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Economic and Emissions Implications of Load-based, Source-based and First-seller Emissions Trading Programs under California AB32

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In response to Assembly Bill 32, the State of California is considering three types of carbon emissions trading programs for the electric power sector: load-based, source-based and first-seller. They differ in terms of their point-of-regulation and in whether in-state-to-out-of-state and out-of-state-to-in-state electricity sales are regulated. In this paper, we formulate a market equilibrium model for each of the three approaches, considering power markets, transmission limitations and emissions trading, and making the simplifying assumption of pure bilateral markets. We analyze the properties of their solutions and show the equivalence of load-based, first-seller and source-based approaches when in-state to out-of-state sales are regulated under the cap. A numeric example illustrates the emissions and economic implications of the models. In the simulated cases, "leakage" eliminates most of the emissions reductions that the regulations attempt to impose. Further, "contract reshuffling" occurs to such an extent that all the apparent emissions reductions resulting from changes in sources of imported power are illusory.

In reality, the three systems would not be equivalent, because there will also be pool-type markets, and the three systems provide different incentives for participating in those markets. However, the equivalence results under our simplifying assumptions show that load-based trading has no inherent advantage compared to other systems in terms of costs to consumers, contrary to claims elsewhere.

Key words: Emissions Trading, Electric Market, CO_2 Emissions, Load-based, First-seller, Source-based
History:

1. Introduction

Due to a lack of federal leadership, a number of states in the U.S. have taken actions to control greenhouse gas (GHG) emissions. The efforts by the western and the eastern states are called the Western Climate Initiative (WCI) and Regional Greenhouse Gas Initiative (RGGI), respectively. Among the WCI states, the State of California is the first state to adopt legislation limiting GHGs. **On September 27, 2006, the state of California passed a comprehensive bill – AB32 "The Global Warming Solutions Act" – that aims at reducing in-state GHG emissions from various sectors to 1990 levels by 2020, or 25% reduction compared to the business-as-usual scenario.** AB32 is the first climate change legislation in the U.S. that would regulate most polluting sectors in an economy. Led by the California Energy Commission (CEC) and the California Air Resources Board (CARB) in consultation with other agencies, a state-wide emission cap is expected to be in effect by 2012. This is expected to be accomplished with a suite of instruments such as a low-carbon fuel standard for vehicles that would reduce GHGs of transportation fuels by at least 10% by 2020 (6).

California has historically been among the states with the lowest GHG emissions per capita in the U.S., roughly 50% below the national average. If apportioned by sectors, approximately 40% of GHG emissions is from transportation, followed by 14% apiece from in-state electricity and imported electricity. Others include industrial and manufacturing for 10%; refining and other industries for 8%; and agriculture, forestry and waste for 6%. The remaining 9% is attributed to commercial, residential and other sources (5). While imported electricity accounts for half of electric sector emissions, such imports represent only 22% of electricity demand in California in 2006 (4). Thus, imported electricity is significantly dirtier than in-state generation since a significant portion is generated by coal plants located in Arizona, New Mexico, Montana and other neighboring states (7).

While AB32 does not explicitly require the implementation of emissions trading programs, the CEC and CARB have been seriously considering using that approach for the electricity sector. A central issue in the recent debate over implementation of AB32 has been over where along the electricity sector supply chain (i.e., fuel to generation to consumption) should limits be imposed; this is the “point-of-regulation” issue. Currently, there are essentially three basic proposals on the table: source-based, load-based and first-seller approaches.

Source-based systems are most popular elsewhere. The conventional point-of-regulation in electric sector cap-and-trade systems (e.g., under the SO_2 Acid Rain Program under the 1990 Clean Air Act Amendments and the USEPA NO_x SIP Call) is at the generator. In source-based systems, a fixed number of emissions allowances are either distributed or sold to generators, who can then trade among themselves. Total emissions across all the regulated entities cannot exceed the number of allowances.

However, the alternative of load-based systems has received much attention in the California debate. Such systems can be viewed as being analogous to tradable renewable energy systems that have been used in other states and Europe, in which the seller of electricity to consumers is required to have a minimum portion of renewable electricity in its portfolio, either as contracts with renewable sources or unbundled “renewable energy credits.” The basic load-based program requires load-serving entities (LSEs) to track the emissions associated with the electricity they purchase from generators and third parties, and to ensure that the sales-weighted carbon emissions rate of its purchases does not exceed a target established by the regulator. **Other variants that unbundle emissions certificates and power are represented by two proposals: the CO_2RC proposal (21) and the “Tradable Emission Attribute Certificates” (TEAC) proposal (14). Objectives of these proposals include the elimination of the need to track emissions associated with energy transactions, which lessens conflicts with pool-type markets, and the encouragement of clean out-of-state generators to participate voluntarily in emissions trading. Elsewhere, under a set of simplifying assumptions (including zero price elasticity, no transmission constraints, and no power imports), it is shown that the CO_2RC and TEAC systems are economically equivalent to source-based systems in which allowances are freely allocated to generators in proportion to their sales (16, 25).**

The final of the three approaches is the first-seller proposal (9). A first-seller is defined as the entity that first contracts to sell electricity in California. Consistent with this, we define generators as the point-of-regulation for in-state sources, while LSEs are designated as the ”first-seller” for out-of-state to in-state sales. Although the first-seller proposal is not explicitly defined by CEC at the moment, the schedule coordinators (SCs) that contract with generators in other states to import electricity into California could be designated as the first-seller instead of the LSE (3). More recently, the CPUC (California Public Utility Commission) released a draft report and expressed their support of a “*first-deliverer*” approach. It is a variant of the first-seller system and the entity that possesses the ownership of the electricity at the first point of delivery in California would be the point of regulation(8).

Table 1 summarizes the jurisdictional coverage of the programs analyzed in this paper. Two versions each of the source- and load-based programs are considered, differing in their treatment of power imports or exports. Together with the first-seller system, this means that we analyze five systems. We refer to the two source-based systems as the "pure" and "modified" systems. The pure source-based system is equivalent to traditional cap-and-trade programs and applies to only generation by California power plants. Meanwhile, the modified source-based approach expands coverage to include imports (i.e., out-of-state to in-state sales). Similarly, the pure load-based approach imposes an emissions cap on California LSEs, including the emissions of power they import, whereas the modified load-based approach, in addition, regulates in-state to out-of-state electricity sales. The modified rather than pure approaches are of most interest here because they both attempt to account for emissions from out-of-state power production, and are therefore the main focus of our analyses. Finally, the first-seller approach is a hybrid of source- and load-based approaches, where the emissions associated with the first-seller's sales come under the cap, with California generators being responsible for their emissions and the LSEs being responsible for emissions associated with imports.

Table 1 Summary of Programs' Jurisdiction Coverage

Programs	Instate to Instate	Instate to Outstate	Outstate to Instate
First-Seller	√	√	√
Modified Load-based	√	√	√
Modified Source-based	√	√	√
Pure Load-based	√		√
Pure Source-based	√	√	

The biggest drawback of the pure source-based approach is its inability to count emissions from imported electricity (i.e., carbon leakage). This "leakage" of emissions turns out to be crucial in regulating regional GHG emissions given that California is a net importer, and its generators are relatively clean compared with other western states. The first-seller and modified source-based approaches represent attempts to correct that problem. As our simulations show later in this paper, however, these approaches are not necessarily effective in preventing leakage. In contrast, while either the modified load-based or pure load-based approaches might address CO_2 leakage, other potential problems with load-based problems have been identified. One issue is whether implementation of a program that places responsibility for compliance upon LSEs would interfere with the operation of the day-ahead and real-time pool-based markets by providing incentives for clean generation to avoid those markets (25). The other concern is whether such an approach would lead to so-called contract shuffling that would eventually result in little or no actual emissions reduction (3). To the extent that there are large amounts of clean energy sources that could compete for contracts to sell into California, the likelihood of shuffling increases. On the other hand, an often cited advantage for load-based approach is that it might increase the incentive for LSEs to invest in demand side management and improved energy efficiency (3). Another claimed advantage is that by paying only a premium for clean power sources, consumer costs would be less than in a source-based system in which the opportunity cost of allowances for marginal power sources would raise the price of all power supply (12). **As shown in the later sections, our findings directly refute such assertion and conclude that load-based approach does not have such advantage over other approaches.** Finally, whether any proposal that would regulate imported power (load-based, modified source-based and first-seller) would survive a challenge based upon the Interstate Commerce Clause is an important question. (Further discussion of the three approaches in terms of potential legal challenges, ability to prevent leakage and potential interference with electricity market operations can be found in elsewhere (3)(17).)

While there is no general consensus about which approach is most appropriate for California, a lack of careful analysis of their economic implications makes the choice even harder. This paper focuses on analyzing the three proposed approaches that include power imports and exports (i.e., modified load-based, modified source-based and first-seller programs), but also considers the pure load- and source-based approaches. We address the following questions assuming that emissions associated with California's electricity consumption are required to meet the same CO_2 emission cap under three approaches. These questions are, first, do the approaches lead to different emissions allowances prices and electricity prices? The second question concerns income distribution: are profits and consumer costs the same among the proposals? Third, how do the proposals compare in terms of contract-shuffling and CO_2 leakage?

To answer these questions, we conduct a general theoretical analysis of the three approaches that regulate emissions of imports and exports and show their equivalence. (An earlier analysis that drew the same conclusion for load- and source-based programs, without considering imports, is Southern California Edison (13).) We also conduct numerical simulations of six cases using a three-zone market, with one capped zone (representing California) and two zones that export to California. Cases 1–3 are the modified source-based, modified load-based and first-seller approaches. Cases 4–6 correspond to pure source-based, pure load-based approaches and no cap (baseline), respectively. The pure cases show how emissions leakage and contract reshuffling can change relative to the modified cases. We report results concerning profits, consumer costs, social welfare, electricity and allowances prices, electricity sales, and CO_2 emissions.

The remainder of the paper is organized as follows. In Section 2, the mathematical formulations of the market equilibrium problems for the modified source-based, modified load-based and first-seller approaches will be introduced. These are complementarity formulations in which each market participant solves an optimization problem (profit-maximization for generators, consumer net benefit maximization for LSEs, and maximization of transmission value for the Independent System Operator (ISO)), and market clearing conditions are imposed. In Section 3, the properties of the solutions of the three models will be discussed. In particular, we show equivalence of the three proposals. We discuss the results of the numerical example in Section 4, focusing on economic and emissions outcomes as well as the issues of CO_2 leakage and contract shuffling. Conclusions and policy implications are addressed in Section 5. The theoretical proofs and data used in the economic and emissions analysis are included in the Appendix A.

2. Models

This section introduces the mathematical formulation of the three main proposals that regulate emissions of imports (modified load-based, modified source-based and first-seller). Within each proposal, optimization problems faced by LSEs, power producers and an ISO are presented. Market clearing conditions will also be specified. The formulations are a generalization of Hobbs (15), accounting for allowances markets and differences in points of regulation among the proposals. Examples of previous studies that also analyze electricity markets using complementarity models include (1, 2, 24). (Note that we also adopt the same hub-spoke type of modeling of a transmission network as in Hobbs (15). Within such a modeling framework, an arbitrary bus is designated as the hub, and routing from node a to node b is as if from node a to the hub, and then from the hub to node b.)

We first summarize the notation to be used in the models.

Sets, Indices and Dimensions

$\mathcal{N}$	Set of zones in a network, including both power producers and LSEs; $\|\mathcal{N}\| = N$
$\mathcal{N}^C \subset \mathcal{N}$	Set of zones that are under the emissions cap
$\mathcal{N}^{NC} \subset \mathcal{N}$	Set of zones that are not under the emissions cap; $\mathcal{N}^{NC} = \mathcal{N} \backslash \mathcal{N}^C$
$\mathcal{I}$	Set of electricity production zones that are under CO_2 regulation; $\mathcal{I} \subset \mathcal{N}$ and $\|\mathcal{I}\| = I$
$\mathcal{J}$	Set of demand zones that are under CO_2 regulation; $\mathcal{J} \subset \mathcal{N}$ and $\|\mathcal{J}\| = J$
$\mathcal{A}$	Set of **(directional)** transmission links in the network $\mathcal{N}$; $\|\mathcal{A}\| = 2J$
$\mathcal{F}$	Set of power producers; $\|\mathcal{F}\| = F$
$\mathcal{F}^C \subset \mathcal{F}$	Set of power producers under the emissions cap
$\mathcal{H}$	Set of generating units
$i, j \in \mathcal{N}$	Zones i, j are in the power generation network
$f \in \mathcal{F}$	Power producer f
$h \in \mathcal{H}$	Generator h
$h \in H_{i,f} \subset \mathcal{H}$	Generator h belongs to power firm f and is in zone i **(Note that the h index is not essential and can be dropped. This point is illustrated further in Section 3.)**

Parameters

P_j^0	Price intercept of the inverse demand function at zone j [\$/MW(h)]
Q_j^0	Quantity intercept of the inverse demand function at zone j [\$/MW(h)]
K_f^P	Initial free allocation of CO_2 allowances to power producer f [tons]
K_j^{LSE}	Initial free allocation of CO_2 allowances to LSE j [tons]
E_{fih}	Emission rate of firm f's generator h, in zone i [tons/MW(h)]
$\overline{E}$	System-wide CO_2 cap [tons]
X_{fih}	Capacity of firm f's generator h, in zone i [MW]
$C_{fih}(x)$	firm f's (convex) production cost function for generator h, in zone i [\$/MW(h)]
$PTDF_{ki}$	The (k,i)-th element of the power transmission-distribution factor matrix
T_k^+	Transmission capacity bound on link k
T_k^-	Transmission capacity in the reverse direction (with regard to a pre-specified orientation of the network) of link k

Variables

z_{fihj}	LSE j's purchase of power from firm f's generator h in zone i [MW(h)]. (Note that there is assumed to be just one LSE per zone; more general assumptions are possible, but would not change the conclusions of our analysis.)
s_{fj}	Power firm f's sale of electricity to LSE in zone j [MW(h)]
x_{fihj}	Electricity sold to LSE j, produced by firm f with generator h at zone i [MW(h)]
y_i	MWs transmitted from the hub to zone i [MW]
w_i	Fees charged for transmitting electricity from the hub to zone i [\$/MW]
p_j	The price of electricity at zone j [\$/MW(h)]
p_{fihj}	The price of electricity produced by generator (f,i,h) sold to LSE j [\$/MW(h)]
p^{CO_2}	CO_2 price [\$/ton]

2.1. Modified Source-based Emissions Trading Program

Under the modified source-based program, the compliance liability is with generators as long as a producer sells electricity to in-state consumers. In-state sources that export also must comply. This is in contrast to our pure source-based system, in which out-of-state sources are exempted from regulation when selling electricity to in-state customers. In what follows, we summarize the optimization problems faced by LSEs, producers and the ISO, followed by their optimality conditions and market clearing conditions.

