	2505.48	2225.54	2225.54	2225.54	2505.48
		Reduction (tons/yr)	62.88	60.98	66.46	57.79	68.61	64.73	69.28	59.40	63.01
	400	Cost ($/yr)	4950.29	4031.65	4031.65	4031.65	4950.29	4031.65	4950.29	4031.65	4031.65
		Reduction (tons/yr)	73.40	68.64	72.97	65.44	78.12	72.45	80.13	67.08	69.44
	500	Cost ($/yr)	5323.55	4404.91	5323.55	4404.91	5323.55	5323.55	6242.19	4404.91	4404.91
		Reduction (tons/yr)	74.22	69.47	75.96	66.27	78.97	75.40	83.13	67.91	70.31
	600	Cost ($/yr)	5323.55	4404.91	5323.55	4404.91	5323.55	5323.55	6242.19	4404.91	4404.91
		Reduction (tons/yr)	74.22	69.47	75.96	66.27	78.97	75.40	83.13	67.91	70.31
	1000	Cost ($/yr)	5323.55	6242.19	5323.55	4404.91	6242.19	6242.19	6242.19	5323.55	5323.55
		Reduction (tons/yr)	74.22	71.63	75.96	66.27	80.01	76.67	83.13	68.90	71.44
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	76.02	72.74	77.88	68.70	81.15	77.79	84.26	70.71	73.24
4 or 10	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	0.00	140.42	140.42	0.00	12.09	80.99	0.00	71.52	12.09
		Reduction (tons/yr)	0.00	14.68	18.05	0.00	1.58	9.82	0.00	8.21	1.68
	20	Cost ($/yr)	142.96	447.65	447.65	447.65	358.51	447.65	358.51	447.65	447.65
		Reduction (tons/yr)	9.33	34.16	42.46	28.47	25.65	37.17	26.14	32.98	32.57
	50	Cost ($/yr)	625.94	797.94	1257.26	744.80	1214.46	797.94	1085.26	797.94	797.94
		Reduction (tons/yr)	25.37	44.37	66.19	36.61	50.23	47.31	47.39	43.89	42.46
	100	Cost ($/yr)	1406.83	1555.10	1741.73	1544.62	1544.62	1555.10	1544.62	1555.10	1555.10
		Reduction (tons/yr)	35.16	57.03	72.70	49.13	54.59	56.76	53.83	56.37	51.20
	200	Cost ($/yr)	2225.54	2505.48	2505.48	2225.54	2225.54	2505.48	2225.54	2505.48	2225.54
		Reduction (tons/yr)	41.67	64.85	79.22	55.11	60.68	64.63	59.46	64.44	57.53
	400	Cost ($/yr)	3471.75	4031.65	4404.91	4031.65	4950.29	4031.65	3471.75	4031.65	4031.65
		Reduction (tons/yr)	46.80	71.23	87.13	62.30	70.37	71.07	64.81	70.94	65.23
	500	Cost ($/yr)	3471.75	4404.91	5323.55	4404.91	4950.29	4404.91	4950.29	4404.91	4404.91
		Reduction (tons/yr)	46.80	72.09	89.17	63.08	70.37	71.92	68.31	71.87	66.06
	600	Cost ($/yr)	4031.65	4404.91	5323.55	4404.91	6242.19	4404.91	4950.29	4404.91	4404.91
		Reduction (tons/yr)	47.85	72.09	89.17	63.08	72.91	71.92	68.31	71.87	66.06
	1000	Cost ($/yr)	5323.55	6242.19	6242.19	4404.91	6242.19	4404.91	6242.19	6242.19	4404.91
		Reduction (tons/yr)	49.35	74.56	90.16	63.08	72.91	71.92	70.43	74.17	66.06
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	50.90	75.67	91.30	65.72	74.00	74.66	71.47	75.29	68.49

Action ID (a)	Weight (W)	Data (1000/yr)	Observation								
			n = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9
5	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	80.99	80.99	80.99	80.99	80.99	12.09	80.99	12.09	12.09
		Reduction (tons/yr)	9.21	8.73	8.49	9.13	8.58	1.62	8.77	1.74	1.69
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	35.08	34.01	33.57	34.89	33.63	33.21	33.79	33.34	32.86
	50	Cost ($/yr)	1257.26	755.14	755.14	1257.26	797.94	755.14	797.94	797.94	797.94
		Reduction (tons/yr)	56.96	42.99	42.63	55.35	43.56	42.35	43.76	43.45	42.91
	100	Cost ($/yr)	1555.10	1544.62	1544.62	1555.10	1555.10	1544.62	1555.10	1555.10	1555.10
		Reduction (tons/yr)	60.99	56.84	54.86	59.35	54.93	53.25	55.79	53.35	52.08
	200	Cost ($/yr)	2225.54	2225.54	2225.54	2225.54	2225.54	2505.48	2225.54	2505.48	2505.48
		Reduction (tons/yr)	67.38	63.29	61.31	65.74	61.28	61.10	62.12	61.12	59.81
	400	Cost ($/yr)	4950.29	4031.65	4031.65	4950.29	4031.65	4031.65	4031.65	4031.65	4031.65
		Reduction (tons/yr)	78.85	71.01	69.04	76.10	69.01	67.43	69.82	67.46	66.13
	500	Cost ($/yr)	5323.55	5323.55	4404.91	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	79.67	73.69	69.88	76.93	69.84	68.28	70.65	68.31	66.98
	600	Cost ($/yr)	5323.55	5323.55	4404.91	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	79.67	73.69	69.88	76.93	69.84	68.28	70.65	68.31	66.98
	1000	Cost ($/yr)	6242.19	6242.19	6242.19	6242.19	6242.19	5323.55	6242.19	5323.55	4404.91
		Reduction (tons/yr)	81.00	74.83	72.06	78.19	71.99	69.23	73.10	69.25	66.98
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	82.13	75.95	73.18	79.32	73.11	71.05	74.22	70.98	69.40
6 or 11	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	80.99	140.42	140.42	12.09	12.09	12.09	12.09	12.09	12.09
		Reduction (tons/yr)	9.26	15.21	15.27	1.79	1.64	1.64	1.74	1.51	1.50
	20	Cost ($/yr)	447.65	447.65	447.65	358.51	447.65	447.65	142.96	142.96	142.96
		Reduction (tons/yr)	35.25	35.42	35.54	26.70	30.84	30.96	9.78	9.30	9.31
	50	Cost ($/yr)	1257.26	755.14	755.14	1257.26	755.14	755.14	500.80	636.28	636.28
		Reduction (tons/yr)	57.53	45.21	45.45	49.08	39.35	39.56	22.89	24.44	24.46
	100	Cost ($/yr)	1555.10	1544.62	1544.62	1555.10	1544.62	1544.62	957.99	947.51	947.51
		Reduction (tons/yr)	61.58	56.82	56.59	52.86	48.89	48.74	30.22	28.97	29.00
	200	Cost ($/yr)	2225.54	2505.48	2505.48	2225.54	2225.54	2225.54	2225.54	1766.22	1766.22
		Reduction (tons/yr)	67.97	64.91	64.71	58.99	55.19	55.06	41.30	35.58	35.62
	400	Cost ($/yr)	4950.29	4031.65	4031.65	4950.29	4031.65	4031.65	4390.39	3471.75	3471.75
		Reduction (tons/yr)	79.28	71.38	71.20	69.63	62.67	62.58	49.31	43.07	42.96
	500	Cost ($/yr)	5323.55	4404.91	4404.91	4950.29	4404.91	4404.91	4390.39	3471.75	3471.75
		Reduction (tons/yr)	80.10	72.28	72.11	69.63	63.46	63.37	49.31	43.07	42.96
	600	Cost ($/yr)	5323.55	4404.91	4404.91	5323.55	4404.91	4404.91	4950.29	4031.65	4031.65
		Reduction (tons/yr)	80.10	72.28	72.11	70.35	63.46	63.37	50.30	44.10	43.99
	1000	Cost ($/yr)	6242.19	5323.55	5323.55	6242.19	4404.91	4404.91	5323.55	4404.91	4404.91
		Reduction (tons/yr)	81.46	73.30	73.11	71.45	63.46	63.37	50.80	44.61	44.51
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	7633.47	7633.47
		Reduction (tons/yr)	82.59	74.84	74.51	72.57	65.74	65.53	52.54	46.19	46.10

