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Leader-follower equilibria for electric power and NO_x allowances markets

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Abstract This paper investigates the ability of the largest producer in an electricity market to manipulate both the electricity and emission allowances markets to its advantage. A Stackelberg game to analyze this situation is constructed in which the largest firm plays the role of the leader, while the medium-sized firms are treated as Cournot followers with price-taking fringes that behave competitively in both markets. Since there is no explicit representation of the best-reply function for each follower, this Stackelberg game is formulated as a large-scale mathematical program with equilibrium constraints. The best-reply functions are implicitly represented by a set of nonlinear complementarity conditions. Analysis of the computed solution for the Pennsylvania–New

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Jersey–Maryland electricity market shows that the leader can gain substantial profits by withholding allowances and driving up NO_x allowance costs for rival producers. The allowances price is higher than the corresponding price in the Nash–Cournot case, although the electricity prices are essentially the same.

Keywords Mathematical programs with equilibrium constraints (MPEC) · Game theory (Stackelberg game) · Economic market modelling · Optimization algorithm · Electric power
MSC Classification: 91B26 Market Model

1 Introduction

Market power is defined as the ability of players in a market – producers and consumers, for example – to unilaterally or collectively maintain prices above the competitive level. The exercise of market power can result in price distortions, production inefficiencies, and a redistribution of income among consumers and producers. The electricity market is especially vulnerable to the exercise of market power by the producers for three reasons. First, short-term demands for electricity are very inelastic, largely because consumers are shielded from fluctuations in real-time prices. Second, network limitations lead to market separation if transmission lines are congested. Third, supply curves steepen when output is near capacity, implying that the marginal cost increases drastically in segments where the electricity price is determined during peak periods.

Pollution control regulation can significantly increase production costs in electricity markets. The NO_x allowances program in the eastern United States, for example, is a cap-and-trade program administered by the US. Environmental Protection Agency (USEPA). The amount of NO_x released into the atmosphere under this program is controlled by distributing allowances to the producers that must be redeemed to cover actual emissions. These allowances can be traded in a secondary market or banked for future use. The theoretical efficiency of cap-and-trade programs is well documented in the economics literature. Under certain assumptions, the absence of market power, for example, the programs achieve predetermined emission reductions at least cost (Newell and Stavins 2003; Stavins 1995; Tietenberg 2003). However, market power can interfere with the promised efficiency, yielding higher costs for both emission control and commodity production. An example of such market power would be the ability of producers to use allowances as a vehicle to affect the costs of rivals. The consequences of exercising market power can be complicated because of the interaction between the electricity and allowances markets. Empirical analysis of the 2000–2001 California power crisis, for example, suggests that in addition to demand growth, a shortage of hydropower, and excessive reliance on spot markets, some price increases were caused by a large producer that intentionally consumed more allowances than necessary, raising the costs for rival producers that were short of allowances (Kolstad and Wolak 2003).

Sartzetakis (1997) investigated the incentive for a producer to raise the costs of its rivals by withholding allowances in a simple market model. The conclusion reached was that competition in the commodity market can be weakened. In a more recent analysis of a large-scale market with thousands of variables, Chen and Hobbs (2005) used a heuristic solution algorithm to explore the profitability of a dominant producer that expands generation, overconsumes allowances, and suppresses the output of the other producers, where the producers were assumed to follow a Cournot strategy in the energy market. The analysis failed to identify the optimal joint emissions and electricity strategy for the dominant producer, possibly underestimating the magnitude of its market power. In this paper, we formulate a Stackelberg game to investigate the consequences of exercising market power in an electricity market with a secondary emissions market.

The Stackelberg game was first proposed in 1934, and the formulation is especially appropriate for studying a game with a sequential move or a leader-follower relationship. Examples can be found in Fudenberg and Tirole (1991); Gibbson (1992); Tirole (1998). The standard backward induction procedure to solve such games initially fixes the decisions made by the leader in the first stage and then derives the best response of each follower. The optimal decisions for the leader are then found by solving an optimization problem with constraints for the derived response of the followers. For applications with capacity constraints, the optimality conditions for the followers must be written as a system of complementarity conditions, leading to a mathematical program with equilibrium constraints (MPEC).

A number of practical Stackelberg problems, including discrete transit planning and facility location and production, have been modeled as MPECs (Luo et al. 1997). In particular, examples of MPEC formulations of such games in energy markets include Entriken and Wan (2003a,b), Hobbs et al. (2000), Hu et al. (2004), Pepermans and Willems (2005), Shanbhag et al. (2004), Yao et al. (2005), and Wolf and Smeers (1997). Since the feasible region is nonconvex, a guaranteed global solution cannot be found by standard algorithms even if the objective function is strongly convex. Moreover, solving MPECs is difficult because *any* smooth reformulation of the complementarity constraints violates the Mangasarian-Fromovitz constraint qualification, a key ingredient for stability. Nevertheless, recent developments indicate that the sequential quadratic programming approach can compute local stationary points to MPECs when using a smooth reformulation of the complementarity constraints with only mild assumptions (Fletcher et al. 2002; Fletcher and Leyffer 2004; Leyffer 2003 a,b). These developments suggest that an MPEC can be a numerically tractable tool to solve large-scale Stackelberg games. This approach is taken here.

Specifically, we construct a Stackelberg game for the Pennsylvania–New Jersey–Maryland Interconnection (PJM) electricity market. This model differs from other oligopolistic models in the following ways. First, interaction between the emissions and electricity markets is explicitly represented in the model. In particular, the allowances price is endogenously determined, as opposed to being an exogenous quantity as in other models. Second, the model is developed from the bottom up and is based on detailed engineering data for a power system with 14 nodes, 18 arcs, and 5 periods. The data incorporated in the model includes heat rates, emission rates, fuel costs, location, and ownership for each generator. This approach allows for a more realistic estimation of the market power associated with the location of a generator in the network. Moreover, the power flow in the network is represented by a linearized direct-current (DC) load flow model in which the Kirchhoff current and voltage laws account for quadratic transmission losses. Although some small alternating-current oligopolistic models have been formulated with transmission losses, quadratic transmission losses have not been previously considered in large-scale oligopolistic models.