Under a source-based approach within a perfectly competitive market, an LSE is simply a price taker. The LSE is assumed to act to maximize the net benefits received by its consumers; if demand was perfectly inelastic, this would be the same as minimizing consumer costs. The benefit function is the integral of the consumers' demand curve, assumed to equal their willingness-to-pay for electricity. The demand curve is:

$$p_j = P_j^0 - \frac{P_j^0}{Q_j^0}\sum_f s_{fj}. \tag{1}$$

In a source-based system, the LSE's costs include only the expense of power. Power is procured through bilateral contracts.

Under the modified source-based program, all producers selling in the regulated zones have to buy allowances. Since importers are also subject to the emissions cap, out-of-state imports need to buy CO_2 allowances from the allowances market to cover the electricity they deliver to California. (An important policy issue under this approach would be whether the CO_2 allowances for in-state sources should be auctioned. It is likely to be argued on discrimination grounds that if allowances are allocated for free to in-state sources, then imports should also receive allowances. A system that allocates allowances for free only to in-state generation would be subject to a commerce clause-based challenge. Avoiding this by allocating allowances to imports, however, would be difficult because CO_2 emissions associated with imports cannot be unambiguously determined. (How to allocate allowances and how to distribute economic rents associated with those allowances are issues that are beyond the scope of this paper.) Here, we only consider a scenario in which in-state power producers have an initial fixed endowment of allowances (obtained either by auctions or other means), while imports have to purchase allowances. It turns out the initial fixed endowments do not affect power producers' decisions, given that the producers behave competitively in both the electricity and allowances markets.

Producer f solves the following optimization problem.

$$\begin{aligned}
\underset{s_{fj},\ x_{fihj}}{\text{maximize}}\ & \sum_{j\in\mathcal{N}} p_j s_{fj} - \sum_{i\in\mathcal{N},h\in H_{i,f}} C_{fih}\left(\sum_{j\in\mathcal{N}} x_{fihj}\right) - \sum_{j\in\mathcal{N}}\left[w_j s_{fj} - \left(\sum_{i\in\mathcal{N},h\in H_{i,f}} w_i x_{fihj}\right)\right] \\
& - p^{CO_2}\left(\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}} x_{fihj}E_{fih} + \sum_{i\in\mathcal{N}^{NC},h\in H_{i,f},j\in\mathcal{N}^C} x_{fihj}E_{fih} - K_f^P\right) \\
\text{subject to}\ & \sum_{j\in\mathcal{N}} x_{fihj} \le X_{fih},\ \forall\ i\in\mathcal{N}, h\in H_{i,f},\ (\rho_{fih}), \\
& \sum_{i\in\mathcal{N},h\in H_{i,f}} x_{fihj} = s_{fj}, \forall\ j\in\mathcal{N},\ (\theta_{fj}), \\
& x_{fihj} \ge 0\ , \forall\ i\in\mathcal{N}, h\in H_{i,f}.
\end{aligned} \tag{2}$$

The first term within the last parenthesis (i.e., $\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}} x_{fihj}E_{fih}$) is the emissions associated with in-state sources, no matter where the electricity is sold; whereas the second term (i.e., $\sum_{i\in\mathcal{N}^{NC},h\in H_{i,f},j\in\mathcal{N}^C} x_{fihj}E_{fih}$) is the emissions of imported electricity. **Because of linearity in the**

DC network, consistent with the formulation elsewhere (15), all generation and sales can be modeled as being routed through the hub zone. Thus, a generator receives w_i for providing counterflow to the hub from its location and then pays w_j to deliver power for sale from the hub to customers at j.

The two constraints are capacity and sales balance constraints. Within this paper, we make the blanket assumption that the cost function C_{fih} is convex and differentiable for each $f \in \mathcal{F}$, $i \in \mathcal{N}$, and $h \in \mathcal{H}$. It follows that Problem (2) is convex in its own variables (s, x) for each $f \in \mathcal{F}$ (that is, any local optimum to the problem is also a global optimum). (Here we assume that producers absorb all costs incurred by transmission congestion and add the costs to the market price of electricity. Alternative assumptions could be made, but would not affect our fundamental equivalence results.) In addition, since the constraints in a firm's profit optimization problem are all linear, the linear constraint qualification holds at any feasible point. Hence, a necessary and sufficient condition for an optimal solution to satisfy is given by the following first-order conditions:

$$
\begin{aligned}
0 \le x_{fihj} \perp \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) + p^{CO_2} E_{fih} - w_i + \rho_{fih} - \theta_{fj} \ge 0, \\
\forall\, i \in \mathcal{N}, h \in H_{i,f}, j \in \mathcal{N}^C, \\
0 \le x_{fihj} \perp \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} - \theta_{fj} \ge 0,\ \forall\, i \in \mathcal{N}^{NC}, h \in H_{i,f}, j \in \mathcal{N}^{NC}, \\
0 \le s_{fj} \perp -p_j + w_j + \theta_{fj} \ge 0,\ \forall\, j \in \mathcal{N}, \\
0 \le \rho_{fih} \perp X_{fih} - \sum_j x_{fihj} \ge 0,\ \forall\, i \in \mathcal{N}, h \in H_{i,f}, \\
\theta_{fj} \text{ free}, \qquad \sum_{i,h \in H_{i,f}} x_{fihj} - s_{fj} = 0, \forall\, j \in \mathcal{N}.
\end{aligned}
\tag{3}
$$

If x_{fihj} is greater than zero (therefore, $s_{fj} > 0$) and subject to an emissions cap, the above conditions indicate that the power price in a CO_2-capped region is equal to the sum of marginal cost $(\nabla C_{fih}(\sum_{j \in \mathcal{N}} x_{fihj}))$, the transmission charge (w_i), the CO_2 cost $(p^{CO_2} E_{fih})$ and a scarcity rent if the output is at its upper bound (ρ_{fih}). The term $p^{CO_2} E_{fih}$ is omitted when x_{fihj} is not subject to the emissions cap. Notice that the allowance position (i.e., K_f^P), a firm's initial possession of allowances, does not enter into the optimality conditions. Hence, as long as firms treat their grant of allowances as exogenous and their allocations are not linked to firms' future generation, how allowances are allocated does not affect the firms' behavior. That said, allocation of the allowances does affect the distribution of economic rents.

The system operator is assumed to maximize the value of the transmission network, without strategically manipulating power flows to earn profits. We use the same formulation as in Metzler et al. (19).

$$
\begin{aligned}
\underset{y_i}{\text{maximize}} \quad & \sum_{i \in \mathcal{N}} w_i y_i \\
\text{subject to} \quad & \sum_{i \in \mathcal{N}} y_i = 0, \quad (\eta) \\
& \sum_{i \in \mathcal{N}} PTDF_{ki} y_i \le T_k^+,\ \forall\, k \in \mathcal{A}, \quad (\lambda_k^+) \\
& \sum_{i \in \mathcal{N}} PTDF_{ki} y_i \ge -T_k^-,\ \forall\, k \in \mathcal{A}, \quad (\lambda_k^-)
\end{aligned}
\tag{4}
$$

ISO's first-order conditions are formulated below.

$$\begin{aligned}
&y_i \text{ free}, \quad w_i - \sum_{k\in\mathcal{A}} PTDF_{ki}\lambda_k + \eta = 0,\ \forall\ i \in \mathcal{N}, \\
&\eta \text{ free}, \quad \sum_{i\in\mathcal{N}} y_i = 0, \\
&0 \le \lambda_k \perp T_k - \sum_i PTDF_{ki} y_i - T_k \ge 0,\ \forall\ k \in \mathcal{A}.
\end{aligned} \tag{5}$$

Two sets of market clearing conditions are associated with this problem. Respectively, they determine the charges for transmission by acting as a central auctioneer to auction off scarce transmission capacity and CO_2 allowance prices. Note that $\overline{E}$ could be equal to $\sum_f K_f^P$, but in general they could differ if some of the allowances are retained by the government for other purposes, such as price control or reservation for new entrants.

$$\begin{aligned}
&w_i \text{ free}, \quad y_i = \sum_f s_{fi} - \sum_{f,j,h\in H_{f,i}} x_{fihj},\ \forall\ i \in \mathcal{N}, \\
&0 \le p^{CO_2} \perp \overline{E} - \sum_{f,h,j,i\in\mathcal{N}^C} x_{fihj}E_{fih} - \sum_{f,h,j\in\mathcal{N}^C, i\in\mathcal{N}^{NC}} x_{fihj}E_{fih} \ge 0.
\end{aligned} \tag{6}$$

The first line in (6) is a mass-balance equation that defines the nodal withdraw or injection. Pre-multiply y_i with PTDF matrix would give the flows along each transmission line in the network. Note that the first equation in (6) implies that $\sum_{i\in\mathcal{N}} y_i = 0$. Hence, the corresponding equation in formulation (5) is redundant. We refer to the system consisting of formulations (1), (3), (5) and (6) as the modified source-based model, or Model SB.

2.2. Modified Load-based Emissions Trading Program

A pure load-based emissions trading program assigns compliance requirements to in-state LSEs. Consequently, an in-state generation source would not be subject to any emissions regulation if it exports electricity to out-of-state loads. Hence, total in-state GHG emissions may not be reduced. Such an outcome would conflict with the program's purpose to curb GHG emissions through a cap-and-trade program. One remedy is to put in-state generating sources that export to out-state loads, along with in-state LSEs, under GHG regulation. This is what we refer to as the "modified" load-based approach, and it is the main focus of this subsection. To our knowledge, there is no proposed load-based program that attempts to regulate in-state to out-of-state sales. It might be a precautionary requirement, however, to prevent producers from taking advantage of programs to increase their exports or to do "round-trip" trades.

When considering the modified load-based approach, we partition LSEs into two groups: those under cap-and-trade programs (i.e., $j \in \mathcal{N}^C$) and those not under cap-and-trade (i.e., $j \in \mathcal{N}^N$). We follow the same assumptions as in Section 2.1 and model a market in which LSEs buy power through bilateral contracts with generators. However, a subtle difference remains regarding bilateral markets under the source-based approach versus the load-based approach. Strictly speaking, the load-based scheme requires LSEs to trace the exact source of each MWh of electricity procured, to accurately account for the emissions rate, while the source-based scheme has no such requirement. As a result, we were able to use a more general term s_{fj} to represent firm f's sales to demand zone j in the source-based approach, but under the load-based approach, we need to use variables with finer granularity, z_{fihj} and x_{fihj}, to model both the LSEs' problems and power producers' problems, respectively.

An LSE in load zone $j \in \mathcal{N}$ attempts to maximize benefits of consumption (integral of the demand curve) minus payments to producers for power and, CO_2 emission allowances. For a zone that is subject to a CO_2 emissions cap (that is, $j \in \mathcal{N}^C$), an LSE's problem is as follows.

$$\begin{aligned} \underset{z_{fihj}}{\text{maximize}}\ & P_j^0 \left(\sum_{i,f,h} z_{fihj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{i,f,h} z_{fihj} \right)^2 \\ & - \sum_{i,f,h \in H_{i,f}} p_{fihj} z_{fihj} - p^{CO_2} \left(\sum_{f,i,h} z_{fihj:j\in\mathcal{N}^C} E_{fih} - K_j^{LSE} \right) \\ \text{subject to } & z_{fihj} \geq 0,\ \forall\ f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}. \end{aligned} \tag{7}$$

However, such formulation does not imply that LSEs behave as a monopsony. The last two terms together can be interpreted as "pseudo supply curve," which incorporates costs of acquiring electricity from producers and pollution emissions. The objective function then becomes maximization of "pseudo social welfare," which the equilibrium is equivalent to perfect competition outcomes. The term, $p^{CO_2}(\sum_{f,i,h} z_{fihj:j\in\mathcal{J}^C} E_{fih} - K_j^{LSE})$, on or eight would be omitted when an LSE is not subject to emissions cap. Hence, this yields first-order conditions of two types for the LSEs as follows:

$$\begin{aligned} & 0 \leq z_{fihj} \perp -P_j^0 + \frac{P_j^0}{Q_j^0} \left(\sum_{i',f',h' \in H_{i',f'}} z_{f'i'h'j} \right) + p_{fihj} + p^{CO_2} E_{fih} \geq 0, \\ & \qquad \forall\ j \in \mathcal{N}^C, f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}, \\ & 0 \leq z_{fihj} \perp -P_j^0 + \frac{P_j^0}{Q_j^0} \left(\sum_{i',f',h' \in H_{i',f'}} z_{f'i'h'j} \right) + p_{fihj} \geq 0,\ \forall\ j \in \mathcal{N}^{NC}, f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}. \end{aligned} \tag{8}$$

For all $j \in \mathcal{N}^C$ with $z_{fihj} > 0$, based on the complementarity constraints (8) the following condition needs to be satisfied:

$$p_j \equiv P_j^0 - \frac{P_j^0}{Q_j^0}(\sum_{i,f,h\in H_{i,f}} z_{fihj}) = p_{fihj} + p^{CO_2} E_{fih},$$

where the term p_j represents the marginal benefit (per MWh) of consumers residing in $j \in \mathcal{N}^C$ for electricity, regardless of its emissions rate. In other words, individual consumers are assumed to view electricity as a homogeneous product, regardless of the type of technology used to generate it. Therefore, their marginal benefit is always p_j. In contrast, an LSE is subject to an emissions cap and treats electricity as a differential product with regard to emissions rates. However, LSEs are allowed to pass on CO_2 costs to consumers in the price of electricity. The term p_{fihj} ($= p_j - p^{CO_2} E_{fih}$) is the expenditure by an LSE to purchase one MWh of electricity from a specific generator with an emission rate of E_{fih}. Thus, if a generator has a high emissions rate, it would receive less revenue per MWh compared to cleaner sources. Finally, in a program that instead awards allowances to LSEs in proportion to their load, the terms in the last parenthesis of the objective function in Problem (7) will be replaced with $\sum_{f,i,h} z_{fihj:j\in\mathcal{N}^C}(E_{fih} - R^L)$, where R^L is the target or default emissions rate.