Action ID (a)	Weight (W)	Data (1000/yr)	Observation $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
12	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	140.42	140.42	140.42	12.09	12.09	12.09	12.09	12.09	12.09
		Reduction (tons/yr)	16.16	15.45	14.68	1.70	1.67	1.65	1.65	1.63	1.52
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	358.51	358.51	358.51	142.96
		Reduction (tons/yr)	37.82	36.06	34.17	32.70	31.27	25.86	24.57	23.12	9.71
	50	Cost ($/yr)	797.94	797.94	797.94	797.94	755.14	755.14	755.14	636.28	636.28
		Reduction (tons/yr)	49.02	46.75	44.35	42.48	39.79	38.53	36.59	32.06	25.63
	100	Cost ($/yr)	1741.73	1555.10	1555.10	1555.10	1544.62	1544.62	1544.62	947.51	947.51
		Reduction (tons/yr)	64.16	58.76	55.46	52.75	50.36	48.57	46.12	37.27	30.25
	200	Cost ($/yr)	2505.48	2505.48	2505.48	2225.54	2225.54	2225.54	2225.54	2225.54	2225.54
		Reduction (tons/yr)	70.45	66.77	63.28	59.06	56.67	54.82	52.27	49.23	40.16
	400	Cost ($/yr)	4404.91	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	3471.75	3471.75
		Reduction (tons/yr)	78.00	73.27	69.66	66.73	64.18	62.21	59.51	54.93	45.54
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4031.65	4031.65	3471.75
		Reduction (tons/yr)	78.00	74.17	70.52	67.55	64.96	62.97	59.51	56.28	45.54
	600	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4031.65
		Reduction (tons/yr)	78.00	74.17	70.52	67.55	64.96	62.97	60.23	56.96	46.61
	1000	Cost ($/yr)	6242.19	6242.19	6242.19	6242.19	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	80.41	76.37	72.56	69.45	64.96	62.97	60.23	56.96	47.14
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	81.53	77.49	73.67	70.56	67.89	65.79	63.00	59.58	49.42
13 or 14	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	140.42	140.42	12.09	12.09	140.42	140.42	140.42	12.09	140.42
		Reduction (tons/yr)	17.01	16.55	1.83	1.62	17.92	16.55	17.25	1.72	16.95
	20	Cost ($/yr)	447.65	447.65	447.65	358.51	447.65	447.65	447.65	358.51	447.65
		Reduction (tons/yr)	39.85	38.80	32.14	25.85	42.09	38.81	40.58	26.46	39.56
	50	Cost ($/yr)	797.94	1257.26	797.94	755.14	1257.26	1257.26	1257.26	797.94	797.94
		Reduction (tons/yr)	51.79	59.93	41.61	38.62	64.89	60.09	63.12	39.90	51.46
	100	Cost ($/yr)	1741.73	1741.73	1555.10	1544.62	1983.63	1741.73	1741.73	1555.10	1741.73
		Reduction (tons/yr)	66.40	66.05	52.52	48.78	73.83	66.21	69.43	50.71	65.26
	200	Cost ($/yr)	2505.48	2505.48	2225.54	2225.54	2940.92	2505.48	2505.48	2225.54	2505.48
		Reduction (tons/yr)	72.80	72.40	58.83	55.01	80.16	72.55	75.87	56.88	71.73
	400	Cost ($/yr)	4404.91	4404.91	4031.65	4031.65	4404.91	4404.91	4404.91	4031.65	4404.91
		Reduction (tons/yr)	80.57	80.02	66.44	62.41	85.93	80.17	83.63	64.28	79.54
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	80.57	80.02	67.22	63.17	85.93	80.17	83.63	65.03	79.54
	600	Cost ($/yr)	4404.91	5323.55	5323.55	4404.91	5323.55	5323.55	5323.55	4404.91	4404.91
		Reduction (tons/yr)	80.57	81.57	68.78	63.17	87.50	81.78	85.27	65.03	79.54
	1000	Cost ($/yr)	5323.55	6242.19	6242.19	4404.91	6242.19	6242.19	6242.19	6242.19	5323.55
		Reduction (tons/yr)	81.73	82.81	69.80	63.17	88.89	83.04	86.60	67.47	80.67
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	83.63	83.94	70.94	65.91	90.04	84.18	87.74	68.58	82.59

