Improved Transmission Representations in Oligopolistic Market Models: Quadratic Losses, Phase Shifters, and DC Lines

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***Abstract*—Practical models of competition among power generators who possess market power have generally had to use simplified models of transmission costs and constraints in order to be tractable. In particular, the linearized dc load flow model has been popular in complementarity and other types of oligopoly models. In this paper, we show how such models can be generalized to include quadratic losses, controllable DC lines, and phase shifting transformers. These generalizations preserve convexity of the feasible region, a property that facilitates computation and proof of solution uniqueness and existence. Piecewise and successive linearization formulations are also provided that allow consideration of nonlinear losses in models that require linear constraints. A simple six-bus example illustrates the application of these generalizations. In that example, the impact of losses on prices is much greater under strategic behavior than under competition. Large-scale applications of these approaches to markets in western North America and the European Union illustrate how inclusion of nonlinear losses and controllable DC lines can affect estimates of prices, flows, and economic efficiency indices resulting from oligopoly models.**

***Index Terms*—Economics, optimization methods, power generation, power transmission economics.**

I. Introduction

NUMEROUS models based on game-theoretic concepts have been proposed for simulating power markets in which strategic generators compete and exercise market power [14], [31]. Such market power can be magnified when transmission costs and constraints isolate submarkets and thereby make it more difficult for distant generation to compete effectively in local markets. Therefore, it is important that these costs and constraints be represented realistically.

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Ideally, ac load flow models would be used in such models, as they are increasingly the basis for determining prices that generators receive if they sell to the market operator (in pool-type markets), and the costs they must pay to deliver their power to consumers (in markets that allow bilateral transactions). An example is the proposed California ISO Market Redesign and Technology Upgrade (MRTU), scheduled for implementation in 2008. There, for instance, ac load flow models will be used in an iterative fashion to generate constraints for use in the linearized day-ahead market model.

However, the nonlinear, nonconvex nature of the ac optimal power flow problem is incompatible with the complementarity [13], [14] and optimization formulations usually adopted in oligopolistic market models. (The exception that proves the rule is the work by Bautista *et al.* [5], who demonstrate the feasibility of calculating equilibria on an ac network, as well as the many practical challenges involved.) Computational issues are one reason for this incompatibility, in that solution of large scale ac-based OPFs is impractical for large systems for the dozens or hundreds of periods typically considered in production costing and market simulations. A second reason is the desirability of being able to prove certain properties of model equilibria, including existence and uniqueness; the nonconvexity of the nonlinear equality constraints in ac load flows make this impossible.

For these reasons, several oligopoly models instead use a linearized dc representation of load flow [38].[1] That representation facilitates use of linear optimization and complementarity models that can be readily solved for large systems, while facilitating demonstration of theoretical properties of market equilibria. Most models in the literature disregard phenomena such as voltage constraints, reactive power, resistance losses, and the use of DC lines and FACTS devices to control flow. In this paper, we show how such market models can be enhanced to include quadratic losses, DC lines, and phase shifting transformers, while preserving the linearity that facilitates solution and analysis.

We also consider the practical importance of including those features in market models in applications to Western Electricity Coordinating Council (WECC) and the European Union. It might be argued, for example, that congestion is responsible for most of the price differences among buses in markets, and that therefore the omission of losses from oligopoly models

[1]In this paper, capitalized "DC" refers to direct current lines, while lower case "dc" instead refers to the linearized load flow model.

is not a problem. However, *ex ante* studies of the components of locational marginal prices (LMPs) that would occur in the California ISO MRTU indicate that losses contribute as much, on average, to spatial price variations as congestion [36]. Congestion varies more over time, and causes large spatial price differences during peak periods, but averaged over the year, losses can be just as important. Hence, it is desirable to develop oligopoly market models that include losses.

Furthermore, phase shifters and controllable DC lines are increasingly important options for facilitating power transfers between regions (and thus enhancing competition) and can significantly affect prices [e.g., [29]]. Therefore, it is desirable to have the capability to include those features in market models. For instance, the NORDPOOL region of Europe is connected to continental Europe (the UCTE region) only by DC lines. Because of the complementary resources in the two regions (hydro with a large amount of storage in NORDPOOL, and growing amounts of wind in the northern UCTE region), representation of the controllability of the DC interconnectors between the two markets has become necessary in Europe-wide market models.