In the load-based approach, power producers do not face CO_2 emissions regulation, except for exports from in-state sources to out-of-state consumers. In this situation, in-state generators have to purchase CO_2 allowances to ship power out of the state. This ensures that all emissions within the state are covered by the program. The producer f's profit maximization problem is shown below:

$$\begin{aligned}
\underset{x_{fihj}}{\text{maximize}} \quad & \sum_{i,j\in\mathcal{N},h\in H_{i,f}} p_{fihj}x_{fihj} - \sum_{i\in\mathcal{N},h\in H_{i,f}} C_{fih}\left(\sum_{j\in\mathcal{N}} x_{fihj}\right) \\
& - \sum_{i,j\in\mathcal{N},h\in H_{i,f}} (w_j - w_i)x_{fihj} \\
& - p^{CO_2}\left(\sum_{i\in\mathcal{N}^C,h\in H_{i,f},j\in\mathcal{N}^{NC}} x_{fihj}E_{fih}\right) \\
\text{subject to} \quad & \sum_j x_{fihj} \le X_{fih},\ \forall\, i,j\in\mathcal{N}, h\in H_{i,f},\ (\rho_{fih}) \\
& x_{fihj} \ge 0,\ \forall\, i,j\in\mathcal{N}, h\in H_{i,f}.
\end{aligned} \tag{9}$$

The optimality conditions of Problem (9) are given below.

$$\begin{aligned}
0 \le x_{fihj} \perp & -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j'\in\mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} \ge 0, \\
& \forall\ i \notin \mathcal{N}^C \text{ or } j \notin \mathcal{N}^{NC}, h\in H_{i,f} \\
0 \le x_{fihj} \perp & -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j'\in\mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} + p^{CO_2}E_{fih} \ge 0, \\
& \forall i\in\mathcal{N}^C \text{ and } j\in\mathcal{N}^{NC}, h\in H_{i,f} \\
0 \le \rho_{fih} \perp & X_{fih} - \sum_j x_{fihj} \ge 0,\ \forall\ i\in\mathcal{N}, h\in H_{i,f}.
\end{aligned} \tag{10}$$

The ISO's problem is the same as in the source-based approach (4). The new set of market-clearing conditions are as follows.

$$\begin{aligned}
w_i \text{ free}, \quad & y_i = \sum_{f,i',h\in H_{i',f}} x_{fi'hi} - \sum_{f,j,h\in H_{i,f}} x_{fihj},\ \forall\ i\in\mathcal{N}, \\
0 \le p^{CO_2} \perp\ & \overline{E} - \sum_{f,i,h,j\in\mathcal{N}^C} z_{fihj}E_{fih} - \sum_{f,h,i\in\mathcal{N}^C,j\in\mathcal{N}^{NC}} x_{fihj}E_{fih} \ge 0, \\
p_{fihj} \text{ free}, \quad & z_{fihj} = x_{fihj},\ \forall\ f\in\mathcal{F}, i,j\in\mathcal{N}, h\in\mathcal{H}.
\end{aligned} \tag{11}$$

We refer to the model consisting of equations (8), (10), (5) and (11) as the modified load-based model, or Model LB.

2.3. First-Seller Emissions Trading Scheme

In the first-seller approach we consider here, the point of regulation is the first entity that brings energy into California. Our interpretation is that for the in-state generating sources, the first-seller approach is equivalent to the source-based approach. In our model setting, we do not consider power producers buying electricity from other firms, nor do we consider power marketers. Hence, only LSEs are responsible in our model for bringing out-of-state power into California if imports are needed or more economical. Consequently, in the first-seller approach, imports are treated the same way as in the load-based approach, where the LSEs are the point of regulation. The first-seller approach can therefore be viewed as a hybrid of the source-based and load-based approaches. It could instead be assumed, for example, that a third importing party is responsible for imports (i.e., first-deliverer approach), but this would not materially affect the conclusions of our analysis.

Again we assume that LSEs explicitly sign bilateral contracts with generators to provide electricity to a specified zone $j \in \mathcal{N}$. Also, there are in-state LSEs and out-of-state LSEs. Under the first-seller scheme, an in-state LSE (i.e., $j \in \mathcal{N}^C$) faces a profit optimization problem as follows.

$$\begin{aligned} \underset{z_{fihj}:\ j\in\mathcal{N}^C}{\text{maximize}}\ & P_j^0 \left(\sum_{f\in\mathcal{F}, i\in\mathcal{N}, h\in\mathcal{H}} z_{fihj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{f\in\mathcal{F}, i\in\mathcal{N}, h\in\mathcal{H}} z_{fihj} \right)^2 \\ & - \sum_{f\in\mathcal{F}, i\in\mathcal{N}, h\in\mathcal{H}} (p_{fihj} z_{fihj}) - p^{CO_2} \left(\sum_{f,h,i\in\mathcal{N}^N} z_{fihj} E_{fih} - K_j^{LSE} \right) \\ \text{subject to}\ & z_{fihj} \geq 0,\ \forall f \in \mathcal{F}, i \in \mathcal{N}, h \in \mathcal{H}. \end{aligned} \tag{12}$$

While for an out-of-state LSE, there is no cost or profit associated with CO_2 emissions:

$$\begin{aligned} \underset{z_{fihj}:\ j\in\mathcal{N}^{NC}}{\text{maximize}}\ & P_j^0 \left(\sum_{i,f,h} z_{fihj} \right) - \frac{P_j^0}{2Q_j^0} \left(\sum_{i,f,h} z_{fihj} \right)^2 \\ & - \sum_{i,f,h} (p_{fihj} z_{fihj}) \\ \text{subject to}\ & z_{fihj} \geq 0,\ \forall f, i, h. \end{aligned} \tag{13}$$

The last term within the parenthesis of the objective function in Problem (12) would be modified to $\sum_{i\in\mathcal{N}^C, h\in H_{i,f}, j\in\mathcal{N}^N} x_{fihj}(E_{fih} - R^F)$ when facing a program that awards allowances in proportion to sales. R^F is the default or target emissions under the first-seller program. The same modification should be applied to Problem (15).

The first-order conditions of the LSEs' problems are as follows.

$$\begin{aligned} & 0 \leq z_{fihj} \perp -P_j^0 + \frac{P_j^0}{Q_j^0} \left(\sum_{f',i',h'} z_{f'i'h'j} \right) + p_{fihj} + p^{CO_2} E_{fih} \geq 0, \\ & \qquad \forall\ f \in \mathcal{F}, h \in H_{i,f}, i \in \mathcal{N}^{NC}, j \in \mathcal{N}^C, \\ & 0 \leq z_{fihj} \perp -P_j^0 + \frac{P_j^0}{Q_j^0} \left(\sum_{f',i',h'} z_{f'i'h'j} \right) + p_{fihj} \geq 0, \\ & \qquad \forall\ f \in \mathcal{F}, h \in H_{i,f}, i \in \mathcal{N}, j \in \mathcal{N}^{NC}. \end{aligned} \tag{14}$$

Power producer f's problem under the first-seller approach is stated below. Notice that CO_2-related costs or revenues only apply to in-state generating sources.

$$\begin{aligned} \underset{x_{fihj}}{\text{maximize}}\ & \sum_{i,j\in\mathcal{N}, h\in H_{i,f}} p_{fihj} x_{fihj} - \sum_{i\in\mathcal{N}, h\in H_{i,f}} C_{fih} \left(\sum_{j\in\mathcal{N}} x_{fihj} \right) \\ & - \sum_{i,j\in\mathcal{N}, h\in H_{i,f}} (w_j - w_i) x_{fihj} \\ & - p^{CO_2} \Big(\sum_{i\in\mathcal{N}^C, h\in H_{i,f}, j\in\mathcal{N}} x_{fihj} E_{fih}\ -\ K_f^P \Big) \\ \text{subject to}\ & \sum_j x_{fihj} \leq X_{fih},\ \forall\ i \in \mathcal{N}, h \in H_{i,f}, \quad (\rho_{fih}) \\ & x_{fihj} \geq 0,\ \forall i \in \mathcal{N}, h \in H_{i,f}. \end{aligned} \tag{15}$$

The optimality conditions for a power producer $f \in \mathcal{F}$ are shown below, where the term $p^{CO_2} E_{fih}$ is omitted for out-of-state sources.

$$\begin{aligned}
&0 \leq x_{fihj} \perp -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) + p^{CO_2} E_{fih} - w_i + \rho_{fih} \geq 0, \\
&\qquad \forall\ i \in \mathcal{N}^C, h \in H_{i,f}, j \in \mathcal{N}, \\
&0 \leq x_{fihj} \perp -p_{fihj} + w_j + \nabla_x C_{fih}\left(\sum_{j' \in \mathcal{N}} x_{fihj'}\right) - w_i + \rho_{fih} \geq 0, \\
&\qquad \forall\ i \in \mathcal{N}^{NC}, h \in H_{i,f}, j \in \mathcal{N}, \\
&0 \leq \rho_{fih} \perp X_{fih} - \sum_j x_{fihj} \geq 0,\ \forall\ i \in \mathcal{N}, h \in H_{i,f}.
\end{aligned} \tag{16}$$

The ISO's problem is exactly the same as in the source-based approach.

The market clearing conditions under the first-seller approach are given below.

$$\begin{aligned}
&w_i \text{ free}, \quad y_i = \sum_{f,i',h \in H_{f,j}} x_{fi'hi} - \sum_{f,j',h \in H_{f,i}} x_{fihj'},\ \forall\ i \in \mathcal{N}, \\
&0 \leq p^{CO_2} \perp \overline{E} - \sum_{f,h,i \in \mathcal{N}^{NC}, j \in \mathcal{N}^C} z_{fihj} E_{fih} - \sum_{f,h,i \in \mathcal{N}^C, j \in \mathcal{N}} x_{fihj} E_{fih} \geq 0, \\
&p_{fihj} \text{ free}, \quad x_{fihj} = z_{fihj},\ \forall f \in \mathcal{F}, i, j \in \mathcal{N}, h \in \mathcal{H}.
\end{aligned} \tag{17}$$

We refer to the system consisting of (14), (16), (5) and (17) as the first-seller model, or Model FS.

3. Properties of the Models

In this section, we present some properties of the three market models (modified source- and load-based and first-seller) introduced above. The properties show that under mild conditions, the models constructed in the previous section are well-posed in the sense that an equilibrium exists in each of the models. More importantly, the properties show that although the three approaches to implement a cap-and-trade program are structurally different, electricity producers' generation patterns and costs are the same and so are the total GHG emissions and consumer surpluses. **Robustness of the results with respect to the assumptions is also discussed.**

3.1. Existence and Uniquess

The following results summarize the model properties. For a more succinct presentation, technical proofs appear in the Appendix A. These proofs proceed in the same manner as in Metzler et al. (19). (for easier presentation, in proving the existence and uniqueness results, we make the assumption that $H_{i,f}$ is a singleton for each $f \in \mathcal{F}$ and $i \in \mathcal{N}$. This is not a restrictive assumption. If a zone $i \in \mathcal{N}$ has several generators, we can expand the network $\mathcal{N}$ to make each of the generator a single zone. Denote the newly added zones as $\tilde{\mathcal{N}}(i)$ and the set of all the adjacent zones of i in the original network as $\mathcal{J}(i)$. Then for each $j \in \mathcal{J}(i)$ and $i' \in \tilde{\mathcal{N}}(i)$, connect them with a link with infinite capacity. Then the resulted expanded network will not alter any firms' or consumers' decisions that are made in the original network. Under this simplifying assumption, we can eliminate h indices from all variables and constraints.)