The remainder of this paper is organized as follows. Section 2 provides a brief background regarding the PJM power market and the USEPA NO_x budget program. Section 3 presents the mathematical formulation of the Stackelberg game as an MPEC. Section 4 describes a two-phase strategy used to solve the resulting large-scale MPEC. Section 5 analyzes the solutions found for the model. Section 6 summarizes our work and briefly discusses future research.

2 Background

The PJM began operating as an independent system operator (ISO) in 1998. It runs day-ahead, hourly-ahead, and spot energy markets with an hourly load that ranged from 20,000 to 49,000 MW in 2000. Nuclear and coal plants served this base load, accounting for 57.9% of the total generation capacity. The capacity shares of oil, gas, and hydro plants were 20.8, 18, and 3.3%, respectively. Six large generating companies each own between 6 and 19% of the generating capacity.

The market is moderately concentrated, with an average hourly Hirschman–Herfindahl Index (HHI) of 0.154 (PJM Market Monitoring Unit 2001). The HHI is the sum of the squared market shares. A market with an HHI over 0.18 is considered concentrated by US. antitrust authorities (Viscusi et al. 1995). Although the PJM market monitor reports that prices have generally been near competitive levels, some market power has apparently been experienced in the installed capacity market. Furthermore, other studies indicate that the market concentration is high enough to present a risk of market power being exercised (Hobbs et al. 2000; Mansur 2001).

The PJM transmission network used in the model is spatially represented by 14 nodes, each representing one power control area or portion thereof, and 18 transmission lines. The network topology is shown in Figure 1. The highest average load among the nodes is 5,300 MW for Public Service Electric and Gas Company (PSEG), and the lowest is 1,310 MW for Atlantic Electric Company (AE). Net imports from other regions averaged 800 MW during the ozone season of 2000. For simplicity, imports are fixed in the model. Power transmission among the nodes in the network is represented by a DC load flow with quadratic transmission losses.

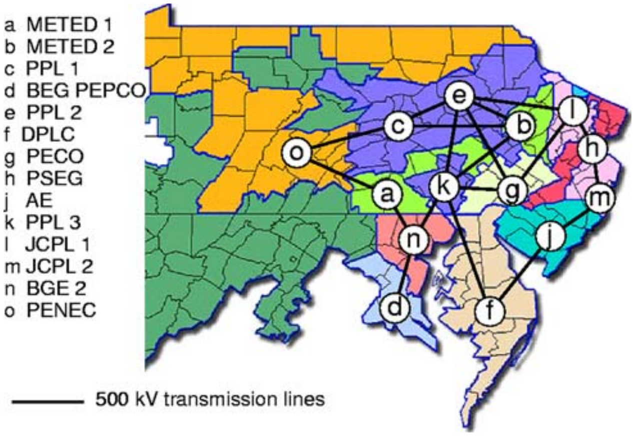

Fig. 1 Network topology of model

The Ozone Transport Commission (OTC) NO_x budget program introduced in 1999 is in effect from May 1 to September 30 of every year. The goal of this program is to reduce summer NO_x emissions throughout the region in order to help the northeastern states attain the National Ambient Air Quality Standard for ground-level ozone. The program has evolved to encompass a larger geographic scope, from an initial 9 states to 19 states in 2004 (Farrel et al. 1999). The mandated NO_x reductions took effect in two phases. The first phase began May 1, 1999, when the program required affected facilities to cut total emission to 219,000 tons, less than half of the 1990 baseline emission of 490,000 tons. The emissions cap was tightened to 143,000 tons in 2003 for the second phase, a reduction of 70%.

The OTC NO_x program is a cap-and-trade program. Every electric generating unit with a rated capacity higher than 25 MW and large industrial process boilers and refineries are subject to this program. The tradable NO_x emission allowances are initially allocated to affected facility owners according to their historical seasonal heat inputs multiplied by a target NO_x emission rate. The participants in the program show compliance by redeeming enough allowances to cover their emissions. The allowance owners can sell excess allowances or bank them for future use. A total of 470 individual sources affiliated with 112 distinct organizations were in the program in 1999. Approximately 90% of NO_x emissions covered by the program are from power generators. More than 70% of generator summer capacity for the PJM market comes under the NO_x budget program, including 422 generators. Non-power sources of NO_x emission are not included in the model because of their small size and because the power industry is the focus of this paper.

The power generators are also subject to the national Clean Air Act SO_2 cap-and-trade program. Because the national market is so large, these costs are treated exogenously by including a SO_2 allowances price of \$140/ton in the production costs.

Even though the NO_x budget program covers a region larger than the PJM market, the use and sales of allowances are modelled only within the PJM market. Therefore, the results may overstate the extent to which market power can be exercised in the NO_x market because the model disregards trading outside the PJM market. Furthermore, concentration in the NO_x market may also be overstated. However, the results illustrate the potential interactions between electricity and allowances markets in the presence of market power.

3 Model statement

The PJM electricity market model builds on the transmission-constrained Cournot models of Hobbs (2001) and Chen and Hobbs (2005). These models are generalized to allow for Stackelberg leader-follower relationships. The sets and indices, parameters, and variables used in the model are given in Tables 1, 2, 3, respectively. Parameters are denoted by capital letters, and variables and multipliers are denoted by lower-case letters throughout. Complementarity is indicated by a $\perp$ sign between two quantities; $0 \leq x \perp y \geq 0$ means that $x \geq 0$, $y \geq 0$, and $x^{\mathrm{T}}y = 0$.

The leader in the Stackelberg game, usually the largest producer, maximizes its profit subject to capacity constraints and subject to the condition that the followers act optimally given the strategy chosen by the leader. In this way, the Stackelberg game can be viewed as a general bilevel optimization problem. If the lower-level optimization problems are convex and satisfy a constraint qualification, then the optimization problems can be replaced by their first-order optimality conditions. This substitution leads to an MPEC.