The features of quadratic losses and FACTS devices have long been included in models of production costs and OPFs, and also in models that simulate competitive markets. Our contribution is to show how these features can be included in easily solved complementarity models of oligopolistic markets, facilitating more realistic analyses of such markets. Regarding losses, an early analysis of the impact of losses on costs is by Boice *et al.* [6]. Haiku [34] and POEMS [41], which are two national models of competitive markets (in which firms bid marginal costs), consider linear and nonlinear relationships, respectively, between interregional flows and losses. The loss and congestion components of LMPs in, e.g., the Nordpool and California markets have been studied in detail using OPF models [25], [36], as has the issue of loss allocation in competitive markets [12]. Ivanic *et al.* [24] have previously formulated a complementarity model of oligopolistic markets with linear losses; our models (this paper, and a previous application in [11]) are the first to include quadratic losses in complementarity model of an oligopolistic equilibrium with a linearized dc load flow. Green [18] has considered interactions of oligopolists on a lossy linearized dc network, but had to solve his model by iterating among players rather than directly obtaining equilibrium conditions. As mentioned above, Bautista *et al.* [5] show how oligopolistic equilibria can be calculated, with significant computational effort, for a full ac network. Turning to FACTS devices, numerous papers show how they can be modeled in production costing and planning models (e.g., [30], [44]) and in competitive market models (e.g., [1], [2], [39]). Baldick and Kahn [4] describe how phase shifter settings could be manipulated to favor certain generators, but do not consider their use in an equilibrium model. The below analysis shows for the first time how phase shifters and controllable DC lines can be included in oligopolistic equilibrium models.

In the next section we summarize a Nash–Cournot oligopoly market model with a typical formulation of the ISO's problem based on the linearized dc load flow. In Section III, we then present a generalization that includes nonlinear losses, controllable DC lines, and phase shifters while preserving the convexity of the ISO's load flow problem. This formulation is readily implemented in oligopoly models formulated as complementarity or nonlinear programming problems. A successive linearization approach is then presented in Section IV for including quadratic losses within linear complementarity and optimization models. An application to competitive and oligopolistic market simulation on a six-bus network is summarized, highlighting the effects of considering congestion, losses, and a phase shifter (Section V). Large scale applications to the WECC and the European Union then illustrate the effect of including losses and controllable DC lines, respectively (Sections VI and VII). Section VIII presents some conclusions.

II. Basic Cournot Model With Linearized DC Load Flow

The model formulation presented here represents a system in which bilateral transactions between generators and consumers dominate, with the ISO providing transmission services. The ISO is assumed to set prices to efficiently clear the market for scarce transmission capacity; this is equivalent to the ISO solving a problem in which it maximizes the value of transmission services while taking the prices of transmission as fixed. In this model, the ISO also performs an arbitrage (or market splitting) function, in which it can buy power from one location and sell it in another if differences in locational prices make such transactions profitable. As a result, differences in prices across buses are the same as the fees charged to generators for transmitting power, consistent with practice in the US ISOs and the FERC Wholesale Market Platform.

The oligopoly model consists of three sets of components: the generator's profit maximization model, the ISO model, and a set of market clearing and consistency conditions. A more complete explanation and analysis of this type of model is presented elsewhere [e.g., [20], [21], [32]]; only the basic model is presented here so that we can show how it can be modified to include losses, controllable DC lines, and phase shifters.

We start by defining the following sets:

$f \in F$ set of generating firms in the market;

$i \in I$ set of buses in the market;

$h \in H(f,i)$ set of generators owned by f at bus i;

$k \in K$ set of transmission flow constraints in the ISO model.

Following [20], each generator f chooses its sales s_{fi} at each network bus i and the generation g_{fih} from each of its units h in order to solve the following profit maximization problem:

$$
\begin{aligned}
&\underline{\text{Cournot Generator model for } f \in F}:\\
&\underline{\text{Choose } \{s_{fi}, g_{fih}\} \text{ in order to}:}\\
&\text{MAX } \Sigma_{i\in I}[P_i(s_{fi} + \Sigma_{j\in F, j\notin f} s_{ji} + a_i) - w_i]s_{fi}\\
&\quad - \Sigma_{i\in I}\Sigma_{h\in H(f,i)}[C_{fih}(g_{fih}) - w_i g_{fih}]\\
&\text{subject to}:\\
&\text{Energy balances}: \Sigma_{i\in I} s_{fi} - \Sigma_{i\in I}\Sigma_{h\in H(f,i)} g_{fih} = 0\\
&\text{Capacity limits}: g_{fih} \le G_{fih} \quad i \in I, h \in H(f,i)\\
&\qquad \forall g_{fih}, s_{fi} \ge 0.
\end{aligned}
$$

Note that price $P_i()$ is a function of the firm's own sales, as well as sales by its rivals ($\Sigma_{j\in F,j\neq f} s_{ji}$) and the ISO as well in performing its arbitrage function (a_i). In this model, consistent with the Nash–Cournot assumption, the generator views sales by other entities $\Sigma_{j\in F,j\neq f} s_{ji} + a_i$ as fixed, but it recognizes that by changing its own sales s_{fi} it can affect the price through the inverse demand function $P_i()$.