Action ID (a)	Weight (W)	Data (1000/yr)	Observation								
			n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9
15	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	0.00	71.52	80.99	12.09	80.99	140.42	80.99	71.52	12.09
		Reduction (tons/yr)	0.00	7.63	9.23	1.24	8.57	16.43	8.84	8.83	1.72
	20	Cost ($/yr)	358.51	447.65	447.65	142.96	447.65	447.65	358.51	447.65	142.96
		Reduction (tons/yr)	24.13	32.88	34.04	10.10	33.67	39.12	29.44	34.99	11.60
	50	Cost ($/yr)	744.80	755.14	797.94	636.28	797.94	797.94	1214.46	797.94	679.08
		Reduction (tons/yr)	35.82	42.13	44.11	27.20	43.68	49.48	52.41	46.72	31.69
	100	Cost ($/yr)	1544.62	1544.62	1797.01	1406.83	1555.10	1095.78	1544.62	1741.73	957.99
		Reduction (tons/yr)	44.60	55.21	58.57	38.36	54.52	53.83	56.92	60.36	36.15
	200	Cost ($/yr)	2225.54	2505.48	2708.67	2225.54	2505.48	2505.48	2225.54	2505.48	2225.54
		Reduction (tons/yr)	50.53	63.11	65.05	45.22	62.27	65.94	62.90	66.78	46.72
	400	Cost ($/yr)	4031.65	4031.65	4031.65	3471.75	4031.65	4031.65	3471.75	4404.91	3471.75
		Reduction (tons/yr)	57.57	69.48	70.42	50.69	68.60	72.46	68.56	74.51	52.46
	500	Cost ($/yr)	4031.65	4404.91	4404.91	4031.65	4404.91	4404.91	4031.65	4404.91	4031.65
		Reduction (tons/yr)	57.57	70.33	71.23	51.93	69.44	73.33	69.95	74.51	53.78
	600	Cost ($/yr)	4404.91	4404.91	6242.19	4031.65	4404.91	4404.91	5323.55	4404.91	4404.91
		Reduction (tons/yr)	58.28	70.33	74.67	51.93	69.44	73.33	72.18	74.51	54.44
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	4404.91	6242.19	4404.91	6242.19	6242.19	4404.91
		Reduction (tons/yr)	58.28	72.72	74.67	52.55	71.43	73.33	73.71	76.56	54.44
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	60.21	73.83	75.83	55.27	72.54	75.74	74.79	77.70	56.69
16 or 25	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	12.09	12.09	12.09	140.42	140.42	140.42	140.42
		Reduction (tons/yr)	1.57	1.62	1.63	1.65	1.69	14.85	15.04	15.60	16.48
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	31.32	32.25	32.51	32.83	33.50	34.51	35.00	36.35	38.61
	50	Cost ($/yr)	755.14	755.14	755.14	755.14	797.94	797.94	797.94	797.94	1257.26
		Reduction (tons/yr)	39.95	41.10	41.43	41.83	43.52	44.80	45.37	47.01	62.05
	100	Cost ($/yr)	1544.62	1544.62	1544.62	1544.62	1555.10	1555.10	1555.10	1555.10	2038.91
		Reduction (tons/yr)	50.51	52.01	52.50	52.97	54.16	55.88	56.90	59.46	71.26
	200	Cost ($/yr)	2225.54	2225.54	2225.54	2225.54	2505.48	2505.48	2505.48	2505.48	4342.69
		Reduction (tons/yr)	56.76	58.35	58.86	59.37	61.92	63.76	64.83	67.54	84.04
	400	Cost ($/yr)	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4950.29
		Reduction (tons/yr)	64.23	65.94	66.48	67.04	68.27	70.20	71.31	74.12	86.09
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	5323.55
		Reduction (tons/yr)	65.02	66.75	67.31	67.86	69.11	71.06	72.18	75.01	87.00
	600	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	5323.55	5323.55
		Reduction (tons/yr)	65.02	66.75	67.31	67.86	69.11	71.06	72.18	76.64	87.00
	1000	Cost ($/yr)	4404.91	5323.55	5323.55	5323.55	5323.55	6242.19	6242.19	6242.19	6242.19
		Reduction (tons/yr)	65.02	67.68	68.26	68.81	70.10	73.11	74.44	77.81	88.48
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.12
		Reduction (tons/yr)	67.60	69.48	70.10	70.69	72.06	74.23	75.58	78.95	89.65

Action ID (a)	Weight (W)	Data (1000/yr)	Observation $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
17 or 27	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	80.99	80.99	12.09	12.09	140.42	12.09	12.09	140.42
		Reduction (tons/yr)	1.63	8.60	9.40	1.60	1.67	15.67	1.58	1.65	15.66
	20	Cost ($/yr)	358.51	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	27.28	32.91	35.75	31.86	33.21	36.46	31.69	33.01	36.37
	50	Cost ($/yr)	1214.46	1257.26	1257.26	755.14	797.94	797.94	755.14	755.14	797.94
		Reduction (tons/yr)	50.18	53.10	58.14	40.60	43.15	47.23	40.50	42.15	47.27
	100	Cost ($/yr)	1544.62	1555.10	1555.10	1544.62	1555.10	1555.10	1544.62	1544.62	1555.10
		Reduction (tons/yr)	54.67	56.96	62.22	51.50	53.67	58.98	50.64	52.65	57.87
	200	Cost ($/yr)	2225.54	2225.54	2225.54	2225.54	2225.54	2505.48	2225.54	2505.48	2505.48
		Reduction (tons/yr)	60.87	63.15	68.66	57.80	60.00	67.11	56.94	60.49	66.04
	400	Cost ($/yr)	4031.65	4950.29	4950.29	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65
		Reduction (tons/yr)	68.21	73.32	80.42	65.33	67.71	73.72	64.49	66.81	72.67
	500	Cost ($/yr)	5323.55	5323.55	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	71.17	74.10	81.26	66.13	68.55	74.62	65.30	67.65	73.59
	600	Cost ($/yr)	5323.55	5323.55	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	71.17	74.10	81.26	66.13	68.55	74.62	65.30	67.65	73.59
	1000	Cost ($/yr)	6242.19	6242.19	6242.19	5323.55	5323.55	6242.19	4404.91	5323.55	5323.55
		Reduction (tons/yr)	72.29	75.28	82.63	67.05	69.52	76.98	65.30	68.58	74.65
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	73.38	76.39	83.77	68.80	71.41	78.13	67.72	70.24	76.60
18 or 26	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	80.99	80.99	140.42	12.09	12.09	140.42	12.09	12.09	140.42
		Reduction (tons/yr)	8.60	9.00	16.02	1.60	1.66	15.56	1.57	1.64	15.48
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	33.02	34.45	37.60	31.77	33.03	36.24	31.43	32.86	35.97
	50	Cost ($/yr)	1257.26	1257.26	1257.26	755.14	755.14	797.94	755.14	755.14	797.94
		Reduction (tons/yr)	53.10	55.47	61.67	40.53	42.08	46.91	40.20	41.99	46.70
	100	Cost ($/yr)	1555.10	1555.10	2038.91	1544.62	1544.62	1555.10	1544.62	1544.62	1555.10
		Reduction (tons/yr)	56.97	59.45	70.77	51.08	53.17	59.02	49.99	52.25	57.73
	200	Cost ($/yr)	2225.54	2225.54	3376.35	2225.54	2225.54	2505.48	2225.54	2505.48	2505.48
		Reduction (tons/yr)	63.18	65.79	78.33	57.38	59.59	67.10	56.27	60.07	65.83
	400	Cost ($/yr)	4950.29	4950.29	4950.29	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65
		Reduction (tons/yr)	74.25	77.14	84.46	64.91	67.28	73.67	63.80	66.38	72.41
	500	Cost ($/yr)	5323.55	5323.55	6242.19	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	75.04	77.96	87.33	65.71	68.11	74.57	64.60	67.22	73.31
	600	Cost ($/yr)	5323.55	5323.55	6242.19	4404.91	4404.91	5323.55	4404.91	4404.91	5323.55
		Reduction (tons/yr)	75.04	77.96	87.33	65.71	68.11	76.22	64.60	67.22	74.88
	1000	Cost ($/yr)	5323.55	6242.19	6242.19	4404.91	4404.91	5323.55	4404.91	4404.91	5323.55
		Reduction (tons/yr)	75.04	78.95	87.33	65.71	68.11	76.22	64.60	67.22	74.88
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	76.95	80.07	88.49	68.21	70.83	78.27	66.82	69.64	76.62