The remainder of this section is organized as follows. We first develop a collection of optimization problems and market-clearing conditions for a Nash–Cournot game for the electricity market with a fixed amount of NO_x emissions

Table 1 Sets and indices

$f, g \in F$	Generating firm
F^c(or F^p) $\subset F$	Set of Cournot (or pricing-taking) firms, $F^c \cap F^p = \varnothing$
$f^s \in F$	Stackelberg leader firm, $f^s \cap F^c = \varnothing, f^s \cap F^p = \varnothing$, and $f^s \cup F^c \cup F^p = F$
$h \in H$	Generating unit
$h \in H^{\mathrm{OTC}}$	Generating unit subject to NO_x cap
$H(i,f) \subset H$	Set of units at node i owned by firm f
$H^{OTC}(i,f) \subset H$	Set of units at node i owned by firm f subject to NO_x cap
$i,j \in I$	Nodes in the network
$j \in J(i)$	Node j adjacent to node i (connected via a single arc)
$k \in K$	Loop for Kirchhoff voltage law in linearized DC model
$t \in T$	Period
$(i,j) \in v(k)$	Set of arcs associated with loop k. These are ordered: for instance, if loop $k = 1$ connects nodes $3 \to 7 \to 4 \to 3$ in that order, then $v(k = 1) \triangleq \{(3,7); (7,4); (4,3)\}$.

allowances withheld. We then give the Stackelberg game as an MPEC and derive important theoretical properties of this MPEC.

3.1 The Nash–Cournot game

The Nash–Cournot game has four types of players: generating firms that decide the amount of power produced; an independent service operator that decides how the power is routed through the transmission network; an arbitrager that exploits price inconsistencies to make a profit; and markets that determine the power price, allowances price, and transmission charges. The power price is a function of the quantity consumed and is derived from the inverse demand

Table 2 Parameters

B_t	Block width of load duration curve in period t (h/year)
C_{fih}	Marginal cost for unit h of firm f at node i (\$/MWh)
E_{fih}	Emission rate for unit h of firm f at node i (tons/MWh)
L_{ij}	Resistant loss coefficient associated with arc *(i, j)*
N_f	Number of allowances initially owned by firm f (tons/year)
P_{it}^0	Vertical intercept of demand curve at node i in period t (\$/MWh)
Q_{it}^0	Horizontal intercept of demand curve at node i in period t (MW)
R_{ij}	Reactance associated with arc *(i, j)*
T_{ij}	Thermal limit of transmission arc *(i, j)* (MW)
X_{fih}	Derated production capacity of plant h of firm f at node i (MW)

Table 3 Variables

a_{it}	Power purchased(-), sold(+) by arbitrager at node i in period t (MW)
n_W	Number of allowances withheld and consumed or sold by leader (tons)
o_{it}	Power consumed by consumers at node i in period t (MW)
p_{it}^E	Power price at node i in period t (\$/MWh)
p_t^H	Power price at arbitrary hub (node PENEC) (\$/MWh)
p^N	NO_x allowances price (\$/ton)
q_{it}	ISO power purchase at node i in period t for resistant losses (MW)
s_{fit}	Power sold by firm f at node i in period t (MW)
t_{ijt}	Power flow from node i to node j in period t (MW)
w_{it}	Wheeling charges for delivering power from hub to node i (\$/MWh)
x_{fiht}	Power output by unit h of firm f at node i in period t (MW)
y_{it}	Power delivered from hub to node i in period t (MW)
ρ_{fiht}	Dual variable associated with capacity constraints for generators
θ_{ft}	Dual variable associated with energy sale/generation balance
γ_{it}	Dual variable associated with Kirchhoff current law
τ_{kt}	Dual variable associated with Kirchhoff voltage law
δ_t	Dual variable associated with net flow balance
λ_{kt}	Dual variable associated with upper limit on power flow

curve:

$$p_{it}^{E} = P_{it}^{0} - \frac{P_{it}^{0}}{Q_{it}^{0}} \quad o_{it} \quad \forall i, t.$$

A sales balance must be maintained at each node in each period, so the energy sold by the producers and arbitragers equals the energy purchased by the ISO and consumers:

$$\sum_{f} s_{fit} + a_{it} = q_{it} + o_{it} \quad \forall i, t. \tag{1}$$

Therefore, o_{it} can be eliminated to yield the inverse demand curve used in the model:

$$p_{it}^{E} = P_{it}^{0} - \frac{P_{it}^{0}}{Q_{it}^{0}} \left(\sum_{g} s_{git} - q_{it} + a_{it} \right) \quad \forall i, t. \tag{2}$$

We next describe the optimization problems solved by each player and the first-order optimality conditions. The variables in parentheses to the right of each constraint are the dual multipliers used when constructing the first-order conditions.

3.1.1 Power generators

Each generating firm maximizes its individual profit, revenue minus costs, by choosing sales s_{fit} and output levels x_{fiht} in each period subject to capacity and energy balance constraints. The generators are divided into two groups: first, Cournot players that can influence the power prices and, indirectly, the NO_x allowances prices and, second, the price-taking fringe players that view the power prices as exogenous quantities. The large producers in the model are designated as Cournot players, while the small producers are price-taking players.

Each Cournot generator $f \in F^c$ solves the following optimization problem:

$$\max_{s_{fit} \geq 0,\, x_{fiht} \geq 0} \left(\begin{array}{l} \sum_{i,t} B_t \left(P_{it}^{0} - \frac{P_{it}^{0}}{Q_{it}^{0}} \left(\sum_{g} s_{git} - q_{it} + a_{it} \right) - w_{it} \right) s_{fit} \\ - \sum_{i,h \in H(i,f),t} B_t \left(C_{fih} - w_{it} \right) x_{fiht} \\ \qquad - p^{N} \left(\sum_{i,h \in H^{OTC}(i,f),t} B_t E_{fih} x_{fiht} - N_f \right) \end{array} \right) \tag{3}$$

$$\begin{aligned} \text{subject to } \; & \sum_{i} B_t s_{fit} = \sum_{i,h \in H(i,f)} B_t x_{fiht} \quad \forall t \qquad (\theta_{ft}) \\ & x_{fiht} \leq X_{fih} \qquad \forall i, h \in H(i,f), t \; (\rho_{fiht}), \end{aligned}$$

where the allowances price p^N, the sales levels s_{-f} for the other generators, and the transmission charges w_{it} are treated as exogenous quantities by firm f.