Sales to bus i are charged a fee of w_i \$/MWh for transmission from an arbitrarily defined hub, while generation is paid the same amount since it provides counterflow (from i to the hub). An important feature of this model is that generators view w_i as fixed. Thus, generators are sophisticated about consumer reactions (recognizing that changes in supply result in changed prices), but are naïve about reactions of the ISO (believing that changed demands for transmission services will not affect transmission fees). This assumption, although simplistic, allows formulation and solution of market models for large complex systems [e.g., [14], [20]], and existence of pure strategy equlibria usually result. In contrast, correct anticipation of ISO responses to changes in generator output requires solution of nonconvex math programs with equilibrium constraints [e.g., [9], [21], [22]], and pure strategy equilibria may not exist [7], [21], [28].

Turning to the ISO, there are two types of primal variables: y_i (the MW of transmission service the ISO provides from an arbitrary hub to i) and a_i (the MW of arbitrage provided by the ISO from the hub to i). The ISO model is

$$\begin{aligned}
&\underline{\text{ISO Model (Linearized dc)}}: \underline{\text{Choose } \{y_i, a_i\} \text{ in order to:}}\\
&\text{MAX}\, \Sigma_{i\in I} w_i y_i + \Sigma_{m\in M}(p_i - p_H)a_i\\
&\text{s.t.}: \text{Transmission capacity:}\\
&\qquad \Sigma_{i\in I} PTDF_{ik}(y_i + a_i) \leq T_k, \qquad\qquad k \in K\\
&\text{Services balance at hub}: \Sigma_{i\in I} y_i = 0\\
&\text{Arbitrage balance at hub}: \Sigma_{i\in I} a_i = 0.
\end{aligned}$$

$PTDF_{ik}$ is the power transmission distribution factor for transmission constraint k for a power injection at the hub bus H and withdrawal at i. Alternative formulations have also been published in which generators sell directly to the ISO, who then resells the power to consumers [43]. The ISO's objective in the latter case is to distribute the power among consumers to maximize the value of consumption, subject to what it views as fixed injections by generators. The more complex transmission representations presented later in this paper can readily be included in that model.

To complete the model, market clearing and definitional conditions are needed as follows:

$$\begin{aligned}
p_i &= P_i(\Sigma_{f\in F} s_{fi} + a_i), && i \in I\\
y_i &= \Sigma_{f\in F}(s_{fi} - \Sigma_{h\in H(f,i)} g_{fih}), && i \in I.
\end{aligned}$$

These are, respectively, the definitions of price and the requirement that transmission services provided to i equal the demand for transmission services from generators.

One way to solve this model for a market equilibrium is to take the first-order (Karush–Kuhn–Tucker, or KKT) conditions for each generator problem, combine them with the KKTs for the ISO model, and finally add the market clearing/definitional conditions. The resulting system should have N complementarity and/or equality conditions and N primal and dual variables, and can be solved using commercial complementarity solvers (such as PATH [15]).

Another way to obtain the equilibrium is to formulate and solve a single nonlinear optimization model whose KKT conditions are equivalent to the complementarity problem that defines the equilibrium. This is not possible for general oligopoly models formulated as complementarity problems, but it is feasible for the above model, if the demand functions are affine

$$P_i(\Sigma_{f\in F} s_{fi} + a_i) = P_i^o - \left(\frac{P_i^o}{Q_i^o}\right)(\Sigma_{f\in F} s_{fi} + a_i)$$

where P_i^o is the price intercept and Q_i^o is the demand intercept for the demand function at i. The following quadratic program can then be derived, taking advantage of Hashimoto's [19] insight that the social welfare (surplus) maximizing problem can be modified to create a Cournot model. The first set of square bracketed terms is the sum of the integrals of the demand functions, while the third constitutes the generation costs; their difference is social welfare (the sum of producer, consumer, and ISO surpluses). The second set of square brackets is the modification that makes the model's KKT conditions the same as the Cournot market equilibrium conditions