Action ID (a)	Weight (W)	Data (1000/yr)	Observation *n* = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9
19 or 28	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	140.42	140.42	140.42	12.09	12.09	140.42	12.09	12.09	12.09
		Reduction (tons/yr)	15.18	16.14	16.72	1.54	1.66	15.30	1.47	1.61	1.76
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	142.96	358.51	358.51
		Reduction (tons/yr)	35.47	37.73	39.15	31.67	33.49	35.59	11.18	23.00	24.44
	50	Cost ($/yr)	755.14	797.94	1257.26	755.14	755.14	797.94	636.28	636.28	797.94
		Reduction (tons/yr)	45.34	48.97	60.26	40.48	42.69	46.11	30.03	31.96	37.13
	100	Cost ($/yr)	1544.62	1741.73	1741.73	1544.62	1544.62	1555.10	947.51	947.51	1555.10
		Reduction (tons/yr)	57.53	63.25	66.44	51.02	53.98	57.75	34.98	37.15	45.74
	200	Cost ($/yr)	2505.48	2505.48	2505.48	2225.54	2505.48	2505.48	2225.54	2225.54	2225.54
		Reduction (tons/yr)	65.55	69.54	72.86	57.28	61.85	65.76	46.22	48.99	51.85
	400	Cost ($/yr)	4031.65	4404.91	4404.91	4031.65	4031.65	4031.65	3471.75	3471.75	4031.65
		Reduction (tons/yr)	71.93	77.11	80.55	64.78	68.19	72.28	51.74	54.69	59.12
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4031.65	4031.65	4031.65
		Reduction (tons/yr)	72.85	77.11	80.55	65.59	69.03	73.16	53.03	56.04	59.12
	600	Cost ($/yr)	4404.91	4404.91	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	72.85	77.11	82.20	65.59	69.03	73.16	53.67	56.72	59.82
	1000	Cost ($/yr)	5323.55	5323.55	6242.19	4404.91	5323.55	6242.19	4404.91	4404.91	5323.55
		Reduction (tons/yr)	73.85	78.23	83.48	65.59	70.01	75.69	53.67	56.72	60.96
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	75.45	80.22	84.62	68.01	71.89	76.83	55.79	59.18	62.93
20 or 30	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	140.42	140.42	140.42	12.09	80.99	80.99	140.42	71.52
		Reduction (tons/yr)	1.48	15.08	16.32	15.90	1.70	8.86	8.65	15.62	8.04
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	30.06	35.14	38.21	36.70	33.38	33.85	33.15	36.41	34.31
	50	Cost ($/yr)	755.14	797.94	1257.26	797.94	797.94	1257.26	1257.26	797.94	797.94
		Reduction (tons/yr)	38.61	45.43	60.25	48.15	43.17	54.34	52.88	47.04	45.01
	100	Cost ($/yr)	1085.30	1555.10	2038.91	1524.31	1555.10	1555.10	1555.10	1555.10	1095.78
		Reduction (tons/yr)	42.84	57.38	69.42	56.57	54.84	58.28	56.76	59.45	48.88
	200	Cost ($/yr)	2225.54	2505.48	3424.05	2940.92	2225.54	2225.54	2225.54	2505.48	2505.48
		Reduction (tons/yr)	53.35	65.29	77.37	66.86	61.14	64.56	62.99	67.52	61.07
	400	Cost ($/yr)	4031.65	4031.65	4950.29	4404.91	4031.65	4950.29	4031.65	4031.65	4031.65
		Reduction (tons/yr)	60.73	71.76	81.97	72.41	68.80	74.64	70.52	74.10	67.59
	500	Cost ($/yr)	4404.91	4404.91	5323.55	4404.91	4404.91	5323.55	5323.55	4404.91	4404.91
		Reduction (tons/yr)	61.52	72.62	82.87	72.41	69.62	75.44	73.48	74.99	68.50
	600	Cost ($/yr)	4404.91	4404.91	5323.55	4404.91	4404.91	5323.55	5323.55	4404.91	4404.91
		Reduction (tons/yr)	61.52	72.62	82.87	72.41	69.62	75.44	73.48	74.99	68.50
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	4404.91	6242.19	6242.19	6242.19	6242.19	4404.91
		Reduction (tons/yr)	61.52	74.89	84.28	72.41	71.73	76.66	74.64	77.54	68.50
	5000	Cost ($/yr)	7633.47	8552.11	8552.11	7633.47	8552.11	8552.11	8552.11	8552.11	7633.47
			63.33	76.03	85.45	74.43	72.85	77.78	75.75	78.69	70.45

Action ID (a)	Weight (W)	Data (1000/yr)	Observation								
			n = 1	n = 2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9
21 or 29	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	140.42	140.42	140.42	12.09	80.99	80.99	140.42	71.52
		Reduction (tons/yr)	1.41	14.99	16.93	15.48	1.69	9.01	8.76	15.54	7.88
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	29.28	34.90	39.65	35.80	33.03	34.57	33.65	36.33	33.48
	50	Cost ($/yr)	755.14	797.94	1257.26	797.94	797.94	1257.26	1257.26	1257.26	755.14
		Reduction (tons/yr)	37.72	45.16	63.30	46.80	42.83	54.86	52.66	56.22	43.10
	100	Cost ($/yr)	1085.30	1555.10	2038.91	1555.10	1555.10	1555.10	1555.10	1555.10	1085.30
		Reduction (tons/yr)	41.84	56.57	72.65	55.56	53.46	58.85	56.57	60.33	47.66
	200	Cost ($/yr)	2225.54	2505.48	3471.75	2505.48	2225.54	2225.54	2225.54	2505.48	2505.48
		Reduction (tons/yr)	51.61	64.47	80.97	63.76	59.76	65.22	62.87	68.35	59.49
	400	Cost ($/yr)	4031.65	4031.65	4950.29	4031.65	4031.65	4950.29	4950.29	4031.65	4031.65
		Reduction (tons/yr)	58.92	70.93	85.23	70.40	67.41	75.77	73.37	74.89	65.93
	500	Cost ($/yr)	4404.91	4404.91	6242.19	4404.91	4404.91	5323.55	5323.55	5323.55	4404.91
		Reduction (tons/yr)	59.70	71.79	88.31	71.33	68.23	76.60	74.18	77.70	66.82
	600	Cost ($/yr)	4404.91	4404.91	6242.19	4404.91	4404.91	5323.55	5323.55	5323.55	4404.91
		Reduction (tons/yr)	59.70	71.79	88.31	71.33	68.23	76.60	74.18	77.70	66.82
	1000	Cost ($/yr)	4404.91	5323.55	6242.19	5323.55	5323.55	5323.55	5323.55	6242.19	4404.91
		Reduction (tons/yr)	59.70	72.98	88.31	72.80	69.43	76.60	74.18	78.92	66.82
	5000	Cost ($/yr)	7633.47	8552.11	8552.11	7633.47	8552.11	8552.11	8552.11	8552.11	7633.47
		Reduction (tons/yr)	61.11	74.90	89.50	73.94	71.03	78.60	75.85	80.07	68.52
22 or 31	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	12.09	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	2.64	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	0.00	140.42	355.97	71.52	12.09	140.42	80.99	71.52	140.42
		Reduction (tons/yr)	0.00	14.75	43.05	7.70	1.78	16.05	8.48	9.38	15.03
	20	Cost ($/yr)	142.96	447.65	447.65	447.65	358.51	447.65	358.51	447.65	447.65
		Reduction (tons/yr)	9.44	34.20	50.09	33.78	24.83	37.90	27.93	34.72	34.77
	50	Cost ($/yr)	631.96	797.94	1257.26	755.14	1257.26	797.94	1095.60	797.94	797.94
		Reduction (tons/yr)	25.45	44.61	77.69	43.31	48.61	48.36	47.88	47.04	45.23
	100	Cost ($/yr)	947.51	1555.10	2225.54	1544.62	1555.10	1095.78	1544.62	1983.63	1579.59
		Reduction (tons/yr)	29.87	55.83	90.21	56.13	52.17	52.60	54.49	62.40	54.12
	200	Cost ($/yr)	2225.54	2505.48	3471.75	2505.48	2225.54	2505.48	2225.54	2940.92	3376.35
		Reduction (tons/yr)	40.00	63.75	97.29	64.19	58.19	64.55	60.24	68.71	65.82
	400	Cost ($/yr)	3471.75	4031.65	6242.19	4031.65	4950.29	4031.65	3471.75	4404.91	4031.65
		Reduction (tons/yr)	45.17	70.19	106.23	70.67	67.74	71.10	65.68	74.41	68.04
	500	Cost ($/yr)	3471.75	4404.91	6242.19	4404.91	4950.29	4404.91	4950.29	4404.91	4404.91
		Reduction (tons/yr)	45.17	71.07	106.23	71.55	67.74	71.97	68.98	74.41	68.91
	600	Cost ($/yr)	4031.65	4404.91	6242.19	5323.55	5323.55	4404.91	5323.55	4404.91	4404.91
		Reduction (tons/yr)	46.25	71.07	106.23	73.08	68.47	71.97	69.62	74.41	68.91
	1000	Cost ($/yr)	4404.91	5323.55	6242.19	5323.55	6242.19	4404.91	6242.19	5323.55	4404.91
		Reduction (tons/yr)	46.79	72.07	106.23	73.08	69.55	71.97	70.62	75.46	68.91
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	48.63	74.03	107.45	75.04	70.63	74.21	71.67	77.48	71.46