The revenue per megawatt–hour for providing electricity to consumers at node i is

$$P_{it}^0 - \frac{P_{it}^0}{Q_{it}^0}\left(\sum_g s_{git} - q_{it} + a_{it}\right) - w_{it}. \tag{4}$$

This quantity includes the price the customers are willing to pay for the energy supplied minus the transmission charges paid to the ISO for sending the energy from the hub to the customers. A price-taking firm replaces (4) with $p_{it}^E - w_{it}$. That is, the prices p_{it}^E are exogenously determined by (2) for the price-taking fringe. The cost of producing electricity per megawatt-hour for unit i is $C_{fit} - w_{it}$, where $-w_{it}$ is the price charged by the ISO to send the power from the generator to the hub. The number of tradable allowances purchased (positive) or sold (negative) over the compliance period is

$$\sum_{i,h \in H^{\text{OTC}}(i,f),t} B_t E_{fih} x_{fiht} - N_f.$$

In addition to nonnegativity constraints, the total power generation and sales have to balance in each period, and the output level for each generator can be no more than the derated capacity.

Lemma 3.1. *The optimization problem* (3) *solved by each generator has the following properties:*

1. *If producer f is a price taker, then the optimization problem has a linear objective function in the decision variables and linear constraints.*
2. *If producer f is a Cournot player and P_{it}^0, Q_{it}^0 and B_t are positive, then the optimization problem has a concave quadratic objective function in the decision variables and linear constraints.*

Proof. Property 1 follows from the fact that (4) is replaced by exogenous $p_{it}^E - w_{it}$ for price takers.

Property 2 follows from writing the first term in the objective function as

$$\sum_{i,t}\left(P_{it}^0 s_{fit} - \frac{P_{it}^0}{Q_{it}^0}\left(s_{fit}^2 + \left(\sum_{g \neq f} s_{git} - q_{it} + a_{it}\right) s_{fit}\right) - w_{it}s_{fit}\right) B_t,$$

which is a concave quadratic function in s_{fit} for positive P_{it}^0, Q_{it}^0 and B_t. □

Lemma 3.1 implies that the first-order optimality conditions for (3) are necessary and sufficient. These conditions are simplified to produce the equilibrium constraints for each generator as follows. The first condition states that power is generated only if the marginal revenue equals marginal cost:

$$
\begin{aligned}
&0 \le s_{fit} \perp -p^E_{it} + \frac{P^0_{it}}{Q^0_{it}} s_{ijt} + w_{it} + \theta_{ft} \ge 0 \quad && \forall f \in F^c, i, t \\
&0 \le s_{fit} \perp -p^E_{it} + w_{it} + \theta_{ft} \ge 0 && \forall f \in F^p, i, t.
\end{aligned} \tag{5}
$$

The first-order conditions associated with x_{fiht} take the form

$$
\begin{aligned}
&0 \quad \le x_{fiht} \perp C_{fih} - w_{it} + p^N E_{fih} - \theta_{ft} + \rho_{fiht} \ge 0, \\
&0 \quad \le x_{fiht} \perp C_{fih} - w_{it} - \theta_{ft} + \frac{\rho_{fiht}}{B_t} \ge 0, \\
&\forall f \ne f^s, \forall i, h \in H^{\mathrm{OTC}}(i,f), \forall.
\end{aligned} \tag{6}
$$

The next constraint states that power generation and sales must balance. The constraint can be written equivalently as

$$
\sum_{i,h \in H(i,f)} x_{fiht} = \sum_i s_{fit} \quad \forall f \ne f^s, t \tag{7}
$$

because $B_t > 0$ for each t. The final constraint is that generation must not exceed capacity:

$$
0 \le \rho_{fiht} \perp -x_{fiht} + X_{fih} \ge 0 \; \forall f \ne f^s, \quad i, h \in H(i,f), t. \tag{8}
$$

3.1.2 Independent system operator

The independent system operator determines the flows in the network to maximize the value received by the users of the network. Because transmission losses represent a significant cost to the overall system, the ISO chooses services that maximize the value provided minus the cost to make up power for losses:

$$
\begin{aligned}
&\max_{y_{it},\, q_{it} \ge 0,\, t_{ijt} \ge 0} && \sum_{i,t} B_t \left(w_{it} y_{it} - p^E_{it} q_{it} \right) \\
&\text{subject to} && y_{it} + \sum_{j \in J(i)} \left(t_{ijt} - t_{jit} + L_{ji} t^2_{jit} \right) \le q_{it} && \forall i, t && (\gamma_{it}) \\
& && \sum_{(i,j) \in v(k)} R_{ij} (t_{ijt} - t_{jit}) = 0 && \forall k, (i,j) \in v(k), t && (\tau_{kt}) \\
& && t_{ijt} \le T_{ij} && \forall i, j \in J(i), t && (\lambda_{ijt}) \\
& && \sum_i y_{it} = 0 && \forall t && (\delta_t),
\end{aligned} \tag{9}
$$

where the transmission price w_{it} and the energy price p^E_{it} are exogenous quantities from the point of view of the ISO. The analogues to the Kirchhoff current and voltage laws are explicitly expressed in the first two constraints (see Schweppe et al (1988), Appendix A), as opposed to using power transfer and distribution factors in the no-loss case (Chen and Hobbs 2005). The third constraint accounts for capacities on the transmission lines. The final constraint states that the total amount of power delivered by the hub (y_{it} positive) equals the amount of power received by the hub (y_{it} negative).

Lemma 3.2. *For nonnegative L_{ij}, the optimization problem* (9) *has a linear objective function and convex constraints.*

Proof. The only nonlinear expression in the optimization problem is the term

$$\sum_{j \in J(i)} L_{ji} t_{jit}^2$$

in the Kirchhoff current law. Since this expression is convex for nonnegative L_{ji}, it follows that the constraints form a convex set. □

The first-order conditions for (9) are sufficient by Lemma 3.2. After simplification, these conditions for the power transferred from the hub to node i and the power purchased from node i are

$$\begin{aligned} &-B_t w_{it} + \gamma_{it} + \delta_t = 0 \qquad && \forall i, t \\ &0 \le q_{it} \perp B_t p_{it}^E - \gamma_{it} \ge 0 && \forall i, t. \end{aligned} \tag{10}$$

The conditions for the transmission variables state that if flow is positive, then the difference between the power prices at two connected nodes adjusted for losses, equals the sum of the relevant dual variables:

$$\begin{aligned} 0 \le t_{ijt} \perp \gamma_{it} + \left(2L_{ij}t_{ijt} - 1\right)\gamma_{jt} \\ + \sum_{k|(i,j)\in v(k)} R_{ij}\tau_{kt} - \sum_{k|(j,i)\in v(k)} R_{ji}\tau_{kt} + \lambda_{ijt} \ge 0 \quad \forall i, j \in J(i), t. \end{aligned} \tag{11}$$

The notation $k|(i,j) \in v(k)$ indicates the set of loops in which arc (i,j) is a member.