$$\begin{aligned}
&\text{MAX}\, \Sigma_{i\in I}\big[P_i^o(\Sigma_{f\in F} s_{fi} + a_i) - (P_i^o/Q_i^o)(\Sigma_{f\in F} s_{fi} + a_i)^2/2\big]\\
&\quad - \big[\Sigma_{i\in I}\Sigma_{f\in F}\,(P_i^o/Q_i^o)\, s_{fi}{}^2/2\big] - [\Sigma_{i\in I}\Sigma_{h\in H(f,i)} C_{fih}(g_{fih})]\\
&\text{s.t.}: \Sigma_{i\in I} PTDF_{ik}(y_i + a_i) \leq T_k, && k \in K\\
&\Sigma_{i\in I} a_i = 0\\
&y_i = \Sigma_{f\in F}(s_{fi} - \Sigma_{h\in H(f,i)} g_{fih}), && i \in I\\
&\Sigma_{i\in I} s_{fi} - \Sigma_{i\in I}\Sigma_{h\in H(f,i)} g_{fih} = 0, && f \in F\\
&g_{fih} \leq G_{fih}, && f \in F, i \in I, h \in H(f,i)\\
&\forall g_{fih}, s_{fi} \geq 0.
\end{aligned}$$

(Note that the transmission services balance constraint is not needed because it is linearly dependent on the set of the generators' generation-sales balance constraints.) The resulting optimal solution can be proven to be equivalent to a Nash–Cournot equilibrium among the generators by showing that the KKT conditions for this model are the same as the conditions defining the equilibrium model described earlier.

III. Nonlinear Load Flow Formulation With Controllable DC Lines and Phase Shifters

We now modify the ISO model above to include nonlinear losses, phase shifters, and controllable DC lines. Because losses will vary with loadings, a constant PTDF-based formulation is not possible, since line loadings will be a nonlinear function of injections at the buses. Instead, we formulate a nonlinear dc load flow model based upon analogues to Kirchhoff's current and voltage laws (KCL, KVL). (See Schweppe *et al.* [38, Appendix] for a rigorous derivation of the linearized dc load flow model with quadratic resistance losses. Linear models of phase shifters and other FACTS devices are proposed in [40], and are the basis of our model.)

The nonlinear formulation includes three new decision variables:

f_{ij} MW (real power) flow from bus i to j on an (uncontrollable) ac line. This is modeled as a nonnegative variable, so that the net flow from i to j is $f_{ij} - f_{ji}$ (neglecting losses);

f_{DCij} MW (real power) flow from bus i to j on a (controllable) DC line. Like flows on ac lines, this too is a nonnegative variable, so that the net flow from i to j is $f_{DCij} - f_{DCji}$ (again neglecting losses);

d_{ij} setting for phase shifter between buses i and j (located on an otherwise uncontrollable ac line).

The following additional sets are needed:

$v \in V$ set of voltage loop constraints (KVL). There should be $N - M + 1$ independent KVL constraints, if N is the number of ac transmission lines and M is the number of buses;

$ij \in IJ$ set of all ac (uncontrollable) transmission flows (from i to j);

$ij \in IJ_{DC}$ set of all DC (controllable) transmission flows (from i to j);

$ij \in IJ_{PS}$ set of all phase shifters;

$ij \in IJ(v)$ ordered set of ac transmission lines comprising voltage loop v (Kirchhoff's voltage law);

$j \in JA(i)$ set of buses j directly linked to bus i by an (uncontrollable) ac line;

$j \in JD(i)$ set of buses j directly linked to bus i by a (controllable) DC line;

$ij \in PS(v)$ set of phase shifters that are modeled as being located on voltage loop v.

The generalized ISO model is as shown at the bottom of the page. Several features of this formulation are notable:

1) The KCL constraint says that the net flow of real power from the grid to bus i (via transmission flows f_{ij} and f_{DCij}) must equal *or exceed* the sum of transmission services plus arbitrage to that bus. The KCL constraint is presented as an inequality rather than equality in order to render the feasible region convex [10], which is desirable from a computational and theoretical point of view. If the marginal value of power (LMP) is positive at a bus i, then this constraint will be binding. It is indeed possible under some low loading or particular network conditions for negative LMPs to occur; this would show up in this model as a nonzero slack on this constraint, which is equivalent to power being thrown away. We disregard that (physically impossible) possibility here.
2) Losses are a *quadratic function* of flow. As a result, real power flow injected into one end of a line f_{ij}, if positive, is greater than flow received at the other end $(1 - \alpha_{ij} f_{ij}) f_{ij}$. The coefficient α_{ij} is a function of line characteristics [38], as is the reactance X_{ij} in the KVL constraint.
3) The net amount of arbitrage no longer has to be zero. Indeed, it cannot be, since the ISO must, on net, buy power to make up for network losses. Because of these losses, the equilibrium price of transmission services w_i will include both the cost of congestion and line losses.
4) The phase shifter acts to add or subtract a change in angle within the KVL analogy. D_{ijv} converts the phase shifter setting into units compatible with the KVL analogy, and is negative or positive depending on whether the direction from i to j conforms to or opposes the direction assumed in the KVL voltage loop v.