Action ID (a)	Weight (W)	Data (1000/yr)	Observation *n* = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9
23	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	80.99	12.09	71.52	140.42	80.99	140.42	140.42
		Reduction (tons/yr)	1.54	1.54	8.70	1.69	7.64	14.91	8.84	15.09	15.73
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	31.18	32.30	33.56	32.35	33.55	34.87	33.61	35.14	36.56
	50	Cost ($/yr)	755.14	755.14	1214.46	797.94	755.14	797.94	1257.26	797.94	797.94
		Reduction (tons/yr)	39.85	41.08	53.42	42.02	42.80	45.00	54.84	45.37	47.40
	100	Cost ($/yr)	1544.62	1544.62	1544.62	1555.10	1544.62	1555.10	1555.10	1555.10	1555.10
		Reduction (tons/yr)	49.95	52.75	58.07	52.77	53.75	57.91	58.76	58.45	59.15
	200	Cost ($/yr)	2225.54	2225.54	3144.18	2225.54	2505.48	2505.48	3144.18	2505.48	2505.48
		Reduction (tons/yr)	56.20	59.08	69.67	59.00	61.65	65.77	70.60	66.32	67.29
	400	Cost ($/yr)	4031.65	4031.65	4950.29	4031.65	4031.65	4950.29	4950.29	4950.29	4031.65
		Reduction (tons/yr)	63.68	66.65	77.25	66.57	68.02	74.93	78.16	75.38	73.91
	500	Cost ($/yr)	4404.91	4404.91	5323.55	4404.91	4404.91	5323.55	5323.55	5323.55	4404.91
		Reduction (tons/yr)	64.47	67.46	78.05	67.38	68.87	75.79	78.95	76.23	74.82
	600	Cost ($/yr)	4404.91	5323.55	5323.55	4404.91	4404.91	5323.55	5323.55	5323.55	4404.91
		Reduction (tons/yr)	64.47	69.15	78.05	67.38	68.87	75.79	78.95	76.23	74.82
	1000	Cost ($/yr)	4404.91	5323.55	5323.55	5323.55	5323.55	5323.55	5323.55	5323.55	6242.19
		Reduction (tons/yr)	64.47	69.15	78.05	68.74	69.80	75.79	78.95	76.23	77.10
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	66.81	70.67	79.68	70.45	71.56	77.50	80.78	78.17	78.24
24 or 32	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	12.09	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	2.49	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	12.09	140.42	140.42	80.99	355.97	140.42	140.42
		Reduction (tons/yr)	1.57	1.45	1.42	19.61	16.88	8.75	48.61	21.27	18.01
	20	Cost ($/yr)	447.65	358.51	142.96	447.65	447.65	447.65	625.94	625.94	447.65
		Reduction (tons/yr)	30.96	23.16	10.88	45.62	39.26	33.92	68.19	58.45	41.95
	50	Cost ($/yr)	755.14	636.28	636.28	797.94	797.94	797.94	1395.06	1257.26	797.94
		Reduction (tons/yr)	39.47	32.01	29.29	59.46	51.05	43.87	88.57	73.83	54.39
	100	Cost ($/yr)	1544.62	1544.62	1406.83	1983.63	1741.73	1555.10	2225.54	2225.54	2225.54
		Reduction (tons/yr)	50.01	44.32	41.50	77.42	65.00	55.70	98.91	86.14	74.74
	200	Cost ($/yr)	2225.54	2225.54	2225.54	2940.92	2505.48	2225.54	4031.65	3471.75	3471.75
		Reduction (tons/yr)	56.25	50.30	48.62	83.96	71.44	62.13	109.42	93.19	81.38
	400	Cost ($/yr)	4031.65	3471.75	4390.39	4404.91	4404.91	4031.65	4404.91	4404.91	4404.91
		Reduction (tons/yr)	63.68	55.96	57.26	90.13	79.21	69.93	110.94	97.02	84.52
	500	Cost ($/yr)	4404.91	4031.65	4950.29	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	64.46	57.29	58.43	90.13	79.21	70.76	110.94	97.02	84.52
	600	Cost ($/yr)	4404.91	4404.91	4950.29	4404.91	4404.91	5323.55	5323.55	4404.91	4404.91
		Reduction (tons/yr)	64.46	57.96	58.43	90.13	79.21	72.34	112.58	97.02	84.52
	1000	Cost ($/yr)	4404.91	5323.55	5323.55	5323.55	5323.55	6242.19	5323.55	5323.55	6242.19
		Reduction (tons/yr)	64.46	58.95	59.01	91.44	80.37	73.41	112.58	98.49	87.09
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	67.06	60.84	60.91	93.17	82.30	74.55	114.33	100.41	88.27