The next constraints are the linearized DC analogue to the Kirchhoff current and voltage laws:

$$\begin{aligned} &0 \le \gamma_{it} \perp q_{it} - y_{it} - \sum_{j \in J(i)} \left(t_{ijt} - t_{jit} + L_{ji}t_{jit}^2\right) \ge 0 \qquad && \forall i, t \\ &\textstyle\sum_{(i,j)\in v(k)} R_{ij}(t_{jit} - t_{ijt}) = 0 && \forall k, t. \end{aligned} \tag{12}$$

Capacity constraints are imposed with the condition

$$0 \le \lambda_{ijt} \perp -t_{ijt} + T_{ij} \ge 0 \quad \forall i, j \in J(i), t. \tag{13}$$

The final constraint is the conservation of the power received and delivered:

$$-\sum_i y_{it} = 0 \quad \forall t. \tag{14}$$

3.1.3 Arbitrager

The arbitrager exploits price differentials among different nodes to buy power from low-price nodes and sell it at high-price nodes to make a profit. This player is assumed to have perfect knowledge of the equilibrium power prices. An exogenous arbitrager formulation is adopted in which an aggregated price-taking agent represents the multiple arbitragers in the market (Metzler et al 2003). Therefore, the arbitrager solves the optimization problem

$$\begin{aligned} \max_{a_{it}} \quad & \sum_{i,t} B_t (p^E_{it} - w_{it}) a_{it} \\ \text{subject to} \quad & \sum_i B_t a_{it} = 0 \quad \forall t \quad (p^H_t), \end{aligned} \tag{15}$$

where the transmission price w_{it} and the energy price p^E_{it} are exogenous quantities.

Lemma 3.3. *The optimization problem* (15) *is a linear program.*

After simplification, the optimality conditions for this problem state that the difference in power price between node i and the hub is the wheeling charge of delivering power from the hub to node i:

$$-p^E_{it} + w_{it} + p^H_t = 0 \quad \forall i, t, \tag{16}$$

and the condition that the power bought equals the power sold:

$$-\sum_i a_{it} = 0 \quad \forall t. \tag{17}$$

3.1.4 Market-clearing conditions

The model includes two sets of market-clearing conditions. The first is a power balance condition at each node stating that the power delivered by the ISO to a node equals the physical consumption, including losses, minus the generation:

$$y_{it} = q_{it} + o_{it} - \sum_{f,i,h \in H(i,f)} x_{fiht} \quad \forall i, t.$$

Using (1), we can restate this equation as follows: the physical power delivered to a node by the ISO equals the sales by the firms and arbitragers minus generation,

$$y_{it} = \sum_f s_{fit} + a_{it} - \sum_{f,i,h \in H(i,f)} x_{fiht} \quad \forall i, t. \tag{18}$$

The second set is a complementarity condition for the NO_x allowances prices. If the demand for allowances equals the available supply, then the price can be positive; otherwise, the price is zero:

$$0 \leq p^N \perp \sum_f N_f - \sum_{f,i,h \in H^{\mathrm{OTC}}(i,f),t} B_t x_{fiht} E_{fih} - n_W \geq 0 \ \forall i,t, \tag{19}$$

where n_W is an exogenous quantity for the amount of allowances withheld from the emissions market by the Stackelberg leader.

3.1.5 Model degeneracy

The Nash–Cournot model obtained by combining all the optimality conditions, market clearing conditions, and energy price constraints is degenerate because (7), (17), and (18) imply that (14) is always satisfied at any feasible point. Hence, constraint (14) is dropped from the model, and one additional condition is added to set the power price at the hub node, PENEC:

$$p_t^H = p_{\mathrm{PENEC},t}^E \quad \forall t. \tag{20}$$

The Nash–Cournot game then consists of the conditions (2), (5)–(8), (10)–(13), and (16)–(20). This model has the same number of variables as equations and complementarity conditions.

3.2 The Stackelberg model

The Stackelberg leader maximizes profit, revenue minus costs, from its participation in the power and NO_x allowances markets by selecting an output level and the number of allowances to withhold given the responses of the followers:

$$\max_{s_{fit} \geq 0,\, x_{fiht} \geq 0,\, n_W \geq 0} \left(\begin{array}{l} \sum_{i,t} B_t \left(p_{it}^E - w_{it}\right) s_{fit} - \sum_{i,h \in H(i,f),t} B_t \left(C_{fih} - w_{it}\right) x_{fiht} \\ \quad - p^N \left(\sum_{i,h \in H^{\mathrm{OTC}}(i,f),t} B_t E_{fih} x_{fiht} - N_f + n_W \right) \end{array} \right) \tag{21}$$

$$\text{subject to} \quad \sum_{i,h \in H(i,f)} x_{fiht} = \sum_i s_{fit} \quad \forall t$$
$$x_{fiht} \leq X_{fih} \quad \forall i,h \in H(i,f),t,$$

along with the solution of the Nash-Cournot game for the rest of the market, including constraints for the price of energy (2); the responses of the generators (5)–(8), independent service operator (10)–(13), and arbitrager (16)–(17); the market clearing conditions (18)–(19); and the price constraint for the hub (20). This MPEC model is summarized in Table 4. From Lemmas 3.1–3.3, the following important result is obtained.

Theorem 3.4. *At any feasible point of the MPEC defined in Table 4, the response of each follower is a global optimum to its optimization problem.*

Table 4 Summary of stackelberg game MPEC model

Type	Description	Equation
Objective	Leader problem	(21)
Constraints	energy prices	(2)
	Follower generators	(5)–(8)
	Follower ISO	(10)–(13)
	Follower arbitrager	(16)–(17)
	Market clearing	(18)–(19)
	Hub prices	(20)

Proof. The proof follows from the convexity of the optimization problems solved by each of the followers. Therefore, the first-order optimality conditions are sufficient for each of the followers. □

The resulting large-scale MPEC was implemented in the AMPL modeling language (Fourer et al. 2003), which provides access to a variety of solvers and has facilities for exchanging information between solvers. The model has approximately 20,000 variables and 10,000 constraints, is highly nonlinear, and is relatively unstructured, with many different types of complementarity constraints. The complete AMPL model is available at `http://www.mcs.anl.gov/~tmunson/models/electric-mpec.zip`.