THEOREM 1. *If* $\max_{j \in \mathcal{N}} \left\{P_j^0 - \nabla_x C_{fi}(0)\right\} \geq 0,\ \forall\ f \in \mathcal{F},\ i \in \mathcal{N}$, *then there exists a solution to Model SB, Model LB and Model FS, respectively.*

THEOREM 2. *Let a vector* $\mathbf{b}$ *represent the common variables shared by the three models; that is,* $\mathbf{b} = (x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$. *Assume that the system-wide* CO_2 *caps among the three models are the same; namely,*

$$\overline{E}^{SB} = \overline{E}^{LB} = \overline{E}^{FS}. \tag{18}$$

Then the following statements are equivalent.
(a) There exist s and θ such that $(s,\ \theta,\ \mathbf{b})$ solves Model SB.
(b) There exist z and p such that $(z,\ \mathbf{b},\ p)$ solves Model LB.
(c) There exist z and p such that $(z,\ \mathbf{b},\ p)$ solves Model FS.

Some of the results to be shown in the next theorem strictly require convexity of the production cost function with respect to total generation. (To be more precise, they are items (b) and (c) in Theorem 3.) Assume that the cost function for each firm $f \in \mathcal{F}$ at a zone $i \in \mathcal{N}$ is as follows.

$$C_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right) = c_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right) + \frac{1}{2} d_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right)^2. \tag{19}$$

In addition, we assume that $d_{fi} > 0$ for each $f \in \mathcal{F}$ and $i \in \mathcal{N}$.

To show the uniqueness and equivalence of consumer surpluses in the equilibria of the three models, we assume that consumers' willingness to pay is captured by a linear inverse demand function, as in Section 2. Consumers' surplus is the integral of the inverse demand function minus the costs for purchasing electricity. LSEs are assumed to charge consumers the marginal cost of supplying electricity, including the shadow price of allowances; in equilibrium, this is the marginal willingness to pay. The resulting expression for consumers' surplus is given below:

$$\begin{aligned} CS^m &\equiv \sum_{j\in\mathcal{N}} \left[P_j^0 \left(\sum_{f\in\mathcal{F}} s_{fj}\right) - \frac{P_j^0}{2Q_j^0}\left(\sum_{f\in\mathcal{F}} s_{fj}\right)^2 - \sum_{f\in\mathcal{F}} \left(P_j^m s_{fj}\right)\right], \qquad m = SB; \\ CS^m &\equiv \sum_{j\in\mathcal{N}} \left[P_j^0 \left(\sum_{f\in\mathcal{F}, i\in\mathcal{N}} z_{fij}\right) - \frac{P_j^0}{2Q_j^0}\left(\sum_{f\in\mathcal{F}, i\in\mathcal{N}} z_{fij}\right)^2 - \sum_{f\in\mathcal{F}, i\in\mathcal{N}} \left(P_j^m z_{fij}\right)\right], \ m = LB \text{ or } FS, \end{aligned} \tag{20}$$

where

$$\begin{aligned} P_j^m &\equiv P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}} s_{fj}\right), \qquad m = SB; \\ P_j^m &\equiv P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}, i\in\mathcal{N}} z_{fij}\right), \ m = LB \text{ or } FS. \end{aligned}$$

An alternative expression for the consumers' surplus could be derived from the assumption that all LSE profits (e.g., resulting from free granting of allowances) are returned to consumers as a lump-sum payment. That is, LSEs are regulated entities (as the largest ones are in California), but lump sum costs or revenues from the fixed allocations of allowances are rebated to consumers in a way that does not distort consumption. Furthermore, an assumption could also be made that all allowances rents earned by producers in the SB and FS models are actually returned to consumers, for example, because allowances are auctioned with the proceeds going to consumers.

Theorem 3. *Assume that the set of solutions to any of the three models is nonempty. Under the same condition (18) as in Theorem 2, together with the cost function (19), the following statements are true for the equilibrium solutions of the three models.*
(a) The total sales to each LSE $j \in \mathcal{N}$ are unique and are the same across the three models.
(b) Firm f's total production in each production zone $i \in \mathcal{N}$ is unique and the same across the three models, for each $f \in \mathcal{F}$;
(c) Total emissions are unique and are the same across the three models.
(d) The consumers' marginal willingness-to-pay (price charged to consumers) is unique at each zone $j \in \mathcal{N}$ and are the same across the three models. in addition, the consumers' surpluses are unique and the same across the three models.

(e) Assume that

$$\overline{E}^{SB} = \overline{E}^{LB} = \overline{E}^{FS} = \overline{E},$$

where $\overline{E}$ is the initial CO_2 allowance. Then the total social welfare (the sum of consumers, producer, ISO and LSE surpluses) is unique and is the same across the three models.

The definition of consumers' surplus in the above theorem assumes that all allowance rents accrue to either LSEs or producers and none to consumers. As noted, regulation of LSEs and auctioning of allowances with the proceeds going to consumers could instead result in all allowances rents being returned to consumers. consumers' surplus would then increase by the amount of the rent, but would still be identical in each of the three systems.

Thus, we have refuted the assertion that has been made (12) that load-based systems result in lower costs to consumers than source-based systems because LSEs are able to price-discriminate among producers. Despite that price discrimination, the costs to consumers (more generally, their surpluses) are the same in all three systems.

Though total social welfare and consumers' surpluses are unique in each of the three equilibrium models, ISO's surplus, producers' surplus, and most interestingly, the market clearing price of CO_2, may not be unique due to possible degeneracies created jointly by capacity constraints, transmission constraints and emissions constraints. An example is shown in the Appendix B. **(Note that this example is to illustrate that even under the conditions of Theorem 3, the above-mentioned 3 quantities are not necessarily unique. More general results regarding to under what conditions that these quantities are unique in an equilibrium are not readily available.)**

3.2. Economic intuition

The equivalence result shown by Theorem may be surprising. However, the economic intuition is relatively simple. In the competitive market setting as laid out in the previous section, market outcomes can also be obtained by maximizing total social welfare, subject to three sets of constraints – capacity constraints, transmission constraints and emissions constraints. There may be internal wealth transfer among producers, LSEs, consumers and the ISO, causing different surplus distribution among these participants across the three proposed cap-and-trade systems. However, the internal wealth transfer would be cancelled out and does not affect aggregate social welfare. That is, the summation of all the surpluses remains the same regardless of the point of regulation of GHG emissions. The capacity and transmission constraints are certainly not affected by point of regulation either. Since all GHG emissions associated with electricity produced within or sold to California are accounted for, though at different points, the emissions constraints of the three GHG cap-and-trade programs (modified source-based, modified load-based, and first-seller) are expected to be equivalent as well. As a result, the equivalence of market outcomes as shown in Theorem 3 follows naturally.

3.3. Robustness of the results

Theorem 1 and 2 are derived under the assumptions that the cost functions of each generator is convex and differentiable. The differentiability should not be restrictive. One would expect that if a realistic cost function is indeed non-differentiable, the non-differentiability should only occur at a finite number of singletons in the form of kinks or jumps. As long as the cost functions are non-decreasing with respect to production quantities, the cost functions with kinks or jumps can be represented by piece-wise functions, with each piece being a differentiable function. Each piece of the piece-wise function can be considered as a cost function associated with a distinctive generator, and then the cost functions of all generators are differentiable (with a expanded set of generators). Consequently, all results presented above follow.

Convexity of cost functions is the key in deriving all the results above. However, we do not feel that this is a restrictive assumption either. Convex cost functions imply non-increasing marginal returns, which correspond to non-decreasing marginal cost functions. Since we do not consider investments or any fixed costs in our models, the marginal costs refer to short-run marginal costs. As illustrated in Considine (10), non-decreasing short-run marginal costs functions of generation units, which lead to convex cost functions, are generally observed and hence, are reasonable assumptions.

For Theorem 3, the assumption of strictly convex quadratic cost functions is indeed only needed for (b) and (c) (as can be seen through the proofs in Appendix A). Dropping such an assumption, we still have the uniqueness and equivalence of total sales of electricity, consumers' surplus and social welfare.

4. Numerical Example

In this section, we present a simple example to illustrate the equivalence of modified load-based, modified source-based and first-seller approaches. We also compare them with three other scenarios: the pure load-based, pure source-based and no-cap scenarios.

4.1. Assumptions

We assume there are three zones ($i = \{$CA,NW,SW) connected with transmission lines with fixed capacities (Figure 1). Zone CA is the regulated zone (referred to below as "*in-state*" or California), and zones NW (Pacific Northwest) and SW (Southwest) are unregulated but trade with zone CA (and are jointly referred to as "*out-of-state*"). A number of generating units ($h = \{1, 2, ..., 10\}$) are located in each zone owned by firms ($f = \{1, 2, 3\}$). We allow firms to own generating assets in different locations. Consumers reside in each zone with their willingness-to-pay represented by linear inverse demand curves. For simplicity, we model the markets for a single hour. A larger model with varying load over a number of hours could readily be solved, but would not provide additional insight.

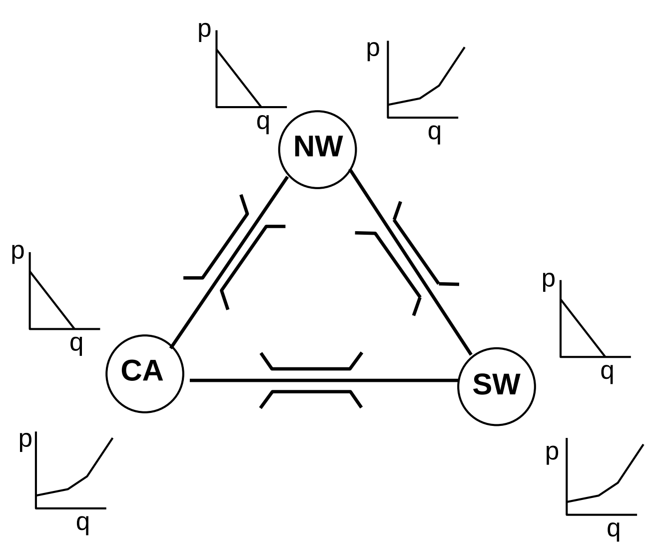

Figure 1 Network of the numeric example.

The specific input assumptions used in this example are presented in the Appendix C. Figure 2 plots marginal cost against cumulative capacity with the corresponding CO_2 emissions rates for zones CA, B and SW, respectively. The marginal cost for zone CA ranges between 35 \$/MWh and 55 \$/MWh with a stable emissions rate around 550 kg/MWh. Zone NW has two types of generating technologies: hydropower with zero variable production cost and zero emissions, and moderately expensive units with a marginal cost and emissions rate of approximately 25 \$/MWh and 550 kgs/MWh, respectively. Zone SW has units with higher emissions but lower costs. The capacity-weighted average CO_2 emissions rate is 582, 367 and 1,171 kg/MWh for zones CA, NW and SW, respectively. **(These numbers are typical values for coal, combined-cycled and combustion turbine technologies.)** Thus, whereas zone CA is designed to resemble California's electricity system primarily with natural gas plants, zones NW and SW represent northwest and southwest states with abundant hydropower and coal plants, respectively. In the baseline (or no-cap) case, roughly 20% and 10% of the electricity demand in zone CA is met by imports from zone NW and SW, respectively. This is an exaggeration of the actual conditions in the western market, but it allows us to investigate possible contract shuffling and carbon leakage. We assume a demand elasticity of -0.2 at the price-quantity pair in the baseline solution.

We simulate six scenarios. In scenarios 1–3, zone CA is subject to an emissions cap of 400 tons under modified load-based, modified source-based, and first-seller approaches, respectively. An additional three scenarios are designed to illustrate the cases when there are pure load- and source-based programs or no cap at all. First, in scenario 4, a 400-ton emissions cap is imposed exclusively on the in-state LSEs with no restriction on emissions associated with exported electricity (*Pure Load-Based*). Second, in scenario 5, we impose a 400-ton cap on just in-state emissions sources (*Pure Source-Based*). Finally, scenario 6 has no emissions cap on either in- and out-of-state sources. In next section, we report the results, including electricity and CO_2 allowances prices, sales, CO_2 emissions, CO_2 leakage and contract shuffling and welfare outcomes. The results of 1–3 and 4-6 scenarios are displayed in Tables 2 and 3, respectively.

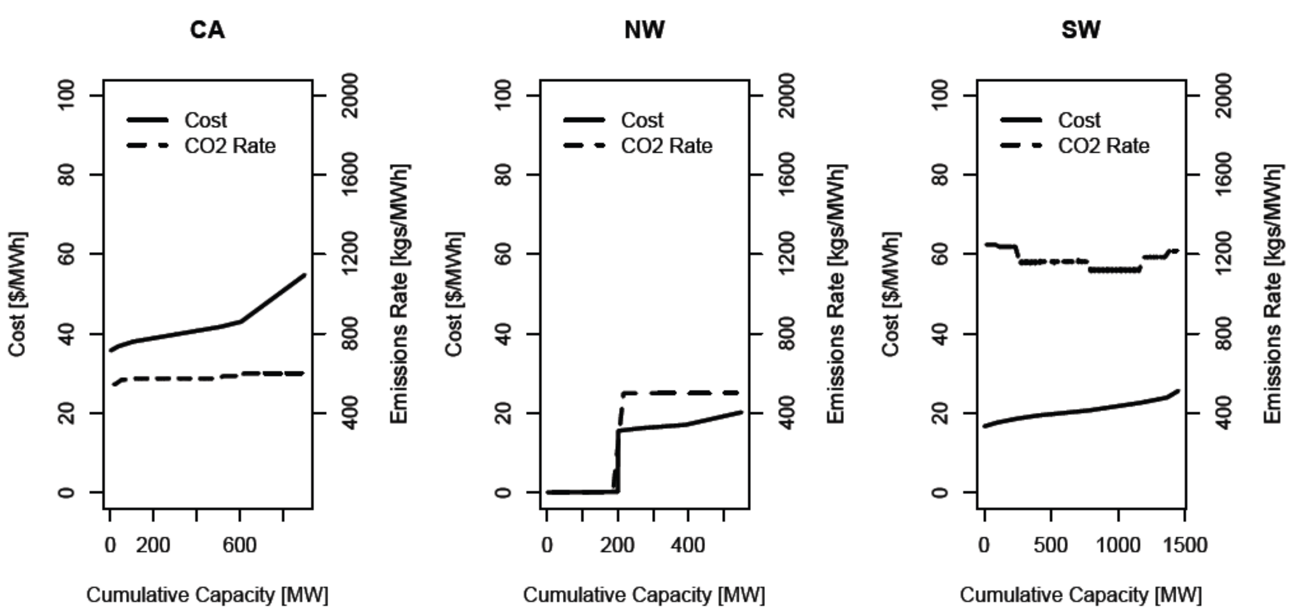

Figure 2 Plots of marginal cost and CO_2 emissions rate against cumulative capacity in zones CA, NW and SW.