Action ID (a)	Weight (W)	Data (1000/yr)	Observation *n* = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9
33	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	12.09
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	2.66
	10	Cost ($/yr)	12.09	12.09	12.09	140.42	140.42	140.42	140.42	140.42	355.97
		Reduction (tons/yr)	1.47	1.57	1.63	14.56	15.06	15.61	16.62	18.03	49.69
	20	Cost ($/yr)	358.51	447.65	447.65	447.65	447.65	447.65	447.65	447.65	625.94
		Reduction (tons/yr)	24.14	30.67	32.06	33.85	35.01	36.29	38.69	42.01	68.69
	50	Cost ($/yr)	755.14	755.14	755.14	797.94	797.94	797.94	797.94	797.94	1395.06
		Reduction (tons/yr)	35.97	39.03	40.80	43.95	45.44	47.12	50.21	54.50	91.40
	100	Cost ($/yr)	1544.62	1544.62	1544.62	1555.10	1555.10	1555.10	1741.73	2225.54	2225.54
		Reduction (tons/yr)	46.10	49.80	51.97	54.90	56.71	58.77	64.51	74.76	101.94
	200	Cost ($/yr)	2225.54	2225.54	2225.54	2505.48	2505.48	2505.48	2505.48	3376.35	4031.65
		Reduction (tons/yr)	52.13	56.03	58.31	62.70	64.64	66.86	70.93	80.90	112.48
	400	Cost ($/yr)	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4404.91	4404.91	4404.91
		Reduction (tons/yr)	59.23	63.43	65.89	69.06	71.09	73.41	78.63	84.55	113.94
	500	Cost ($/yr)	4031.65	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	6242.19
		Reduction (tons/yr)	59.23	64.20	66.69	69.91	71.97	74.32	78.63	84.55	117.96
	600	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	6242.19
		Reduction (tons/yr)	59.94	64.20	66.69	69.91	71.97	74.32	78.63	84.55	117.96
	1000	Cost ($/yr)	4404.91	4404.91	6242.19	6242.19	6242.19	6242.19	6242.19	6242.19	6242.19
		Reduction (tons/yr)	59.94	64.20	68.58	71.94	74.08	76.51	81.00	87.16	117.96
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	62.68	67.10	69.69	73.06	75.20	77.64	82.14	88.32	119.19
34 or 35	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	80.99	140.42	12.09	140.42	140.42	12.09	140.42	140.42
		Reduction (tons/yr)	1.48	8.88	16.29	1.44	14.85	16.80	1.42	14.97	16.99
	20	Cost ($/yr)	358.51	447.65	447.65	358.51	447.65	447.65	358.51	447.65	447.65
		Reduction (tons/yr)	23.36	33.83	38.17	23.49	34.52	39.12	23.38	34.68	39.43
	50	Cost ($/yr)	636.28	1257.26	1257.26	755.14	797.94	797.94	755.14	797.94	797.94
		Reduction (tons/yr)	31.79	54.53	61.72	35.00	44.81	50.75	35.17	45.28	51.44
	100	Cost ($/yr)	1544.62	1555.10	1555.10	1544.62	1555.10	1741.73	1085.30	1555.10	1741.73
		Reduction (tons/yr)	46.99	58.48	65.96	44.82	55.84	64.92	39.24	54.65	63.88
	200	Cost ($/yr)	2225.54	2225.54	2505.48	2225.54	2505.48	2505.48	2225.54	2505.48	2505.48
		Reduction (tons/yr)	52.89	64.78	74.08	50.80	63.71	71.36	49.71	62.63	70.38
	400	Cost ($/yr)	4390.39	4950.29	4950.29	3471.75	4031.65	4404.91	4031.65	4031.65	4404.91
		Reduction (tons/yr)	61.45	75.80	84.31	56.45	70.14	79.10	56.79	69.10	78.22
	500	Cost ($/yr)	4950.29	5323.55	5323.55	4031.65	4404.91	4404.91	4031.65	4404.91	4404.91
		Reduction (tons/yr)	62.72	76.60	85.21	57.83	71.00	79.10	56.79	70.00	78.22
	600	Cost ($/yr)	5323.55	5323.55	5323.55	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	63.35	76.60	85.21	58.52	71.00	79.10	57.49	70.00	78.22
	1000	Cost ($/yr)	5323.55	6242.19	6242.19	4404.91	5323.55	5323.55	4404.91	4404.92	5323.55
		Reduction (tons/yr)	63.35	77.83	86.68	58.52	72.05	80.30	57.49	70.00	79.28
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	7633.48	8552.11	8552.11
		Reduction (tons/yr)	65.33	78.96	87.83	61.01	73.97	82.35	59.29	72.28	80.73

Action ID (a)	Weight (W)	Data (1000/yr)	Observation *n* = 1	*n* = 2	*n* = 3	*n* = 4	*n* = 5	*n* = 6	*n* = 7	*n* = 8	*n* = 9
36	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	140.42	12.09	12.09	140.42	12.09	12.09	12.09
		Reduction (tons/yr)	1.24	1.47	17.20	1.29	1.56	16.09	1.35	1.64	2.15
	20	Cost ($/yr)	358.51	447.65	447.65	142.96	358.51	447.65	142.96	358.51	447.65
		Reduction (tons/yr)	23.26	32.27	40.19	10.83	25.19	37.17	9.79	22.57	33.43
	50	Cost ($/yr)	755.14	755.14	797.94	636.28	755.14	797.94	636.28	636.28	797.94
		Reduction (tons/yr)	34.74	41.07	52.08	29.35	37.56	48.34	26.16	31.30	43.64
	100	Cost ($/yr)	1544.62	1544.62	1741.73	1406.83	1544.62	1797.01	947.51	947.51	1797.01
		Reduction (tons/yr)	45.80	53.54	67.38	39.35	47.76	61.48	30.72	36.47	54.82
	200	Cost ($/yr)	2225.54	2225.54	2505.48	2225.54	2225.54	2940.92	2225.54	2225.54	2660.97
		Reduction (tons/yr)	51.67	59.83	73.86	46.36	53.93	69.49	41.29	48.11	61.08
	400	Cost ($/yr)	3471.75	4031.65	4404.91	3471.75	4031.65	4031.65	3471.75	3471.75	4031.65
		Reduction (tons/yr)	57.23	67.37	81.67	51.81	61.23	73.99	46.64	53.80	66.76
	500	Cost ($/yr)	4031.65	4404.91	4404.91	4031.65	4031.65	4404.91	3471.75	4031.65	4404.91
		Reduction (tons/yr)	58.60	68.19	81.67	53.06	61.23	74.92	46.64	55.11	67.59
	600	Cost ($/yr)	4404.91	4404.91	4404.91	4031.65	4404.91	4404.91	4031.65	4404.91	4404.91
		Reduction (tons/yr)	59.29	68.19	81.67	53.06	61.97	74.92	47.74	55.77	67.59
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	4404.91	4404.91	6242.19	4404.91	4404.91	6242.19
		Reduction (tons/yr)	59.29	70.18	84.19	53.68	61.97	77.10	48.29	55.77	69.50
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	61.95	71.27	85.34	56.08	64.75	78.26	50.54	58.35	70.66
37 or 38	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	80.99	140.42	140.42	12.09	80.99	12.09	140.42	12.09
		Reduction (tons/yr)	1.53	8.62	17.72	17.75	1.66	8.74	1.67	17.45	1.70
	20	Cost ($/yr)	358.51	447.65	447.65	447.65	358.51	358.51	358.51	447.65	447.65
		Reduction (tons/yr)	25.45	33.06	41.67	41.19	26.78	28.34	26.23	40.81	33.65
	50	Cost ($/yr)	755.14	797.94	1257.26	797.94	755.14	1257.26	1095.60	1257.26	797.94
		Reduction (tons/yr)	37.85	42.55	67.51	53.70	39.15	54.13	46.91	63.04	43.74
	100	Cost ($/yr)	1544.62	1555.10	2225.54	2225.54	1544.62	1555.10	1544.62	2225.54	1555.10
		Reduction (tons/yr)	48.62	55.53	78.92	72.31	52.09	58.04	53.78	74.26	54.15
	200	Cost ($/yr)	2225.54	2225.54	3471.75	3376.35	2225.54	2225.54	2225.54	3376.35	2505.48
		Reduction (tons/yr)	54.75	61.80	85.47	78.44	58.35	64.23	59.90	80.29	61.97
	400	Cost ($/yr)	4031.65	4031.65	5323.55	4404.91	4031.65	4950.29	4390.39	4404.91	4031.65
		Reduction (tons/yr)	62.01	69.37	92.27	82.09	65.73	75.37	69.34	83.73	68.37
	500	Cost ($/yr)	4031.65	4404.91	5323.55	4404.91	4031.65	4950.29	4950.29	4404.91	4404.91
		Reduction (tons/yr)	62.01	70.16	92.27	82.09	65.73	75.37	70.69	83.73	69.22
	600	Cost ($/yr)	4404.91	4404.91	6242.19	4404.91	4404.91	5323.55	5323.55	5323.55	4404.91
		Reduction (tons/yr)	62.75	70.16	93.99	82.09	66.47	76.10	71.37	85.28	69.22
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	5323.55	6242.19	6242.19	6242.19	6242.19	5323.55
		Reduction (tons/yr)	62.75	72.79	93.99	83.24	69.02	77.42	72.54	86.68	70.21
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	65.59	73.91	95.17	85.25	70.14	78.54	73.65	87.84	72.19