4 Solution methodology

The generic MPEC is to compute a solution to the optimization problem

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g(x) \leq 0 \\ & h(x) = 0 \\ & 0 \leq x_1 \;\perp\; x_2 \geq 0, \end{array} \tag{22}$$

where $x = (x_0, x_1, x_2)$ is a decomposition of the problem variables and slacks. This problem is reformulated as a nonlinear program by converting the complementarity condition into a nonlinear inequality. The reformulation leads to the optimization problem

$$\begin{array}{ll} \min\limits_{x} & f(x) \\ \text{subject to} & g(x) \leq 0 \\ & h(x) = 0 \\ & x_1^T x_2 \leq 0 \\ & x_1, x_2 \geq 0. \end{array} \tag{23}$$

The nonlinear program (23) violates the Mangasarian–Fromovitz constraint qualification at *any* feasible point for the optimization problem (Scheel and Scholtes 2000). The failure of this constraint qualification has important negative numerical implications: the multiplier set is unbounded, the active constraint normals are linearly dependent, and a linearization of (23) can become inconsistent *arbitrarily close* to a solution to the optimization problem (Fletcher et al. 2002).

Recent theoretical developments and numerical experience have shown that MPECs can be solved using standard nonlinear solvers. These developments build upon the seminal work on stationarity conditions for MPECs by Scheel and Scholtes (Scheel and Scholtes 2000), which we review next: Strong stationarity is equivalent to the existence of multipliers of (23), see Fletcher et al. (2002). A point x^* is called B-stationarity if it is a solution of the linearized MPEC obtained by linearizing the functions $f(x), g(x)$, and $h(x)$ in (22) about x^*. It can be shown that strong stationarity implies B-stationarity, but not vice-versa. Anitescu (2000) builds on (Scheel and Scholtes 2000) and shows that a sequential quadratic programming method with an ℓ_1 penalty formulation of the complementarity error $x_1^T x_2$ converges locally. Fletcher et al. (2002) prove that a sequential quadratic programming method converges quadratically near strongly stationary points. This quadratic rate of convergence is also observed in practice (Fletcher and Leyffer 2004).

Two sequential quadratic programming algorithms, SNOPT (Gill et al. 2002) and FILTER (Fletcher and Leyffer 2002), were applied to the reformulated nonlinear program for the PJM model. These solvers were unable to obtain a feasible solution and instead converged to a local minimum of the constraint violation. This negative result motivated a two-phase solution methodology.

The first phase solves a square nonlinear complementarity problem to compute a feasible point for the MPEC constraints. This complementarity problem is constructed from the Stackelberg game of section 3 by recasting the leader as a Cournot follower and fixing the NO_x withholding by setting $n_W = 0$. The nonlinear complementarity problem is solved by applying the PATH algorithm (Dirkse and Ferris 1995; Ferris and Munson 2000), a generalized Newton method that solves a linear complementarity problem to compute the direction.

The second phase supplies this feasible starting point to one of the nonlinear programming solvers, which computes an optimal solution to the original MPEC. The reformulation of the MPEC used does not lump all complementarity constraints together as in (23). Rather, groups of complementarity constraints corresponding to the different equations are combined. This approach improves the scaling of the model because unbounded multipliers affect fewer variables and constraints.

No single nonlinear programming solver could solve the Stackelberg game even from the feasible starting point provided by the feasibility phase. Instead, the solvers converged to infeasible points and to limit points where the algorithms could not make any progress because of numerical difficulties. These results illustrate the difficulty of the Stackelberg game for the PJM market. The PJM market model was eventually solved by applying the SNOPT and

FILTER algorithms in sequence. SNOPT was used to obtain a solution to the Stackelberg game starting from the initial feasible point provided by PATH for a fixed amount of withholding. The problem was then re-solved by applying the FILTER algorithm for variable withholding n_W.

5 Numerical results and economic analysis

In this section, we state the scenario assumptions, analyse the numerical performance and provide an interpretation of the economic implications of our results.

5.1 Scenario assumptions

Four scenarios – perfect competition, Nash–Cournot oligopolistic competition, and two Stackelberg scenarios – were constructed to quantify the impact of interactions between the energy and allowances markets. In the perfect competition case, all players in the market are assumed to behave competitively. In contrast, in the Nash–Cournot oligopolistic models (Chen and Hobbs 2005; Hobbs 2001), large producers with a capacity share between 6 and 19% are exercising Cournot strategies. The Stackelberg model represents a situation in which a leader exists in energy and emission markets and the remaining suppliers are either Cournot or price-taking followers. The four scenarios are illustrated in Table 5.

The leader in the Stackelberg game is selected based on market share. The underlying assumptions are that a large supplier has an advantage in retaining market-related information and taking early action and that the markup of each supplier, the amount by which a supplier increases its bids over its marginal cost, is monotonic in market share, with the largest firm having the greatest incentive to manipulate prices (Tirole 1998). Two different Stackelberg leaders were used for this study: PECO and PSEG. PECO is the generator with the longest position in the allowances market in the perfect competition case. Therefore, it has an incentive to drive up allowances prices by either overconsuming or with-

Table 5 Scenario assumptions

generators conjecture target	Perfect competition	Cournot competition	Stackelberg Competition		
			Leader	Cournot followers	Price-taking followers
Energy prices/sales by rivals	Bertrand	Cournot	Actual response	Cournot	Bertrand
Transmission prices	Bertrand	Bertrand	Actual response	Bertrand	Bertrand
Emission allowance prices	Bertrand	Bertrand	Actual response	Bertrand	Bertrand

holding allowances if allowed to do so. In contrast, the designation of PSEG as a leader serves as a reference case to determine whether a supplier in a relatively weak position in the NO_x allowances market is profitable enough to undertake a withholding strategy. The followers are the remaining generators, the arbitrager and the ISO. There are three smaller price-taking followers, namely, Conectiv, Allegheny and *Others*, a collected entity representing the set of all small generating firms. All other generators are intermediate in size and are treated as Cournot players in the energy market.