4.2. Results

This section reports the results of the six scenarios. We report price and welfare results in Section 4.2.1 and address the equivalence of three proposed programs in Section 4.2.2, followed by discussion of CO_2 leakage and contract shuffling in Section 4.2.3.

4.2.1. Electricity and Allowances Prices and Social Welfare Analysis The extent to which electricity prices would increase under emissions trading depends on the level of allowances prices as well as the emissions rate of the marginal unit that determines the electricity price. As for the level of allowances prices, it depends on how the programs are designed. In particular, the key parameter is whether the in-state to out-of-state and out-of-state to in-state electricity sales are regulated under the emissions programs. Overall, higher allowances prices (and thus, higher electricity prices) occur in the three approaches that regulate imports to zone CA (modified load- and source-based and first-seller based), as shown in Table 2. **When comparing to pure load-based and pure source-based scenarios, zone CA's electricity price increases by $5.8/MWh (=78.1-72.3) and $24.1/MWh (=78.1-54), respectively.** This is primarily because in-state to out-of-state (e.g., CA to NW) and out-of-state to in-state (e.g., SW to CA) electricity sales are not regulated under the pure load- and source-based programs, respectively, while both types of transactions are regulated under the first-seller and modified load- and modified source-based approaches.

The level of allowances prices under the pure source-based approach relative to the pure load-based program depends on the relative costs of in-state and out-of-state generating sources (i.e., the direction of flow). The allowances price under the first-seller (or modified source-based or load-based) approach is $61.0/ton when cross-state electricity sales in both directions are under regulation. Under the pure load-based approach, exports from from zone CA to zones NW and SW increase (since such exports incur no CO_2 cots); this would suppress the demand for CO_2 allowances, and thus reduce allowance prices. Yet, since zone CA is a net importer, the effect on allowances prices would be minor (i.e., dropping from $61.0/ton to $52.3/ton). **In contrast, increased out-of-state imports to CA that result from not to the cap under a pure source-based approach would significantly suppress demand for CO_2 allowances, and thus, the allowances price drops even further (i.e., from $61.0/ton to $12.5/ton).** In a more extreme case in which the out-of-state sources can meet a significant portion of zone CA's load while transmission constraints are not binding, the allowances price under a pure source-based system that applies just to zone CA could crash to zero.

The welfare analysis shows that the overall social welfare (the sum of consumer, producer, LSE and ISO's surpluses) is higher in the pure load- and source-based programs than in the modified programs. **This is only because emissions externality are *not* included in the welfare calculation, and the emissions reductions are smaller in the pure programs, which reduces production costs.** The pure source-based program has only a slightly lower social welfare than the no-cap baseline. However, this is not to say that the pure programs are superior. If the policy goal is to control CO_2 emissions in the WECC region or beyond California state territory, first-seller, modified load-based and modified source-based approaches would be appropriate since they allow agencies to reach out and to regulate transactions in both directions. (As for allocative inefficiency due to higher power prices under modified approaches, it is in the magnitude of 4.9, 18 and 22% compared to pure load-based, pure source-based and no-cap cases, respectively.

As shown in Table 2, the total welfare gained in the first-seller, modified load-based and modified source-based approaches is the same. This total amount is distributed differently among producers, consumers, LSEs and the ISO because different parties receive free allowances in each of the systems. If instead all allowance rents were returned to consumers, then each market participant would have the same net benefits in each scenario, and consumers' surplus would increase by the amount of that rent (from $109,000 to $133,000). We discuss this particular comparison in more detail in the next section.

4.2.2. Equivalence of First-seller, Modified Load-based and Modified Source-based Approach As indicated in Section 2, the first-seller, modified load-based and modified source-based approaches share the following characteristic: all regulate in-state to out-of-state and out-of-state to in-state electricity sales, even though their point-of-regulation is different. Nevertheless, the three approaches differ in two aspects. First, although the total sales (i.e., *Total Sales* in Table 2), and thus, retail electricity prices are the same in the three approaches, the distribution of bilateral generator-LSE electricity sales could be different as implied by Theorem 2 in Section 3. However, the pattern of generation and total production costs are unaffected.

Second, the distribution of economic rents among market parties is different even though the total social welfare is equal under three cases. This basically results because the distribution of allowance rents ($61/ton times the 400 ton cap = $24,400) differs among the three solutions in Table 2. In particular, the decision as to who should retain the LSEs' profits (i.e., $\sum_{i,f,h \in H_{i,f}} (p_{fihj} z_{fihj}) - p^{CO_2}(\sum_{f,i,h} z_{fihj:j \in \mathcal{N}^C} E_{fih} - K_j^{LSE})$ in equations (7) and (12) in the modified load-based and first-seller approaches and how producers are allocated allowances are important policy questions that affect the distribution of allowance rents. If, on one hand, producers receive all allowances for free and LSEs purchase allowances from producers in compliance with the first-seller requirement, the LSE's economic rents of $24,383 and $5,040 under the modified load-based and first-seller program, respectively, should instead be incorporated into the producers' surplus. On the other hand, if allowances are first given to consumers, and producers and LSEs must then purchase those allowances from consumers (perhaps via a state-administered auction), the allowances rents will accrue to consumers and will partially compensate for the pass-through of CO_2 allowances costs. Hence, those three programs will be equivalent from the perspective of welfare distribution if a consistent set of assumptions is made concerning the distribution of allowances rents. (Wolak et al. (Appendix) analyze load- and source-based programs assuming a closed system with inelastic loads and only bilateral trading. They conclude that if allowances are owned by consumers, and producers need to purchase allowances from consumers, the two systems are economically equivalent (25). But they also point out a number of practical and important disadvantages of load-based systems, including impacts on the efficiency of the pool-typed markets.)

4.2.3. Contract Shuffling and CO_2 Leakage The distribution of emissions among zones under different programs is a consequence of how the programs are designed and the degree of transmission congestion. We examine total emission, contract shuffling and emissions leakage in this section. **Overall emissions reflect how stringent the program is designed, and its impact on the market. As the modied approaches are most stringent, the total emission is 1443.1 tons, which is 2.2, 5.2 and 6.9% less than pure load-based, pure source-based and no-cap scenarios. As discussed below, these differences are remarkably low due to contract shuffling and CO_2 leakage.**

In electricity markets, most sales are financial transactions since actual flows in the network must follow physical laws. Contract shuffling in the current context refers to the situation in which the rearrangement of financial contracts results in no actual emissions reduction in an emissions trading program. Whether contract shuffling would occur under different cases can be analyzed by examining variables z_{fihj} in the models. Here, we sum variables z_{fihj} over subscripts f and h to get electricity sales (s_{ij}) from i to j (see *Electricity Sales* in Tables 2 and 3).

We focus on a comparison of the no-cap case in Table 3 with other approaches in Tables 2 and 3. First, we consider the issue of leakage, and then turn to contract shuffling.

Consistent with definitions elsewhere (22), we defined leakage as the difference between the decrease in regulated emissions and the decrease in total regional emissions. That is, increases in unregulated emissions elsewhere could, to some degree, offset the decline in regulated emissions.

Table 2 Summary of market equilibria under modified source-based, modified load-based and first-seller programs.

	Modified Load-Based			Modified Source-Based			First-Seller		
Emission Cap [tons]			400.0			400.0			400.0
Consumer Surplus [$]			107,953			107,953			107,953
Producer Surplus [$]			25,889			50,272			45,232
ISO's Surplus [$]			4,834			4,834			4,834
LSE's Surplus [$]			24,383			N\A			5,040
Social Welfare [$]			163,059			163,059			163,059
Variable\Zone	CA	NW	SW	CA	NW	SW	CA	NW	SW
Price [$/MWh]	78.1	40.9	20.8	78.1	40.9	20.8	78.1	40.9	20.8
CO_2 Emissions [tons] (from\to)									
CA	317.3	0	0	317.3	0	0	317.3	0	0
NW	82.7	13.4	78.0	82.7	0.8	91.5	82.7	16.8	75.6
SW	0	330.7	620.0	0	368.9	581.8	0	325.2	625.5
Zonal CO_2 Emissions [ton]*	317.3	175	950.7	317.3	175	950.7	317.3	175	950.7
Associated CO_2 Emission [tons]	400	345.1	697.9	400	369.7	673.3	400.0	342	701.0
Import (+)/Export(-) Emissions [ton]	82.7	170.1	-252.79	82.7	194.7	-277.4	82.7	167	-249.7
Total CO_2 Emissions [ton]		1,443.1			1,443.1			1,443.1	
CO_2 Price [$/ton]		61.0			61.0			61.0	
Generating Cost [$]		43,358.0			43,358.0			43,358.0	
Allowances Rents [$]		24,384.0			24,384.0			24,384.0	
Electricity Sales (from\to)	CA	NW	SW	CA	NW	SW	CA	NW	SW
CA	555.1	0	0	555.1	0	0	555.1	0	0
NW	365.4	28.7	155.9	365.4	1.6	183.0	365.4	33.5	151.1
SW	0	274.1	527.9	0	301.2	500.7	0	269.3	532.6
Total Sale [MWh]	920.5	302.8	683.8	920.5	302.8	683.8	920.5	302.8	683.8

*Note: The zonal CO_2 emissions is the row sum of CO_2. For example, the zonal emissions under the modified source-based program for zone CA equals $317.3 + 0 + 0 = 317.3$ tons.

Table 3 Summary of market equilibria under no-cap, pure source-based and pure load-based programs.

	Pure Load-Based			Pure Source-Based			No-Cap		
Emission Cap [tons]			400			400			N\A
Consumer Surplus [$]			113,532			131,854			138,017
Producer Surplus [$]			25,635			28,369			23,464
ISO's Surplus [$]			3,986			6,659			4949
LSE's Surplus [$]			20,934			N\A			0
Social Welfare [$]			164,088			166,882			166,430
Variable\Zone	CA	NW	SW	CA	NW	SW	CA	NW	SW
Price [$/MWh]	72.3	37.4	20.8	54.0	43.1	20.7	48.3	41.4	20.8
CO_2 Emission [tons] (from\to))									
CA	225.0	31.2	87.8	209.2	35.7	155.1	198.8	48.0	179.1
NW	175.0	0.0	0.0	96.6	7.2	71.2	78.7	8.4	87.9
SW	0.0	333.3	622.9	477.7	244.6	224.9	601.3	232.8	115.6
Zonal CO_2 Emissions [ton]	344.0	175.0	956.2	400.0	175.0	947.2	425.8	175.0	950.0
Associated CO_2 Emission [tons]	400.0	364.5	710.7	783.5	287.5	451.2	878.9	290.0	381.9
Import (+)/Export(-) Emissions [ton]	56	189.5	-245.5	383.5	112.5	-496.0	453.1	115.0	-568.1
Total CO_2 Emissions [ton]		1,475.2			1,522.2			1,550.7	
CO_2 Price [$/ton]		52.3			12.5			0	
Generating Cost [$]		45,327			49,365			51,451	
Allowances Rents [$]		20,932			4,988			0	
Electricity Sales (from\to)	CA	NW	SW	CA	NW	SW	CA	NW	SW
CA	400.0	52.0	148.0	362.5	61.9	268.9	343.4	82.9	309.9
NW	550.0	0.0	0.0	302.4	21.4	226.2	248.5	25.2	276.2
SW	0.0	271.2	535.6	403.4	206.7	188.7	511.8	191.9	97.6
Total Sale [MWh]	950.0	323.2	683.6	1,068.3	290.0	683.9	1,103.7	300.1	683.8

To obtain a gross indicator of leakage, we first calculate for each of the load-, source- and first-seller-based proposals the decrease in emissions of generators in zone CA plus imports to zone CA, compared to the no-cap case (row 2 of Table 4, called E_1). These are the emissions that are subject to regulation in the modified and first-seller proposals (Table 2), and for consistency, we also consider them for the pure source- and load-based proposals (Table 3). We then calculate the total decrease in emissions in zones CA+NW+SW relative to the no-cap case (row 4 of Table 4, called E_2). The percentage leakage is then $100\%(1 - E_2/E_1)$.

Table 4 shows that in all cases most of the apparent emission reductions in regulated emissions are lost due to leakage – i.e., increases in nonregulated emissions. For instance, for the modified- and first-seller-based proposals (Table 2), the leakage is 85%. This is because regulated emissions fall from 1,056 tons (no-cap case) to 400 tons (the cap), but total emissions fall by much less (1,551 to 1,443). **Thus, leakage results in overall emissions reduction remarkably low, 108(=1551-1443), 76 and 29 tons for modified-, pure-load, pure-source based approaches, respectively.**

This leakage is in large measure due to contract shuffling. We define contract shuffling as the difference between the apparent decrease in emissions associated with power imports to zone CA (called E_3 in Table 4) and the actual decrease in emissions in zones NW and SW (E_4 in Table 4). In our simulations, emissions in zones NW and SW are actually unchanged, even though the emissions associated with bilateral transactions from zones NW and SW to zone CA decrease. If we define the percentage of contract shuffling as $100\%(1 - E_4/E_3)$, then this percentage is actually equal to 100%. Thus, the apparent emissions decreases due to changes in the composition of imports to zone CA are completely illusory, as emissions elsewhere are actually increasing under regulation.