Action ID (a)	Weight (W)	Data (1000/yr)	Observation $n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 7$	$n = 8$	$n = 9$
39	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	140.42	140.42	140.42	12.09	140.42	80.99	140.42	140.42
		Reduction (tons/yr)	1.55	17.52	20.75	18.18	1.90	18.06	8.88	18.85	17.82
	20	Cost ($/yr)	447.65	447.65	625.94	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	30.80	40.65	56.60	41.74	33.58	42.24	34.17	42.25	41.31
	50	Cost ($/yr)	755.14	797.94	1257.26	797.94	1257.26	797.94	1214.46	797.94	797.94
		Reduction (tons/yr)	39.27	52.89	73.80	55.42	53.60	54.43	52.99	57.01	53.73
	100	Cost ($/yr)	1544.62	2225.54	2225.54	1983.63	1555.10	2225.54	1544.62	2225.54	2225.54
		Reduction (tons/yr)	49.73	72.35	86.14	73.01	57.47	72.31	57.63	76.85	71.60
	200	Cost ($/yr)	2225.54	3424.05	3471.75	2940.92	2505.48	3471.75	2225.54	3471.75	3471.75
		Reduction (tons/yr)	55.97	78.71	93.33	79.59	65.30	78.94	63.90	83.93	78.26
	400	Cost ($/yr)	4031.65	4404.91	4404.91	4404.91	4031.65	4404.91	4031.65	4404.91	4404.91
		Reduction (tons/yr)	63.41	82.04	97.17	85.78	71.71	82.01	71.37	87.69	81.40
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	64.19	82.04	97.17	85.78	72.56	82.01	72.16	87.69	81.40
	600	Cost ($/yr)	4404.91	4404.91	6242.19	4404.91	6242.19	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	64.19	82.04	100.29	85.78	75.76	82.01	72.16	87.69	81.40
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	6242.19	6242.19	6242.19	6242.19	6242.19	6242.19
		Reduction (tons/yr)	64.19	84.43	100.29	87.67	75.76	83.97	75.05	89.81	83.31
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	66.95	85.59	101.50	88.83	76.89	85.15	76.15	91.00	84.48
40 or 43	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	71.52	12.09	80.99	140.42	80.99	80.99	140.42
		Reduction (tons/yr)	1.56	1.61	7.73	1.65	8.64	14.91	8.73	9.06	15.33
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	31.40	32.60	33.33	32.65	33.89	34.66	33.71	34.99	35.78
	50	Cost ($/yr)	755.14	755.14	755.14	755.14	797.94	797.94	1257.26	1257.26	1257.26
		Reduction (tons/yr)	40.25	41.80	42.75	41.51	43.98	44.98	52.86	54.40	55.36
	100	Cost ($/yr)	1544.62	1544.62	1544.62	1544.62	1555.10	1555.10	1555.10	1555.10	1555.10
		Reduction (tons/yr)	49.52	51.11	52.10	53.08	54.81	55.81	56.77	58.41	59.42
	200	Cost ($/yr)	2225.54	2505.48	2505.48	2225.54	2505.48	2505.48	2225.54	2505.48	2505.48
		Reduction (tons/yr)	55.81	58.95	60.05	59.44	62.61	63.72	63.07	66.25	67.37
	400	Cost ($/yr)	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65	4031.65
		Reduction (tons/yr)	63.36	65.26	66.44	67.05	68.98	70.17	70.72	72.67	73.87
	500	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	5323.55	5323.55	5323.55
		Reduction (tons/yr)	64.18	66.11	67.31	67.86	69.83	71.04	73.41	75.41	76.64
	600	Cost ($/yr)	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91	5323.55	5323.55	5323.55
		Reduction (tons/yr)	64.18	66.11	67.31	67.86	69.83	71.04	73.41	75.41	76.64
	1000	Cost ($/yr)	4404.91	4404.92	4404.91	5323.55	5323.55	5323.55	6242.19	6242.19	6242.19
		Reduction (tons/yr)	64.18	66.11	67.31	68.84	70.84	72.07	74.56	76.59	77.85
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	66.29	68.27	69.49	70.74	72.76	74.00	75.68	77.73	78.99