As in the basic model of Section II, a market equilibrium model can be formulated by taking the KKT conditions for the ISO model, and combining them with the generators' KKTs along with the market clearing conditions. The result is a complementarity model that can be solved by standard complementarity solvers. (This is done in [11] for the lossy formulation, but excluding controllable DC lines and phase shifters.)

However, if affine demand curves are assumed, as in the end of Section II, then a single quadratic program (with quadratic and linear constraints) can be formulated whose KKTs are the same as the equilibrium problem. This model is as shown at the top of the next page. This is the model solved in the simple example of Section V.

IV. Linearized Loss Approximations

Because some market models are based upon solvers that require linear constraints, it is sometimes desirable to linearize the

$$
\begin{aligned}
&\underline{\text{ISO Model (Linearized dc)}}: \ \underline{\text{Choose}\,\{y_i, a_i, f_{ij}, f_{DCij}, d_{ij}\}\text{ to}:}\\
&\text{MAX}\, \Sigma_{i\in I} w_i y_i + \Sigma_{m\in M} p_i a_i\\
&\text{s.t.}: \text{KCL Analogy}: \Sigma_{j\in JA(i)}[f_{ij} - (1-\alpha_{ji} f_{ji}) f_{ji}]\\
&\quad + \Sigma_{j\in JD(i)}[f_{DCij} - (1-\alpha_{DCji} f_{DCji}) f_{DCji}] + y_i + a_i < 0,\ i \in I\\
&\text{KVL Analogy}: \ \Sigma_{ij\in IJ(v)} X_{ij}(f_{ij} - f_{ji}) - \Sigma_{ij\in PS(v)} D_{ijv} d_{ij} = 0,\ v \in V\\
&\text{ac transmission capacity}: f_{ij} \le T_{ij}, \quad ij \in IJ\\
&\text{DC transmission capacity}: f_{DCij} \le T_{DCij}, \quad ij \in IJ_{DC}\\
&\text{Phase shifter bounds}: T_{PSij-} \le d_{ij} \le T_{PSij+},\ ij \in IJ_{PS}\\
&\text{Services balance at hub}: \Sigma_{i\in I} y_i = 0\\
&\qquad\qquad \forall f_{ij}, f_{DCij} > 0.
\end{aligned}
$$

$$
\begin{aligned}
&\underline{\text{QP Market Model with Nonlinear Losses and Phase Shifters :}}\\
&\text{MAX}\, \Sigma_{i\in I}\big[P_i^o(\Sigma_{f\in F}s_{fi}+a_i)-(P_i^o/Q_i^o)\,(\Sigma_{f\in F}s_{fi}+a_i)^2/2\big]\\
&\qquad -\big[\Sigma_{i\in I}\Sigma_{f\in F}\,(P_i^o/Q_i^o)\,{s_{fi}}^2/2\big]-\big[\Sigma_{i\in I}\Sigma_{h\in H(f,i)}C_{fih}(g_{fih})\big]\\
&\text{s.t.}:\Sigma_{j\in JA(i)}[f_{ij}-(1-\alpha_{ji}f_{ji})f_{ji}]\\
&\qquad +\Sigma_{j\in JD(i)}[f_{DCij}-(1-\alpha_{DCji}f_{DCji})f_{DCji}]\\
&\qquad +y_i+a_i<0,\ i\in I\\
&\qquad \Sigma_{ij\in IJ(v)}X_{ij}(f_{ij}-f_{ji})-\Sigma_{ij\in PS(v)}D_{ijv}d_{ij}=0,\ v\in V\\
&\qquad f_{ij}\le T_{ij},\ ij\in IJ\\
&\qquad f_{DCij}\le T_{DCij},\ ij\in IJ_{DC}\\
&\qquad T_{PSij-}\le d_{ij}\le T_{PSij+},\ ij\in IJ_{PS}\\
&\qquad y_i=\Sigma_{f\in F}(s_{fi}-\Sigma_{h\in H(f,i)}g_{fih}),\quad i\in I\\
&\qquad \Sigma_{i\in I}s_{fi}-\Sigma_{i\in I}\Sigma_{h\in H(f,i)}g_{fih}=0,\quad f\in F\\
&\qquad g_{fih}\le G_{fih},\quad f\in F, i\in I, h\in H(f,i)\\
&\qquad \forall g_{fih}, s_{fi,}\ge 0.
\end{aligned}
$$

loss expressions in the KCL constraints. We do so here. Controllable DC line flows are omitted below for simplicity, but their losses can be linearized in the same way.