5.2 Algorithm performance

All experiments on the PJM model were performed on a Linux workstation with a 2.5 GHz Intel Pentium 4 processor with a 512 KB cache. The run times and iteration counts reported are intended only to illustrate the level of difficulty of this model. The Nash–Cournot feasibility problem was solved by PATH in 13.2 s. The calculation required a total of 25 major iterations involving the solution of a linear complementarity problem and 13 crash iterations involving the solution of a system of equations. Results for the nonlinear programming algorithms applied to the Stackelberg game during the optimization phase are displayed in Table 6 for the two Stackelberg scenarios considered.

The three solvers have a significant difference in cost per minor iteration, although all three are essentially pivoting algorithms. This difference can be explained by the fact that PATH factors or updates only a single *sparse* matrix per minor iteration, while SNOPT and FILTER, in addition, update a *dense* factorization of the reduced Hessian matrix. Moreover, FILTER uses a less efficient linear algebra package than does SNOPT, explaining the order of magnitude performance difference.

The maximum multiplier value in the two Stackelberg scenarios is 3.2×10^9 and 2.9×10^9, respectively. These large values indicate that the computed solution is probably *not* strongly stationary because the multipliers do not appear to be bounded. The solutions are likely B-stationary, but to test this conjecture is not practical given the size of the problem. Note that the objective value increases from the first nonlinear programming solve with fixed withholding to the second solve with variable withholding (Table 6).

Table 6 Statistics for the Nonlinear Solvers

Solver		PECO Leader	PSEG Leader
SNOPT	CPU time	288 s	293 s
	major/minor iter.	100/17996	153/9248
	final objective	9.5325×10^8	5.7812×10^8
FILTER	CPU time	831 s	1364 s
	major/minor iter.	12/5763	43/11585
	final objective	9.5327×10^8	5.7888×10^8

5.3 Economic analyses

Table 7 summarizes the comparative statics of the four different scenarios. Tables 8–11 summarize the results of the four scenarios, including the overall market equilibrium and the profile for each individual producer. Negative values in the "Allowance Traded" column refer to allowances sold.

In the following subsections, we contrast the Stackelberg solution with PECO as the leader with the perfect and Nash–Cournot competition solutions. This discussion initially concentrates on the market equilibrium and welfare analysis, equilibrium prices, consumer and producer surplus, and the NO_x trading volume. We then discuss the response of the followers to the strategy chosen by the leader. We then compare the two Stackelberg scenarios.

5.3.1 Stackelberg (PECO) versus perfect and Nash–Cournot competition

The consumer surplus in the Stackelberg solution is only marginally different from that of the Cournot case (Table 7). However, the consumer surplus exhibits a 10.2% decline, from \$9,521M to \$8,549M, when compared to the perfect competition scenario (Tables 7, 8). The optimal strategy is for PECO to with-

Table 7 Summary of comparative statics

	Competition:		Leader:	
	Perfect	Nash–Cournot	PECO	PSEG
Average power price (\$/MWh)	31.3	39.8	39.6	39.6
Price of allowances (\$/ton)	1,197	0	1,173	663.9
Allowances withheld (tons)	N/A	N/A	5,536	0
Importer revenue (\$M)	99	130	128	129
ISO revenue (\$M)	72	37	60	42
Transmission loss (10^6 MWh)	0.46	0.42	0.41	0.40
Consumer surplus (\$M)	9,521	8,535	8,549	8,552
Social welfare (\$M)	12,133	11,990	11,980	11,955

Table 8 Perfect competition: detailed results

Supplier	Profit (\$M)	Allowance traded (tons)	Total sales (10^6 MWh)	Var. gen. cost (\$M)
Conectiv	34.0	−1,436	2.0	36.7
Constellation	310.0	1,294	16.4	179.0
Mirant	133.7	0	10.7	202.1
PECO	752.9	−9,357	29.1	101.6
PPL	374.4	11,320	17.6	134.2
PSEG	451.9	3,320	18.4	126.2
Reliant	98.4	2,230	6.2	90.5
Allegheny	23.7	138	1.1	7.7
Others	262.1	−7,509	17.7	326.2
Total	2441.0	0	119.2	1,204.0

Table 9 Cournot competition: dtailed results

Supplier	Profit ($M)	Allowance traded (tons)	Total sales (10^6 MWh)	Var. gen. cost ($M)
Conectiv	60.9	2,144	3.9	101.1
Constellation	418.8	1,451	12.9	100.6
Mirant	214.3	0	9.2	142.7
PECO	893.7	−15,108	24.6	41.2
PPL	503.4	7,511	15.5	103.6
PSEG	552.8	−1,402	15.9	69.9
Reliant	170.0	5,379	7.7	113.7
Allegheny	33.6	177	1.1	8.0
Others	440.3	−1,267	21.4	432.3
Total	3,287.6	−1,115	112.0	1,113.2

Table 10 Stackelberg Model with PECO Leader: Detailed Results

Supplier	Profit ($M)	Allowance traded (tons)	Total sales (10^6 MWh)	Var. gen. cost ($M)
Conectiv	56.9	1,686	3.6	88.6
Constellation	403.8	−296	12.3	98.6
Mirant	210.3	0.0	9.0	139.3
PECO	969.5	−4,527	28.7	94.0
PPL	457.9	5,747	14.5	101.7
PSEG	540.7	−3,457	15.1	58.2
Reliant	144.8	3,657	7.0	112.8
Allegheny	31.7	177	1.1	8.5
Others	427.3	−2,987	20.9	425.0
Total	3,242.9	0	112.2	1,126.8

hold 5,536 tons of NO_x allowances, 7% of the total allowances available in the market. By doing so, PECO is able to drive up the NO_x allowances price to $1,173/ton, almost as high as the perfect competition solution. The power prices are maintained at the Cournot levels. Furthermore, the efficiency of the NO_x program in the Stackelberg case deteriorates as measured by the total NO_x trading volume: a drop of 38% compared with the perfect competition case. Since the leader creates more congestion than in the Cournot competition case, the ISO collects an additional $23M in revenue, even though the total power sold is the same as in the Cournot case. The social welfare is slightly lower than the Cournot level. One of the unique features of this model is the inclusion of a quadratic transmission loss. The solutions show that the transmission loss amounts to 0.4–0.5 $\times 10^6$ MWh in all cases, about 0.4% of generation.

We also ran a scenario in which the leader PECO did not have the ability to withhold NO_x permits. In this scenario, PECO's profit drops from 969 M$ to 932 M$. This indicates to what extend the Stackelberg leader PECO can benefit by withholding permits.