Action ID (a)	Weight (W)	Data (1000/yr)	Observation								
			n **= 1**	***n*** **= 2**	***n*** **= 3**	***n*** **= 4**	***n*** **= 5**	***n*** **= 6**	***n*** **= 7**	***n*** **= 8**	***n*** **= 9**
41	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	140.42	140.42	80.99	140.42	140.42	140.42	140.42	80.99
		Reduction (tons/yr)	1.61	15.12	17.37	9.09	15.52	15.83	15.79	14.85	9.93
	20	Cost ($/yr)	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	32.37	35.35	40.92	34.86	36.34	36.99	37.05	34.40	37.15
	50	Cost ($/yr)	755.14	1257.26	1257.26	1257.26	1257.26	1257.26	1257.26	797.94	1257.26
		Reduction (tons/yr)	41.30	55.00	69.07	54.71	56.14	58.08	58.40	44.83	62.51
	100	Cost ($/yr)	1544.62	1555.10	2038.91	1555.10	1555.10	1555.10	1797.01	1555.10	1797.01
		Reduction (tons/yr)	52.02	59.03	78.61	58.71	60.25	62.25	65.02	54.80	69.15
	200	Cost ($/yr)	2225.54	2505.48	4390.39	2225.54	2505.48	3424.12	2940.92	2505.48	3579.61
		Reduction (tons/yr)	58.38	66.90	92.76	65.11	68.27	76.24	72.83	62.74	81.21
	400	Cost ($/yr)	4031.65	4031.65	5868.93	4950.29	4031.65	4950.29	4950.29	4031.65	4950.29
		Reduction (tons/yr)	66.00	73.35	97.10	75.44	74.82	82.81	79.78	69.19	86.93
	500	Cost ($/yr)	4404.91	4404.91	6242.19	5323.55	4404.91	5323.55	5323.55	4404.91	5323.55
		Reduction (tons/yr)	66.82	74.20	98.01	76.27	75.70	83.69	80.65	70.07	87.77
	600	Cost ($/yr)	4404.91	5323.55	6242.19	5323.55	5323.55	5323.55	5323.55	4404.91	5323.55
		Reduction (tons/yr)	66.82	75.76	98.01	76.27	77.27	83.69	80.65	70.07	87.77
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	5323.55	6242.19	5323.55	6242.19	4404.91	6242.19
		Reduction (tons/yr)	66.82	77.00	98.01	76.27	78.51	83.69	81.89	70.07	89.29
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.12	8552.11	8552.11	8552.11
		Reduction (tons/yr)	69.49	78.14	99.20	78.28	79.66	85.50	83.04	72.77	90.45
42 or 44	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	0.00	0.00	12.09	0.00	12.09	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	2.59	0.00	2.45	0.00	0.00
	10	Cost ($/yr)	12.09	140.42	140.42	71.52	140.42	0.00	140.42	59.43	80.99
		Reduction (tons/yr)	1.33	15.34	21.76	8.01	21.68	0.00	17.48	7.42	10.37
	20	Cost ($/yr)	358.51	447.65	625.94	447.65	625.94	358.51	447.65	447.65	447.65
		Reduction (tons/yr)	23.96	35.96	59.76	34.08	59.06	25.97	40.48	33.48	38.78
	50	Cost ($/yr)	755.14	797.94	1257.26	1257.26	797.94	750.82	797.94	1210.14	797.94
		Reduction (tons/yr)	36.17	46.43	81.04	59.33	65.32	37.96	52.03	57.39	48.80
	100	Cost ($/yr)	1544.62	1555.10	2225.54	1555.10	2225.54	1544.62	2225.54	1731.25	1555.10
		Reduction (tons/yr)	45.96	58.58	93.69	63.18	86.13	48.06	70.22	63.70	61.11
	200	Cost ($/yr)	2225.54	2505.48	3471.75	2505.48	3471.75	2225.54	3471.75	2505.48	2225.54
		Reduction (tons/yr)	51.99	66.56	100.91	71.16	93.36	54.11	76.81	70.06	67.53
	400	Cost ($/yr)	4031.65	4031.65	5323.55	4950.29	4404.91	4031.65	5323.55	4404.91	4031.65
		Reduction (tons/yr)	59.14	73.06	107.27	80.03	97.20	61.27	82.05	77.74	75.33
	500	Cost ($/yr)	4031.65	4404.91	6242.19	5323.55	4404.91	4031.65	5323.55	4404.91	4404.91
		Reduction (tons/yr)	59.14	73.95	109.37	80.93	97.20	61.27	82.05	77.74	76.16
	600	Cost ($/yr)	4404.91	4404.91	6242.19	6242.19	4404.91	4404.91	5323.55	5323.55	4404.91
		Reduction (tons/yr)	59.87	73.95	109.37	82.70	97.20	62.00	82.05	79.33	76.16
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	6242.19	6242.19	4404.91	5323.55	5323.55	6242.19
		Reduction (tons/yr)	59.87	76.02	109.37	82.70	99.47	62.00	82.05	79.33	78.51
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	62.15	77.15	110.58	83.82	100.69	64.30	84.16	80.76	79.64

Action ID (a)	Weight (W)	Data (1000/yr)	Observation $n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
45	1	Cost ($)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5	Cost ($)	0.00	0.00	12.09	0.00	0.00	0.00	0.00	0.00	0.00
		Reduction (tons)	0.00	0.00	2.60	0.00	0.00	0.00	0.00	0.00	0.00
	10	Cost ($/yr)	12.09	12.09	140.42	140.42	80.99	71.52	140.42	71.52	12.09
		Reduction (tons/yr)	1.33	1.73	21.56	15.48	9.63	9.07	17.65	8.71	1.42
	20	Cost ($/yr)	358.51	447.65	447.65	447.65	447.65	447.65	447.65	447.65	447.65
		Reduction (tons/yr)	22.63	33.43	49.99	36.61	35.79	35.59	41.85	33.84	31.63
	50	Cost ($/yr)	636.28	797.94	1257.26	755.14	797.94	1257.26	797.94	797.94	1214.46
		Reduction (tons/yr)	31.26	43.47	76.39	46.09	46.01	58.99	52.56	45.45	51.51
	100	Cost ($/yr)	1406.83	1555.10	2225.54	1544.62	1797.01	1741.73	2038.91	1741.73	1544.62
		Reduction (tons/yr)	41.01	53.87	88.92	56.05	58.93	64.92	68.63	56.16	55.85
	200	Cost ($/yr)	2225.54	2505.48	3471.75	2505.48	2660.97	2505.48	3424.05	2505.48	2225.54
		Reduction (tons/yr)	48.24	61.66	96.07	64.08	65.19	71.39	76.49	62.56	62.10
	400	Cost ($/yr)	3471.75	4031.65	4404.91	4031.65	4031.65	4404.91	4031.65	4404.91	4031.65
		Reduction (tons/yr)	53.80	68.04	99.81	70.53	70.86	79.24	78.46	70.27	69.59
	500	Cost ($/yr)	4031.65	4404.91	6242.19	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	55.11	68.88	103.73	71.41	71.70	79.24	79.33	70.27	70.39
	600	Cost ($/yr)	4404.91	4404.91	6242.19	4404.91	4404.91	4404.91	4404.91	4404.91	4404.91
		Reduction (tons/yr)	55.76	68.88	103.73	71.41	71.70	79.24	79.33	70.27	70.39
	1000	Cost ($/yr)	4404.91	6242.19	6242.19	4404.91	6242.19	6242.19	6242.19	4404.91	6242.19
		Reduction (tons/yr)	55.76	70.74	103.73	71.41	74.25	81.88	82.23	70.27	73.17
	5000	Cost ($/yr)	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11	8552.11
		Reduction (tons/yr)	58.07	71.87	104.95	73.79	75.40	83.01	83.41	72.53	74.26
0	1	Cost ($)	0.00								
		Reduction (tons)	0.00								
	5	Cost ($)	0.00								
		Reduction (tons)	0.00								
	10	Cost ($/yr)	12.09								
		Reduction (tons/yr)	1.69								
	20	Cost ($/yr)	447.65								
		Reduction (tons/yr)	33.47								
	50	Cost ($/yr)	797.94								
		Reduction (tons/yr)	43.46								
	100	Cost ($/yr)	1555.10								
		Reduction (tons/yr)	54.31								
	200	Cost ($/yr)	2505.48								
		Reduction (tons/yr)	62.06								
	400	Cost ($/yr)	4031.65								
		Reduction (tons/yr)	68.39								
	500	Cost ($/yr)	4404.91								
		Reduction (tons/yr)	69.23								
	600	Cost ($/yr)	4404.91								
		Reduction (tons/yr)	69.23								
	1000	Cost ($/yr)	6242.19								
		Reduction (tons/yr)	71.23								
	5000	Cost ($/yr)	8552.11								
		Reduction (tons/yr)	72.35								

The values of the decision variables for each run of the multiobjective linear program are available on CD from the author by request.

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CURRICULUM VITA

Sarah Keenan Jacobi was born in Chicago, Illinois during the winter of 1978. She spent her childhood in suburban Chicago and stayed close to home to pursue her undergraduate degree at Northwestern University. A year later she transferred to the University of Illinois at Urbana-Champaign, where she studied to be an Environmental Engineer. It was during this time that Sarah found the field of environmental system analysis. After venturing out in the real world for a short time, Sarah returned to Johns Hopkins where, as a Clare Boothe Luce Fellow and a National Science Foundation Graduate Research Fellow, she received two Master's degrees and worked towards her Ph.D. Upon completion of her doctorate, Sarah will apply environmental systems and decision analysis to the field of biological conservation as a David H. Smith Conservation Research Fellow at the Lincoln Park Zoological Society.