A. Piecewise Linearization

One linearization approach is piecewise linearization, in which f_{ij} is divided into several segments $f_{ijl}, l=1,2,\ldots,L$, and each segment has a different linear loss coefficient α_{jil}

$$
\begin{aligned}
&\text{KCL Analogy :}\\
&\quad \Sigma_{j\in JA(i)}[\Sigma_l f_{ijl}-\Sigma_l(1-\alpha_{jil})f_{jil}]+y_i+a_i=0,\ i\in I.
\end{aligned}
$$

This constraint can now be an equality and still define a convex feasible region. The terms $\Sigma_l f_{ijl}$ are also substituted for f_{ij} in the transmission flow bounds and KVL constraints.

To our knowledge, the use of piecewise linear formulation of losses for market models was first proposed by [35]. Since then it has also been described in [17] and implemented in market software in Australia and New Zealand [16] where each nonlinear term is approximated by ten pieces. Assuming that the LMPs are positive, then the OPF will choose the segments with the lowest loss rates first, as it should. (If LMPs are negative, then segments will enter in the wrong order, exaggerating losses.) The disadvantage of this formulation is that the number of flow variables grows by a factor of L, which may cause computational problems or necessitate simplification of other aspects of the model.

B. Successive Linearization

An alternative approach is successive linearization of the nonlinear term in the KCL constraint. The nonlinear term is linearized around a starting solution f_{ji}^o using a first-order Taylor's series expression.[2] This results in

[2]Other successive linearization approaches could also be used [26].

$$
\begin{aligned}
\text{KCL}:\Sigma_{j\in JA(i)}[f_{ij}-\alpha_{ji}(f_{ji}^o)^2-(1-2\alpha_{ji}f_{ji}^o)f_{ji}]&\\
+y_i+a_i=0,\ i\in I.&
\end{aligned}
$$

Note that the $\alpha_{ji}(f_{ji}^o)^2$ and $(1-2\alpha_{ji}f_{ji}^o)$ terms are constants, so the power arriving at an importing bus $\alpha_{ji}(f_{ji}^o)^2+(1-2\alpha_{ji}f_{ji}^o)f_{ji}$ is now a linear function of the power injected at the exporting bus f_{ji}. The correctness of this local approximation can be confirmed by noting that if $f_{ji}=f_{ji}^o$, then the total power arriving at the importing bus is correct, as is its first derivative with respect to the exporting bus's flow f_{ji}. As a result, the prices calculated based upon the dual variables to the KCL constraints will correctly reflect marginal losses.

Given this approximation, the now linear model can be solved, and then relinearized around the newest solution by substituting the latest f_{ji} into f_{ji}^o. This process can be repeated until convergence is achieved. Because the losses are usually a small fraction of the flows, convergence is likely to be quite rapid. Convergence results from successive linear programming theory apply [33] which basically state that under certain conditions, the successive linearization of a nonlinear program with a convex nonlinear feasible region and a linear objective can be shown to converge to the optimal solution. The advantage of successive linearization is that the model size is smaller than if piecewise linearization is adopted. However, the disadvantage is the need to solve the model more than once in order to obtain convergence.

V. Simple Example of Nonlinear Losses and Phase Shifter in a Cournot Model

The six-bus system portrayed in Fig. 1 is used to illustrate the application of the model of Section III with nonlinear losses and phase shifters. Normalized reactances X_{ij} on all lines are identical and equal to 2. The loss coefficients α_{ij} equal 0.0002

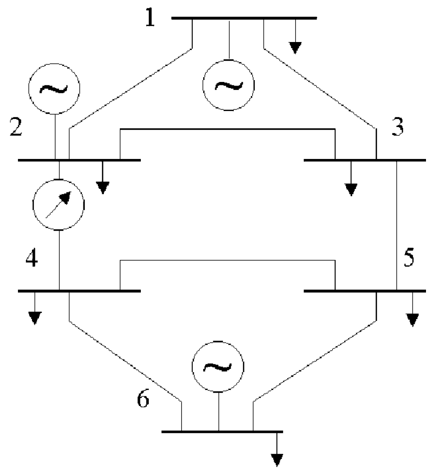

Fig. 1. Six-bus, three-generator, single-phase shifter network example.

for all lines. Each line has a limit of 300 MW. A phase shifter is inserted on the line between buses 2 and 4, and its angle can vary between -100 and $+100$. (Given that the p.u. reactances of the lines are 2, this can be viewed as equivalent to being able to shift flows by approximately ± 50 MW $= \pm 100/2$.)