Table 11 Stackelberg model with PSEG leader: detailed results

Supplier	Profit ($M)	Allowance Traded (tons)	Total Sales (10^6 MWh)	Var. Gen. Cost ($M)
Conectiv	58.0	1,686	3.6	88.6
Constellation	407.5	450	12.5	100.7
Mirant	212.4	0.0	9.1	141.5
PECO	888.1	−15,390	24.2	38.1
PPL	473.5	6,160	14.8	105.5
PSEG	578.9	4,163	18.7	132.4
Reliant	150.6	4,324	7.2	115.1
Allegheny	32.5	177	1.1	8.5
Others	430.1	−1,570	21.0	422.6
Total	3,231.6	0	112.2	1,153.2

Unlike the counterintuitive response of some Cournot producers in the pure Cournot competition case, where they take advantage of the zero allowances price and expand their output (Chen and Hobbs 2005), all other producers contract their output by a total of 6.6×10^6 MWh compared to the perfect competition case. The reason is that, on average, the action taken by the leader raises their production cost by $2.60 per MWh, assuming the average emission rate is 2.0 kg/MWh. The restriction of output by the following producers, in turn, creates an upward pressure on power prices. PECO recognizes this opportunity and expands its sale by 16.7% (4.1×10^6 MWh), increasing its market share from 22% in the Cournot to 26% in the Stackelberg scenario. In comparison with the pure Cournot solution, this strategy leads to an additional profit of $75.8M for PECO, at the cost of other producers, whose profits fall by $120.7M. Therefore, in contrast to a pure Cournot model, the dominant role of the leader in a Stackelberg model allows one producer to extract more rent from the market at the expense of other producers. However, consumers benefit only very slightly, unlike the classic Stackelberg model without an allowances market, in which commodity prices are generally significantly lower than in the Cournot market (Gibbson 1992; Tirole 1998). The interactions of energy, allowances, and transmission mean that other producers who are long in allowances do not necessarily benefit from a higher NO_x price. For example, the *Others* price-taking producer sells 1,720 tons more NO_x allowances in the Stackelberg case than in the Cournot solution, thereby earning an extra $2.0M from the NO_x allowances market. However, the loss associated with the contraction of generation and the higher charges for transmission service offset the additional profits of selling allowances, resulting in a net decrease of $13M in its profit.

5.3.2 Comparison of stackelberg Scenarios

By comparing the two Stackelberg scenarios in Table 10 (PECO) and Table 11 (PSEG), we can explore the relationship between market power potential and the net positions in the power and NO_x allowances markets. In the perfect competition scenario, PSEG is the second largest producer in the power market and has a short position in the NO_x allowances market. In the Stackelberg scenario, the solutions show that, as a leader, its optimal strategy is not to withhold any NO_x allowances at all, unlike PECO, but to acquire more allowances while expanding its power output. This strategy benefits it by $127.0M, $26.1M, and $38.2M relative to the perfect, Cournot, and Stackelberg (PECO) competition solutions, respectively. PECO finds it optimal to sell 2.4 times more allowances at the NO_x allowances price level of $663.9/ton, with an additional gain of $4.9M from the NO_x allowances market. To sell NO_x allowances, however, it produces less output in the power market, since extra allowances are required to cover the emissions. Consequently, the output for PECO shrinks by 4.5×10^6 MWh, and its profit drops by $81.4M.

In summary, as long as there is market power in the markets, the overall social welfare is less than its counterpart in the perfect competition scenario. Because the difference in power prices is only marginal between the Cournot and Stackelberg cases, the overall impact on consumers is essentially the same. Thus, the effect of a firm taking a leadership role is to reshuffle the producer surplus among the producers: that is, the leader gains at the expense of the other producers. The comparison of two Stackelberg scenarios shows that the appeal of withholding NO_x allowances depends on the market share in the power market of the leader and its net position in the NO_x allowances markets.

6 Conclusions and future work

The solutions to the Stackelberg game for the PJM electricity market show that the leader can gain substantial profit through the exercise of market power at the expense of other producers. Whether the withholding allowances strategy is profitable depends on, among other factors, the net position of the leader in the NO_x allowances market. According to this model, PECO may be in a position to profit from withholding allowances; however, it is not optimal for PSEG to undertake such practices. This computational experience is promising for policy modelers interested in investigating the complicated interactions among imperfectly competitive markets.

The model in this paper of the PJM electricity market is subject to three sets of simplifying assumptions that possibly overestimate the potential of market power. First, the model assumes there is no vertical integration in the power market and all energy transactions take place in the spot market. The PJM power market was actually highly integrated or forward contracted during 2000. According to Mansur (2003), only 10–15% of power supply is from the spot market; 30% is from short- or long-term bilateral contracts, 53–59% is self-supplied, and the remaining 1–2% is imported. However, firm-level information about forward contract data is generally proprietary and not publicly available. Clearly, whether a supplier has an incentive to exercise market power depends on its net position in the market. If it possesses significant excess capacity, the incentive can be substantial. The current model can be expanded to represent this situation by explicitly introducing two additional fixed parameters (Green 1999): forward contracts (s^F_{fit}), where positive (negative) value of s^F_{fit} implies sales (purchases) of contracts, and forward contract prices (p^F_{fit}).

The second assumption that may overestimate the market power potential in PJM is the fixing of imported power. If the supply of imported power is price responsive, the quantity of imported power can increase in the face of higher power prices, dampening market power. This can be represented using price responsive demand curves for exports and/or supply curves for imports (Bushnell et al. 2005). However, the best way in theory to handle this issue is to expand the geographic scope of the model to include nearby markets. For instance, given that substantial energy trading occurs between New York ISO (NYISO) and PJM, including NYISO in the model will more correctly represent price-responsiveness of energy imports from or exports to NYISO.

The third assumption is that, in effect, no allowances are imported or exported from outside PJM. Since the OTC NO_x market is somewhat larger than PJM, this assumption may overstate the amount of market power in the NO_x market. Similar to the power import/export issue, demand curves for exports/supply curves for imports of allowances could be defined or, better yet, the geographical scope of the market could be expanded. The residual demand for permits could also account for opportunities to bank permits, a possibility that a multiyear model would more realistically represent. Such formulations could be used to explore the potential for allowance banking to enhance or dampen market power.

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