For example, the percentage of contract reshuffling for the modified- and first-seller-based proposals (Table 2) is 100% (last row, Table 4). This is because although emissions associated with imports by zone CA fall from 680 tons (no-cap) to 83 tons (Table 2), emissions in zones NW and SW actually approximately unchanged (1,125 tons=78.7 (from B)+601.3 (from C)). This occurs because the large amounts of dirty power that used to be exported from zone SW to zone CA in the no-cap case (512 MWh, Table 3) is diverted to zone NW under regulation. As a result, zone NW is then able export more of its clean power to zone CA (compare Tables 2 and 3). Of course, these are financial transactions, and the shifts in physical power flows are much less dramatic. (The distribution of bilateral contracts in the no-cap model is not unique, which implies that the calculations of leakage and contract shuffling in this section are also not unique. For instance, if a no-cap solution with more zone NW to CA bilateral contracts was instead used as the baseline, then the estimated amounts of leakage and shuffling would be smaller. However, these illustrative calculations demonstrate that large amounts of leakage and contract shuffling are very likely to occur under these regulatory proposals.)

5. Conclusion and Discussion

California is considering three proposals to regulate greenhouse gasses emitted by electric power plants under the Assembly Bill 32: source-based, load-based and first-seller approaches. These three proposals differ by their point-of-regulation and possibly the "jurisdiction" of the regulation. Our interpretation is that the three proposals will regulate emissions associated with in-state and cross-state electricity transactions. However, if in-state to out-of-state sales or vice versa are not subject to a cap, greater amounts of contract shuffling and CO_2 leakage with fewer real emissions reductions could possibly occur as shown in the pure load-based and pure source-based scenarios simulated in Section 4.

We have formulated equilibrium models for the three proposals. In addition to existence and uniqueness properties of the solutions, this paper shows that the three proposals are essentially equivalent in that they produce the same zonal sales, the same production and emission patterns,

Table 4 Leakage and Reshuffling Results, Scenarios 1-6

	Table 2 Regulations[a]	Pure Load-Based	Pure Source-Based	No-Cap
Emissions of CA + Imports to A[tons]	400	519	974	1,106
Δ Relative to No-Cap (E_1)[tons]	-706	-587	-132	
Total Emissions [tons]	1,443	1,475	1,522	1,551
Δ Relative to No-Cap (E_2)[tons]	-108	-76	-29	
Leakage[b][%]	85%	87%	78%	
Emissions Imports to CA[tons]	83	175	574	680
Δ Relative to No-Cap (E_3)[tons]	-597	-505	-106	
Emissions NW+SW[tons]	1,125	1,131	1,122	1,125
Δ Relative to No-Cap (E_4)[tons]	0	6	-3	
Reshuffling[c][%]	100%	101%	97%	

Notes: a. Modified Source- and Load-Based, and First- Seller

b. Leakage [%] = $(1 - E1/E2) * 100\%$

c. Reshuffling [%] = $(1 - E3/E4) * 100\%$

the same allowance and retail electricity prices and the same social surplus. This occurs under the assumption that imports to California would be subject to the emissions cap, as well as sales from California plants to other states.

However, the proposals can differ in the distribution of social surplus; when this occurs, it is because of assumptions concerning the distribution of allowance rents. If the markets are designed so that all emissions allowance rents accrue to consumers, then the consumer surplus is the same in all three proposals.

This points out the importance of the issue of allowance allocation. The value of those allowances could represent a significant amount of wealth transfer and might also affect program's political feasibility and acceptability. For instance, it has been shown that significant windfall profits were earned by generators in the European Union Emissions Trading Scheme in 2005 given that generators received most allowances for free (23). Additionally, for first-seller and source-based approaches to survive a legal challenge, it is reasonable to conjecture that allowances need to be distributed by auctions. **That is, it will be less contestable under the interstate commence laws.** Otherwise, in-state sources would be given a financial windfall that their counterparts in neighboring states would not obtain if allowances are grandfathered.

Another assumption made in this paper is that the CO_2 intensity of imported electricity can be unambiguously determined. This may prove to be a challenging task in reality given the complicated operations of the competitive electricity markets. **At least two proposals (e.g., TEAC and CO_2RC) have suggested using a default emission rate.** In this modelling framework, this is equivalent to assigning a uniform emissions rate to all the out-of-state sources. However, the conclusion about the equivalence of three proposed programs will remain intact.

As California has traditionally been a leader in pioneering new environmental policies in the U.S., if the policy goal is to control regional greenhouse gas emissions in the West, a California-only emissions trading program may prove to have little impact on overall greenhouse gas emissions while possibly disrupting electricity market operations, as argued by Wolak et al.(25). In contrast, if the policy goal to take the lead with the hope that a comprehensive federal CO_2 cap-and-trade will follow in near future, careful consideration should be given to how the AB32 program should be designed to minimize interference with the operations of competitive electricity markets and reduce the chance that California consumers will suffer from additional electricity price increases.

Appendix A: Proofs

To prove the results presented in Section 2.4, we first provide an exact definition of a mixed linear complementarity problem, which plays a central role in all the proofs.

DEFINITION 1. Let $M \in \Re^{n\times n}$, $N \in \Re^{n\times m}$, $q \in \Re^n$, $U \in \Re^{m\times n}$, $V \in \Re^{m\times m}$, and $r \in \Re^m$ be all given. A mixed linear complementarity problem (MLCP) is to find a vector $x \in \Re^n$ and $y \in \Re^m$ such that

$$\begin{array}{c} 0 \le x \perp Mx + Ny + q \ge 0; \\ 0 = Ux + Vy + r. \end{array} \tag{21}$$

We denote the above problem as $MLCP(M, N, q, U, V, r)$.

A special case of an MLCP is $MLCP(M, N, q, -N^T, 0, r)$, which has some important properties as shown below. We summarize some known results and facts that are used to prove the main theorems.

LEMMA 1. *(Metzler (20)) For an MLCP problem $MLCP(M, N, q, -N^T, 0, r)$, if M is positive semi-definite, and the problem is feasible, then a solution exists to the MLCP.*

LEMMA 2. *Suppose that $MLCP(M, N, q, -N^T, 0, r)$ is feasible. If M is positive semi-definite, then for any two solutions of the MLCP, (x^1, y^1) and (x^2, y^2), the following is true.*

$$(M + M^T)x^1 = (M + M^T)x^2.$$

Proof. By Lemma 1 we know that a solution exists to $MLCP(M, N, q, -N^T, 0, r)$. The rest of the proof is the same as that of Theorem 3.1.7 in Cottle et al. (11). □

For the remaining section, we assume that the cost function is as given in Section 3 (equation (19)), with $d_{fi} > 0$ for each $f \in \mathcal{F}$ and $i \in \mathcal{N}$. To prove the theorems presented in Section (3), we first introduce the MLCP formulation of the modified source-based, modified load-based and first-seller models. Note that the variable representing transmission charges, w, can be replaced by variables λ and η, and hence, eliminated from any of the three models. Similarly, the variables of power shipped over transmission lines, y, can be replaced by the sales variables (s or x) , according to the market clearing conditions. Let $\{SB, LB, FS\}$ denote the set of the three models. Then for $m \in \{SB, LB, FS\}$, the three models can be represented by an MLCP with the same structure.

$$\begin{array}{c} 0 \le \mathbf{a}^m \perp q^m + M^m \mathbf{a}^m + \Delta^m \theta^m \ge 0, \\ 0 = \Delta^{m^T} \mathbf{a}^m. \end{array} \tag{22}$$

The variables in the MLCP are as follows.

$$\mathbf{a}^{SB} = \begin{bmatrix} \lambda \\ s \\ x \\ \rho \\ p^{CO_2} \end{bmatrix} \in \Re^{2K+2FN+FN^2+1}, \ \mathbf{a}^{LB} = \mathbf{a}^{FS} = \begin{bmatrix} \lambda \\ z \\ x \\ \rho \\ p^{CO_2} \end{bmatrix} \in \Re^{2K+FN+2FN^2+1};$$

$\theta^{SB} = [\theta'_{fj}] \in \Re^{FN}$, $\theta^{LB} = \theta^{FS} = [p_{fij}] \in \Re^{FN^2}$, where $\theta'_{fj} = \theta_{fj} - \eta$ for all $f \in \mathcal{F}$ and $i \in \mathcal{N}$.

The parameters are q^m, M^m and Δ^m, with q^m being a vector of the same dimension as $\mathbf{a}^m$, M^m being a square matrix of the appropriate dimension, and $\Delta^m \in \Re^{(2K+2FN+FN^2+1)\times FN}$ for $m = SB$, and $\Delta^m \in \Re^{(2K+FN+2FN^2+1)\times FN^2}$ for $m = LB,\ FS$. Define the following vectors: $T = [T_k] \in \Re^{2K}$,

$P^0 = [P_j^0] \in \Re^N$, $C = [c_{fi}] \in \Re^{FN}$, $X = [X_{fi}] \in \Re^{FN}$. Then the detailed expression of the parameters are as follows.

$$q^{SB} = \begin{pmatrix} T \\ -P^0 \\ C \\ X \\ \overline{E}^{SB} \end{pmatrix}, \; q^{LB} = \begin{pmatrix} T \\ -P^0 \\ C \\ X \\ \overline{E}^{LB} \end{pmatrix}, \; q^{FS} = \begin{pmatrix} T \\ -P^0 \\ C \\ X \\ \overline{E}^{FS} \end{pmatrix};$$

$$\Delta^{SB} = \begin{pmatrix} 0 \\ I_{FN\times FN} \\ -J_{FN^2\times FN} \\ 0 \\ 0 \end{pmatrix}, \; \Delta^{LB} = \Delta^{FS} = \begin{pmatrix} 0 \\ I_{FN^2\times FN^2} \\ -I_{FN^2\times FN^2} \\ 0 \\ 0 \end{pmatrix},$$

where I represents the identity matrix of appropriate dimensions, and

$$J = \begin{bmatrix} \mathbf{1}_{N\times 1} & 0 & \cdots & 0 \\ 0 & \mathbf{1}_{N\times 1} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \mathbf{1}_{N\times 1} \end{bmatrix} \in \Re^{(FN^2)\times(FN)};$$

$$M^{SB} = \begin{bmatrix} 0 & M_{\lambda s} & M_{\lambda x}^{SB} & 0 & 0 \\ -M_{\lambda s}^T & M_s & 0 & 0 & 0 \\ -M_{\lambda x}^{SB^T} & 0 & M_x & M_{x\rho} & \mathcal{E}_x^{SB} \\ 0 & 0 & -M_{x\rho}^T & 0 & 0 \\ 0 & 0 & -\mathcal{E}_x^{SB^T} & 0 & 0 \end{bmatrix};$$

$$M^m = \begin{bmatrix} 0 & 0 & M_{\lambda x}^m & 0 & 0 \\ 0 & M_z^m & 0 & 0 & \mathcal{E}_z^m \\ -M_{\lambda x}^{m^T} & 0 & M_x & M_{x\rho} & \mathcal{E}_x^m \\ 0 & 0 & -M_{x\rho}^T & 0 & 0 \\ 0 & -\mathcal{E}_z^{m^T} & -\mathcal{E}_x^{m^T} & 0 & 0 \end{bmatrix}, \text{where } m \in \{LB, FS\}.$$

We first provide details on the off-diagonal block matrices of the M matrices.

Let $\Pi \equiv [PTDF_{ki}] \in \Re^{2L\times N}$ denote the matrix of $PTDF$'s. Then

$$\begin{aligned} M_{\lambda s} &= -[\Pi \; \ldots \; \Pi\,] \qquad\;\; \in \Re^{2L\times FN}, \\ M_{\lambda x}^{SB} &= \;\;[\Pi \; \ldots \; \Pi \; \ldots \; \Pi] \in \Re^{2L\times FN^2}. \end{aligned}$$

Assume that the vector $x = [x_{fij}] \in \Re^{FN^2}$ is arranged first by the j index, then by the i index, and then by the f index. The $M_{\lambda x}^m$ matrix, for $m = \{LB, FS\}$, is given as follows

$$M_{\lambda x}^m = [\Pi \; \ldots \; \Pi \; \ldots \; \Pi] - [\Pi \; \ldots \; \Pi \; \ldots \; \Pi]\, P \in \Re^{2L\times FN^2},$$

where $P \in \Re^{FN^2\times FN^2}$ is a permutation matrix that re-groups the matrix $[\Pi \; \ldots \; \Pi \; \ldots \; \Pi]$ corresponding to index i of the vector x.

$$M_{x\rho} = \begin{bmatrix} \mathbf{1}_{N\times 1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{1}_{N\times 1} \end{bmatrix} \in \Re^{FN^2\times FN}.$$

Further assume that the vector $x = [x_{fij}]$ is arranged by $j \in \mathcal{N}^C$ first, then by $j \in \mathcal{N}^N$, $i \in \mathcal{N}^C$, $i \in \mathcal{N}^N$, and $f \in \mathcal{F}$. Then $\mathcal{E}_x^m$'s are as follows.

$$\mathcal{E}_x^m = \left[\mathcal{E}_{x_1}^{m^T} \ldots \mathcal{E}_{x_f}^{m^T} \ldots \mathcal{E}_{x_F}^{m^T}\right] \in \Re^{FN^2},$$

where for each $f \in \mathcal{F}$, $\mathcal{E}_{x_f}^{m^T} = \left[E_{fi}\tilde{I}_i^m\right]_{i\in\mathcal{N}} \cdot \mathbf{1}_{N^2\times 1} \in \Re^{N^2}$. $\tilde{I}_i^m$ are diagonal matrices for each $m \in \{SB, LB, FS\}$ and $i \in \mathcal{N}$, with diagonal entries being either 0s or 1s. Their detailed forms are as follows.

$$\tilde{I}_i^{SB} = \begin{cases} \text{identity matrix of size } N, & i \in \mathcal{N}^C, \\ \text{1's for the first } J \text{ diagonal entries, 0's otherwise}, & i \in \mathcal{N}^{NC}. \end{cases}$$

$$\tilde{I}_i^{LB} = \begin{cases} \text{0's for the first } J \text{ diagonal entries, 1's for the rest diagonal entries}, & i \in \mathcal{N}^C, \\ \text{0 matrix of size } N, & i \in \mathcal{N}^{NC}. \end{cases}$$

$$\tilde{I}_i^{FS} = \begin{cases} \text{identity matrix of size } N, & i \in \mathcal{N}^C, \\ \text{0 matrix of size } N, & i \in \mathcal{N}^{NC}. \end{cases}$$

$\mathcal{E}_z^m$ are defined in a similar fashion as $\mathcal{E}_x^m$ and are omitted here.