There are generators located sited at buses 1, 2, and 6 with constant marginal costs of 50, 60, and 20 \$/MWh, respectively. Each has capacity 1000 MW. The generators at buses 1 and 2 are owned by the same company ($f = 1$), and the generator at 6 is owned by another ($f = 2$), rendering this a duopoly market. Consumption occurs at each of the six buses, with identical demand curves having price intercept $P_i^o = \$200/\text{MWh}$ and quantity intercept $Q_i^o = 500$ MW.

In Tables I and II, we compare "Base Case" market equilibria (including losses, congestion, and the phase shifter) with equilibria that lack one of those features (either no losses, unlimited transmission capacity, or no phase shifter). The tables show generation by each plant, flows on each line (at the injection end of the line, so losses on that line are not subtracted out), the phase shifter setting, total losses, and the energy price at each bus. Table I shows the competitive market solutions, in which each generator is a price-taker (no strategic behavior); while Table II contains the Nash–Cournot solutions. The competitive solutions are obtained by solving the optimization model at the end of Section III without the Hashimoto term in the objective function (the second square bracketed term), while the Cournot solutions are the result of a model that includes that term.

The two tables show that the competitive and oligopoly solutions differ greatly in prices, quantities demanded, generation patterns, and flows. This is not surprising, since this is a two-firm market, and there would be a strong incentive for each duopolist to restrict output and raise prices. Generally, the oligopoly prices are up to twice as high as the competitive prices (e.g., see bus 1), which arises because each duopolist restricts its output by one-quarter to one-third, resulting in much higher profits (not shown). Firm 2 shuts down one of its plants entirely (at bus 2) in the Nash–Cournot case. Quantities demanded by consumers are less, consistent with our assumption of price-responsive demand. Another important difference between the competitive and oligopoly cases is that the smaller amount of supply in the latter case results in less congestion and fewer losses, as indicated by the proportionally smaller dispersion of prices. The highest price in the base competitive case is almost twice as high as at the lowest cost bus (bus 6), but the ratio of highest to lowest prices in the base Nash–Cournot case is only 1.3:1.

TABLE I
COMPETITIVE SOLUTIONS: PRIMAL VARIABLES AND PRICES

Variable		Base Case	No Losses	No Congestion	No Phase Shifter
Generation MW	$g_{f=1,i=1}$	890.0	925.0	1000.0	902.8
	$g_{f=2,i=2}$	300.7	200.0	201.8	293.8
	$g_{f=2,i=6}$	1000.0	1000.0	1000.0	1000.0
Flows, MW	$f_{1,2}$	215.0	250.0	288.9	227.9
	$f_{1,3}$	300.0	300.0	343.6	300.0
	$f_{2,3}$	85.0	50.0	54.6	72.1
	$f_{2,4}$	71.5	50.0	69.5	89.1
	$f_{3,5}$	37.8	25.0	27.5	26.4
	$f_{4,5}$	1.3	0.0	14.0	9.4
	$f_{6,4}$	298.7	300.0	310.5	290.6
	$f_{6,5}$	300.0	300.0	324.6	300.0
Ph. Shifter	$d_{2,4}$	-100.0	-50.0	2.7	0.0
Losses, MW		65.8	0.0	82.4	66.1
Nodal prices, \$/MWh	LMP 1	50.0	50.0	53.0	50.0
	LMP 2	60.0	60.0	60.0	60.0
	LMP 3	68.9	70.0	61.4	69.3
	LMP 4	60.0	60.0	61.7	59.3
	LMP 5	71.7	70.0	62.1	72.9
	LMP 6	39.5	40.0	54.0	36.2

TABLE II
NASH–COURNOT SOLUTIONS: PRIMAL VARIABLES AND PRICES

Variable		Base Case	No Losses	No Congestion	No Phase Shifter
Generation MW	$g_{f=1,i=1}$	681.7	695.5	648.8	681.7
	$g_{f=2,i=2}$	0.0	0.0	0.0	0.0
	$g_{f=2,i=6}$	900.0	900.0	972.5	900.0
Flows, MW	$f_{1,2}$	210.2	218.2	190.9	210.1
	$f_{1,3}$	210.1	218.2	190.9	210.1
	$f_{2,3}$	0.1	0.0	0.0	0.0
	$f_{2,4}$	38.4	40.9	64.8	38.5
	$f_{3,5}$	38.6	41.0	64.8	38.5
	$f_{4,5}$	0.0	0.0	0.0	0.0
	$f_{6,4}$	300.0	300.0	342.4	300.0
	$f_{6,5}$	300.0	300.0	342.4	300.0
Ph. Shifter	$d_{2,4}$	0.8	0.2	-0.1	0.0
Losses, MW		54.2	0.0	63.1	54.2
Nodal prices, \$/MWh	LMP 1	95.4	96.4	93.2	95.4
	LMP 2	104.2	96.4	101.0	104.2
	LMP 3	104.2	96.4	101.0	104.2
	LMP 4	102.5	96.4	98.3	102.6
	LMP 5	102.6	96.4	98.3	102.6
	LMP 6	80.0	80.0	84.9	80.0