For the diagonal block matrices in the M matrices, first denote a diagonal matrix $Diag(\frac{P_j^0}{Q_j^0})_{j\in\mathcal{N}}$ as matrix D. Then $M_s = D \otimes \mathbf{1}_{F\times F}$, and $M_z^m = D \otimes \mathbf{1}_{FN\times FN}$, where $\mathbf{1}$ represents a matrix of all 1s with proper dimensions, and $\otimes$ denotes the Kronecker product. The M_x matrix is the same across the three models, and it is a block-diagonal matrix as follows.

$$M_x = \begin{bmatrix} d_{11}\mathbf{1}_{N\times N} & & & & 0 \\ & \ddots & & & \\ & & d_{fi}\mathbf{1}_{N\times N} & & \\ & & & \ddots & \\ 0 & & & & d_{FN}\mathbf{1}_{N\times N} \end{bmatrix} \in \Re^{FN^2\times FN^2}.$$

LEMMA 3. *The matrix M^m, for each $m \in \{SB, LB, FS\}$, is positive semi-definite.*

Proof. It is easy to see that M^m is positive semi-definite if only M_x, M_s and M_z are positive semi-definite matrices. M_x is trivially seen to be positive semi-definite, given the assumption that $d_{fi} > 0$ for each $f \in \mathcal{F}$, $i \in \mathcal{N}$. Since the structure of M_s and M_z^m are the same, we need only consider one of them. Here we consider M_s. By the formulation introduced above, M_s is the Kronecker product of the matrices D and $\mathbf{1}_{F\times F}$. Since both matrices are positive semi-definite, by a known result from matrix analysis, (see, for example, Horn and Johnson (18)), M_s, as the Kronecker product of two positive semi-definite matrices, is also positive semi-definite. □

Theorem 1.
Proof. For Model SB, choose $s = x = 0$, $\lambda = 0$, $y = w = 0$, $p^{CO_2} = 0$, $\theta'_{fj} = \theta_{fj} - \eta = P_j^0$, $\forall\ f \in \mathcal{F},\ j \in \mathcal{N}$, and $\rho_{fi} = \max_{j\in\mathcal{N}} \{P_j^0 - C_{fi}\}$, $\forall\ f \in \mathcal{F}$ and $i \in \mathcal{N}$. Then the tuple $(s,\ \theta,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$ is feasible to Model SB. By Lemma 1 and 3, Model SB, written as $MLCP(M^{SB}, \Delta, q^{SB}, -\Delta^T, 0, 0)$, has a solution. Existence of a solution to Model LB and Model FS can be shown in the similar way. □

Theorem 2.
Proof. (a) $\Rightarrow$ (b). Suppose that $(s,\ \theta,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$ solves Model SB. First consider a

$s_{fj}>0$, with $f\in\mathcal{F}$ and $j\in\mathcal{N}$. By complementarity in firm f's first order condition (3), we have that $\theta_{fj}=\tilde{p}_j-w_j$, and for the particular f and j,

$$0\le x_{fij}\perp c_{fi}+d_{fi}\sum_{j'\in\mathcal{N}}(x_{fij'})+p^{CO_2}E_{fi}-w_i+\rho_{fi}-\tilde{p}_j+w_j\ge 0,\ \forall\ i\in\mathcal{N}\text{ if }j\in\mathcal{N}^C,$$
$$0\le x_{fij}\perp c_{fi}+d_{fi}\sum_{j'\in\mathcal{N}}(x_{fij'})-w_i+\rho_{fih}-\tilde{p}_j+w_j\ge 0,\ \forall\ i\in\mathcal{N}^{NC},\text{if }j\in\mathcal{N}^{NC},$$

where $\tilde{p}_j = P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_f s_{fj}\right) = P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} x_{fihj}\right)$.

For a $s_{fj}=0$, by nonnegativity of x_{fij}, we have that $x_{fij}=0$, $\forall\ i\in\mathcal{N}$.

Now for each $f\in\mathcal{F}$, $i,j\in\mathcal{N}$, let $z_{fij}=x_{fij}$ and define p_{fij} as follows

$$p_{fij}=\begin{cases}\tilde{p}_j, & \text{if } j\in\mathcal{N}^{NC};\\ \tilde{p}_j-p^{CO_2}E_{fi}, & \text{if } j\in\mathcal{N}^C.\end{cases}$$

Let $z=[z_{fij}]$ and $p=[p_{fij}]$. Then clearly $(z,x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},p)$ satisfies all the equality and complementarity conditions of Model LB. Hence, it solves Model LB.

(b) $\Rightarrow$ (c). Assume that $(z,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},\ p)$ solves Model LB, where $p=[p_{fij}]$. For each $f\in\mathcal{F}$, $i,j\in\mathcal{N}$, define $\hat{p}_{fij}$ as follows

$$\hat{p}_{fij}=\begin{cases}p_{fij}+p^{CO_2}E_{fi}, & \text{if } i,j\in\mathcal{N}^C;\\ p_{fij}, & \text{otherwise.}\end{cases}$$

Let $\hat{p}=[\hat{p}_{fij}]$. Then it is easy to see that $(z,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},\ \hat{p})$ solves Model FS.

(c) $\Rightarrow$ (a). Assume that $(z,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2},\ p)$ solves Model FS, where $p=[p_{fij}]$. For each $f\in\mathcal{F}$, $i\in\mathcal{N}^{NC}$ and $j\in\mathcal{N}^C$ with $z_{fij}>0$, by the LSE j's first-order condition, we have that $p_{fij}+p^{CO_2}E_{fi}=P_j^0-\frac{P_j^0}{Q_j^0}(\sum_{f,i,h\in H_{i,f}} z_{fij})$. Notice that the right-hand-side is independent of index f, i and h. Similarly, for $i\in\mathcal{N}^C$ or $j\in\mathcal{N}^{NC}$ with $z_{fij}>0$, $p_{fij}=P_j^0-\frac{P_j^0}{Q_j^0}(\sum_{f,i,h\in H_{i,f}} z_{fij})$. By letting $\tilde{p}_j$, $j\in\mathcal{N}$ denote $P_j^0-\frac{P_j^0}{Q_j^0}(\sum_{f,i,h\in H_{i,f}} z_{fij})$, we have the following.

$$\tilde{p}_j=\begin{cases}p_{fij}+p^{CO_2}E_{fi}, & \text{if } i\in\mathcal{N}^{NC}\text{ and }j\in\mathcal{N}^C;\\ p_{fij}, & \text{otherwise.}\end{cases}$$

Further define that for each $f\in\mathcal{F}$ and $j\in\mathcal{N}$, $s_{fj}=\sum_{i\in\mathcal{N}}x_{fij}$ and $\theta_{fj}=\tilde{p}_j-w_j$. The complementarity constraints on s variables in formulation (3) are be seen to be satisfied. It is then trivial to show that $(s,\ \theta,\ x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$ solves Model SB. □

Theorem 3.

Proof. (a) Let $SOL(MLCP^m)$ denote the set of solutions to the MLCP model i, where $m\in\{SB,LB,FS\}$. First consider the source-based model. Let $\mathbf{a}^{SB^1}$, $\mathbf{a}^{SB^2}\in SOL(MLCP^{SB})$. By Lemma (2), we have the following.

$$\begin{aligned}&(M^{SB}+M^{SB^T})\mathbf{a}^{SB^1}=(M^{SB}+M^{SB^T})\mathbf{a}^{SB^2}\\ \iff& M_s^{SB}s^1=M_s^{SB}s^2\\ \iff& \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}}s_{fj}^1\right)=\frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}}s_{fj}^2\right),\ \forall j\in\mathcal{N}.\end{aligned}\tag{23}$$

The last equality constraint implies that among solutions to Model SB, total sales at zone $j \in \mathcal{N}$ in the source-based model is unique. The uniqueness of total sales $\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}$ for each $j \in \mathcal{N}$ in solutions for Model LB and FS can be shown similarly.

Define a vector $\mathbf{b}$ the same way as in Theorem 2; namely, $\mathbf{b} = (x,\ \rho,\ w,\ y,\ \eta,\ \lambda,\ p^{CO_2})$. By Theorem 2, we know that if $(s,\theta,\mathbf{b}) \in SOL(MLCP^{SB})$, there exist z and p such that $(z,\mathbf{b},p) \in SOL(MLCP^{LB})$, and z' and p' such that $(z',\mathbf{b},p') \in SOL(MLCP^{FS})$. Then we have the following.

$$\begin{array}{ccccc} \sum_{f\in\mathcal{F}} s_{fj} & = & \sum_{f\in\mathcal{F},i\in\mathcal{N}} x_{fij} & = & \sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij},\ \forall\ j \in \mathcal{N}. \\ & & \| & & \\ & & \sum_{f\in\mathcal{F},i\in\mathcal{N}} z'_{fij} & & \end{array}$$

It has just been shown that in a solution to Model SB, the total sale $\sum_{f\in\mathcal{F}} s_{fj}$ is unique at each zone $j \in \mathcal{N}$; similarly, the total sale $\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}$ is unique for solutions to Model LB, and $\sum_{f\in\mathcal{F},i\in\mathcal{N}} z'_{fij}$ is unique for Model FS. Hence, we have shown that among solutions to the three models, total sale at each region $j \in \mathcal{N}$ is the same.

(b) Following similar arguments in (a), we have that

$$\begin{aligned} & (M^{SB} + M^{SB^T})\mathbf{a}^{SB^1} = (M^{SB} + M^{SB^T})\mathbf{a}^{SB^2} \\ \Longleftrightarrow\ & M_x x^1 = M_x x^2 \\ \Longleftrightarrow\ & d_{fi}\left(\sum_{j'\in\mathcal{N}} x^1_{fij'}\right) = d_{fi}\left(\sum_{j'\in\mathcal{N}} x^2_{fij'}\right),\ \forall f \in \mathcal{F}, i \in \mathcal{N}. \end{aligned} \tag{24}$$

Hence, $\sum_{j\in\mathcal{N}} x_{fij}$, the total production at zone i for firm f is unique in an equilibrium of Model SB. Uniqueness of the corresponding quantity in the equilibria of Model LB and Mode FS can then be shown with the same agrement. By Theorem 2, the equivalence of the equilibrium quantity across the three models follows.

(c) Total emissions in the entire system under consideration are given by $\sum_{f\in\mathcal{F},i,j\in\mathcal{N}} (x_{fij}E_{fi})$. By re-arranging the indices, we have the following relationship

$$\sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} (x_{fij}E_{fi}) = \sum_{f\in\mathcal{F}}\sum_{i\in\mathcal{N}} E_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right).$$

By the uniqueness and equivalence results of $\sum_{j\in\mathcal{N}} x_{fij}$ shown in Part (b), the same results hold true for total emissions.

(d) consumers surpluses, under linear inverse demand function, are given in Section 3 (20). For illustration purpose, they are written out again.

$$\begin{aligned} CS^m &\equiv \sum_{j\in\mathcal{N}}\left[P^0_j\left(\sum_{f\in\mathcal{F}} s_{fj}\right) - \frac{P^0_j}{2Q^0_j}\left(\sum_{f\in\mathcal{F}} s_{fj}\right)^2 - \sum_{f\in\mathcal{F}}\left(P^m_j s_{fj}\right)\right], \qquad m = SB; \\ CS^m &\equiv \sum_{j\in\mathcal{N}}\left[P^0_j\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right) - \frac{P^0_j}{2Q^0_j}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right)^2 - \sum_{f\in\mathcal{F},i\in\mathcal{N}}\left(P^m_j z_{fij}\right)\right],\ m = LB \text{ or } FS, \end{aligned}$$

with

$$P_j^m \equiv P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F}} s_{fj}\right), \qquad m = SB;$$
$$P_j^m \equiv P_j^0 - \frac{P_j^0}{Q_j^0}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right), \; m = LB \text{ or } FS.$$

Then the uniqueness of P_j^m for each $j \in \mathcal{N}$ and CS^m at an equilibrium for each $m \in \{SB, LB, FS\}$, and the equivalence of P_j^m and CS^m across the models follow directly from (a).

(e) The social welfare under the modeling context is the sum of consumers, LSEs, producers and ISO surpluses. The mathematical expressions of the surpluses are given below. (The expressions for the three models are similar. To avoid lengthy presentation, we only show those for the first-seller model to illustrate the concepts.)

$$\begin{aligned}
\text{Consumer surplus} &= \sum_{j\in\mathcal{N}}\left[P_j^0\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right) - \frac{P_j^0}{2Q_j^0}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right)^2 - (P_j^0 - \frac{P_j^0}{Q_j^0}\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij})(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij})\right];\\
\text{LSE surplus} &= \sum_{j\in\mathcal{N}}\left[(P_j^0 - \frac{P_j^0}{Q_j^0}\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij})(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij})\right] - \sum_{f\in\mathcal{F},i,j\in\mathcal{N}}(p_{fij}z_{fij});\\
&\quad -P^{CO_2}\left[\sum_{f\in\mathcal{F},i\in\mathcal{N}^{NC},j\in\mathcal{N}^C}(z_{fij}E_{fi}) - K_j^{LSE}\right];\\
\text{Producer surplus} &= \sum_{f\in\mathcal{F},i,j\in\mathcal{N}} p_{fij}x_{fij} - \sum_{i\in\mathcal{N}} C_{fi}\left(\sum_{j\in\mathcal{N}} x_{fij}\right) - \sum_{i,j\in\mathcal{N}}(w_j - w_i)x_{fij}\\
&\quad - p^{CO_2}\left[\sum_{f\in\mathcal{F},i\in\mathcal{N}^C,j\in\mathcal{N}}(x_{fij}E_{fi}) \; - \; K_f^P\right];\\
\text{ISO surplus} &= \sum_{i\in\mathcal{N}} w_i y_i.
\end{aligned} \tag{25}$$

By adding the surpluses from the four parties, we note that the following pairs of quantities can be canceled in an equilibrium: consumers' costs and LSEs' revenue from power purchase and sales, LSE's cost and producers' revenue from power purchase and sales, producers payments on transmission services and ISO's revenue of auctioning off transmission services, and CO_2 allowance purchases and sales. The CO_2 allowance purchases and sales can be canceled due to the marketing clearing condition (17) for the first-seller model, (condition (6) for the modified source-based model and condition (11) for the modified load-based model) and the assumption

$$\overline{E}^{SB} = \overline{E}^{LB} = \overline{E}^{FS} = \overline{E}.$$

As a result, we arrive at the following expressions for social welfare, denoted as SW.

$$SW^{SB} \equiv \sum_{j\in\mathcal{N}}\left[P_j^0\left(\sum_{f\in\mathcal{F}} s_{fj}\right) - \frac{P_j^0}{2Q_j^0}\left(\sum_{f\in\mathcal{F}} s_{fj}\right)^2\right] - \sum_{f\in\mathcal{F},i\in\mathcal{N}} C_{fi}(\sum_{j\in\mathcal{N}} x_{fij}),$$
$$SW^{LB,\;\; FS} \equiv \sum_{j\in\mathcal{N}}\left[P_j^0\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right) - \frac{P_j^0}{2Q_j^0}\left(\sum_{f\in\mathcal{F},i\in\mathcal{N}} z_{fij}\right)^2\right] - \sum_{f\in\mathcal{F},i\in\mathcal{N}} C_{fi}(\sum_{j\in\mathcal{N}} x_{fij}).$$

By the uniqueness of the total sales at a zone j and total generation at a zone i in an equilibrium to each of the three models, the uniqueness of social welfare follows. The equivalence of social welfare across the equilibria to the three models also readily follow by the results shown in (a) and (b). □

Appendix B: A Degenerate Example

In the following we present an example of a degenerate network in which ISO's surplus, producers' surplus and the market-clearing price of CO_2 may not be unique. Consider a network consisting of two nodes and one transmission line that connects them, as illustrated in Figure 3. The line

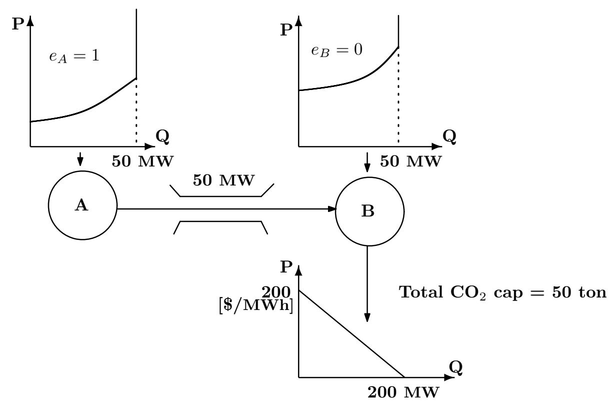

Figure 3 An example of a degenerate market equilibrium.

is subject to a 50 MW transfer capacity limit. The load is at Zone B, with an inverse demand function of $200 - D$, with D MW of electricity consumption. Two electricity power plants serve the load, sitting at Zones A and B, respectively. Assume that the cost functions for plants A and B are as follows

$$C_A(x) = 10x + 0.2x^2; \; C_B(x) = 30x + 0.1x^2. \tag{26}$$

Suppose each plant has a 50 MW capacity constraint. Then the marginal cost of producer A is strictly less than that of producer B, given a generation level $x \in [0, 50 \text{ MW}]$. Further assume that the emissions rate for plant A is 1 ton/MWh, while 0 for plant B. Hence, plant B is more expensive, but cleaner, than plant A.

For illustration purposes, we only consider the market equilibrium under the source-based approach. Equilibrium conditions for the other two CO_2 cap-and-trade schemes can be derived in the same manner. Let K_A and K_B denote the initial CO_2 allocation for plants A and B, and assume the total cap of CO_2 emissions is 50 tons. Then their profit optimization problems in the market setting depicted in Figure 3 are as follows.

$$\begin{aligned} \underset{s_{AB},\, x_A}{\text{maximize}} \;& p_B s_{AB} - (10x_A + 0.2x_A^2) + w_A x_A - p^{CO_2}(x_A - K_A) \\ \text{subject to } & x_A = s_{AB}, \; (\theta_{AB}), \\ & x_A \le 50, \; (\rho_A), \\ & x_A, \; s_{AB} \ge 0. \end{aligned} \qquad \textit{Plant A}$$

$$\begin{aligned}\underset{s_{BB},\ x_B}{\text{maximize}}\ & p_B s_{BB} - (30x_B + 0.1x_B^2) - p^{CO_2}(x_B - K_B)\\ \text{subject to}\ & x_B = s_{BB},\ (\theta_{BB}),\\ & x_B \le 50,\ (\rho_B),\\ & x_B,\ s_{BB} \ge 0.\end{aligned} \qquad \textit{Plant B}$$

The ISO's profit maximization problem is

$$\begin{aligned}\underset{y_A,\ y_B}{\text{maximize}}\ & w_A y_A + w_B y_B\\ \text{subject to}\ & y_A \le 50,\ (\lambda^+)\\ & -y_A \le 50,\ (\lambda^-).\end{aligned} \qquad \textit{ISO}$$

Consumers' willingness-to-pay at zone B is

$$p_B = 200 - (s_{AB} + s_{BB}).$$

The market-clearing conditions are

$$\begin{aligned} y_A &= -x_A\\ y_B &= s_{AB} + s_{BB} - x_B\\ 0 \le p^{CO_2} &\perp 50 - x_A \ge 0.\end{aligned}$$

$(s_{AB},\ g_A,\ s_{BB},\ g_B,\ y_A,\ y_B,\ w_B,\ p_B)$ are unique in an equilibrium of the above system, with values at $(50,\ 50,\ 50,\ 50,\ -50,\ 50,\ 0,\ 100)$. $(w_A,\ p^{CO_2})$, however, are not unique in equilibria. This is so because the generation capacity constraint for plant A, the transmission constraint, and the emissions cap constraint all create the same constraint $x_A \le 50$. This degeneracy leads to non-unique wheeling charge at zone CA, w_A, which ranges continuously within $[0, \$70]$; and CO_2 price ranges within $[0, 60\$/\text{ton}]$ in equilibria. This example shows that unless stronger regularity conditions are imposed to the equilibrium system, uniqueness for ISO's profit, power producers' surplus, and CO_2 prices does not hold in general.

Appendix C: Generator Characteristics and Transmission Line Data

Table 5 Generating Data

Plant	Firm	Location	c [\$/(MWh)]	d [\$/(MWh^2)]	CO_2 Rate[kg/MWh]	Capacity[MW]
1	3	A	38.00	0.02	580	250
2	1	A	35.72	0.03	545	200
3	2	A	36.80	0.04	600	450
4	1	B	15.52	0.01	500	150
5	2	B	16.20	0.02	500	200
6	3	B	0.00	0.001	0	200
7	1	C	17.60	0.02	1216	400
8	1	C	16.64	0.01	1249	400
9	1	C	19.40	0.01	1171	450
10	3	C	18.60	0.02	924	200

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Table 6 PTDF Values

Link\Zone	A	B	C
1	-0.3333	0.5	0
2	-0.6667	-0.5	0
3	-0.3333	-0.5	0

Table 7 Transmission Line Capacities

Link	Capacity [MW]
1	255
2	120
3	30

References

[1] L. Adler, S. Oren, and J. Yao. Modeling and computing two-settlement oligopolistic equilibrium in a congested electricity network. *Operations Research*, 56(1):34–47, 2008.

[2] J. Bushnell. A mixed complementarity model of hydro-thermal electricity competition in the western U.S. *Operations Research*, 51(1):81–93, 2003.

[3] J. Bushnell. The implementation of California AB32 and its impact on wholesale electricity markets. Technical Report WP-170, University of California Energy Institute, 2007.

[4] California Energy Commission. 2006 net system power report. Technical Report CEC-300-2007-007, California Energy Commission, 2006.

[5] California Energy Commission. Inventory of California greenhouse gas emissions and sinks: 1990 to 2004. Technical Report CEC-600-2006-013-SF, California Energy Commission, 2006.

[6] California Energy Commission. Low-carbon fuel standard, 12 2007. `http://www.energy.ca.gov/low_carbon_fuel_standard/index.html`.

[7] California Energy Commission. Revised methodology to estimate the generation resources mix of California electricity imports. Technical Report CEC-700-2007-007, California Energy Commission, 2007.

[8] California Public Utility Commission. Interim opioion on greenhouse gas regulatory strategies, 2008. `http://docs.cpuc.ca.gov/efile/PD/78643.pdf`.

[9] Market Advisory Committee. Market Advisory Committee to the California Air Resources Board: Recommendation for desgining a greenhouse gas cap-and-trade system for California, June 2007.

[10] T. J. Considine. Economies of scale and asset values in power production. *The Electricity Journal*, 12(10):37–42, 1999.

[11] R. W. Cottle, J.-S. Pang, and R. E. Stone. *The Linear Complementarity Problem*. Academic Press, Boston, 1992.

[12] Synapse Energy Economics. Exploration of costs for load side and supply side carbon caps for California. Joint En Banc Hearing of PUC and CEC on Point of Regualtion in the Electricity Sector, 2007. `http://www.synapse-energy.com`.

[13] Southern California Edison. Presentation, CAISO Market Surveillance Committee Meeting, California Air Resources Board, Sacramento, CA. June 8, 2007. `http://www.caiso.com/1bf7/1bf76c5424a10.pdf`.

[14] M. Gillenwater and C. Breidenich. Internalizing carbon costs in electricity markets: Using certificates in a load-based emissions trading scheme. Technical report, Science, Technology and Environmental Policy Program, Woodrow Wilson School of Public and International Affairs, Princeton University, Princeton, NJ, 2007.

[15] B. F. Hobbs. Linear complementarity models of Nash-Cournot competition in bilateral and poolco power markets. *IEEE Transactions on Power System*, 16(2):194–202, 2001.

[16] B. F. Hobbs. Theoretical equivalence of CO_2RC and source-based CO_2 trading. Mimeo, Department of Geography & Environmental Engineering, The Johns Hopkins University, 2007.

[17] A. Van Horn and E. Remedios. A comparison of three cap-and-trade market designs and incentives for new technologies to reduce greenhouse gases. *The Electricity Journal*, 12(2):51–62, 2008.

[18] R. A. Horn and C. R. Johnson. *Topics in Matrix Analysis.* Cambridge University Press, 1991.

[19] C. Metzler, B. Hobbs, and J.-S. Pang. Nash-Cournot equilibria in power markets on a linearized DC network with arbitrage: Formulations and properties. *Networks & Spatial Economics*, 3(2):123 – 150, 2003.

[20] C. B. Metzler. *Complementarity Models of Competitive Oligopolistic Electric Power Generation Markets.* PhD thesis, Johns Hopkins University, 2000.

[21] S. Michel and J. Nielsen. An alternative load-based CO_2 cap & trade for the West. Western Resource Advocate, October 2007.

[22] M. Ruth, K.M. Conklin, K. Palmer, D. Burtraw, A. Paul, Y. Chen, B.F. Hobbs, D. Irani, J. Michael, K. M. Ross, R. Conklin, and J. Miller. Economic and energy impacts from participation in the regional greenhouse gas initiative: A case study of the state of Maryland. *Energy Policy*, 36(6):2279–2289, 2008.

[23] J. Sijm, K. Neuhoff, and Y. Chen. CO_2 costs pass through and windfall profits in power sector. *Climate Policy*, 6(1):119–129, 2006.

[24] J.-Y. Wei. and Y. Smeers. Spatial oligopolistic electricity models with Cournot generators and regulated transmission price. *Operations Research*, 47(1):102–112, 1999.

[25] F. Wolak, J. Bushnell, and B. F. Hobbs. Opinion on load-based and source-based trading of Carbon Dioxide in California. Technical report, Market Surveillance Committee of the California ISO, November 2007.