The losses amount to roughly 3% of generation. This may seem small, but comparison of the base case and no-losses solutions (first and second columns) show that including losses can significantly affect the solution. For instance, comparing the competitive solutions with and without losses shows that considering losses—and thus the loss of transmission capacity because lines must carry those losses—results in a significant shift in generation between generators at buses 1 and 2. Prices increase at only one bus in the competitive case, and there only by 2%. This is part because the 6–4 transmission constraint becomes uncongested in the lossy competitive base case, changing the pattern of congestion costs among buses.

But in the Cournot cases, losses cause prices to rise 6% or more at nongeneration buses. This occurs for two reasons. One is the high market price that the ISO must pay for power to make up for losses (variables $a_1 - a_6$), as those prices are roughly twice as high as competitive prices. The second and more significant reason is that Firm 2, whose generator is at bus 6, can ship power to other buses only through heavily loaded lines 6–4 and 6–5 (each carrying 300 MW). Consequently, when losses are considered, the ISO's charge to Firm 2 to deliver power to other buses significantly increases to account for those losses, and that firm is less competitive at buses 1–5. This allows Firm 1 to exercise more market power, and it does so by shrinking its output and raising prices at buses 1–5. (Firm 1 generates 13.8 MW less power in the lossy Cournot base case than in the no losses case, driving most prices up significantly in buses 2–5; see Table II.) Thus, the interaction of transmission losses and competition can be complex, and strategic behavior can significantly alter the impact of losses on prices.

Losses actually cause more differences in nodal prices than congestion in the Cournot solution. If the transmission flow constraints of 300 MW are disregarded (third column), prices only converge somewhat, showing that most of the price differences are due to losses rather than congestion. On the other hand, congestion is relatively more important in the competitive solutions, in part because more power is sold at lower prices, resulting in higher flows that exacerbate congestion.

Finally, the phase shifter makes only a small difference in the competitive solution. Comparing the first and last columns of Table I, we see that the phase shifter has resulted in a small (about 20 MW) decrease in flow on the line that the shifter is placed on (line 2–4). The largest price difference is at Bus 6, a supply bus where congestion has resulted in low prices. Use of the phase shifter relieves the congestion somewhat, allowing the price there to rise from 36.2 \$/MWh (no shifter) to 39.5 \$/MWh (with the shifter). In contrast, the phase shifter makes no difference at all in the Cournot solution, because the pattern of flows in that case is such that the phase shifter cannot relieve congestion.

VI. Successive Linearization of Losses: The Western North American Market

In this section, we consider how the inclusion of quadratic losses can affect the prices calculated from a large-scale oligopoly market model for western North America. The approach of successive linearization is used to calculate the quadratic loss terms within a linearized dc load flow.

A. Model Summary

The California ISO (CAISO) has implemented a methodology for evaluating the economic benefits of transmission reinforcements that accounts for how changes in import capability can affect bidding behavior by large generators and resulting market prices [3], [8]. The methodology is called Transmission Economic Evaluation Methodology (TEAM). TEAM recognizes that a large portion of benefits of transmission can result from increasing the competitiveness of markets, yielding more cost-reflective bids and lower prices for consumers. TEAM's assessment of market power involves four basic steps:

1) estimation of historical relationships between market conditions and deviations of prices from marginal cost within zones;

2) assessment of hourly market conditions;

3) calculation of resulting hourly bids based on the historical relationships; and

4) simulation of the market, including calculation of locational marginal prices, based on the projected bids.

The main indicator of market conditions is the Residual Supply Index (*RSI*), which is defined as:

$$\text{RSI} = (US_{tot} - \frac{US_{LS})}{UL}$$

where

US_{tot}	total uncontracted supply available to serve a particular zone, including potential imports;
US_{LS}	uncontracted supply of largest supplier within the zone;
UL	uncontracted load within the zone.

RSI can be viewed as an index of the extent to which the largest supplier is pivotal in the zone's market; a value of *RSI* less than 1 indicates that the largest supplier's output is required to meet load, and that supplier is able to charge whatever price it wishes, subject to the bid cap. The CAISO experience indicates that values of $RSI < 1.2$ are associated with significant mark-ups. The CAISO has developed statistical relationships between mark-ups and *RSI*, as well as other independent variables such as seasonal dummy variables [3], [8]. Adding transmission affects mark-ups by increasing the amount of US_{tot} available to the zone, thus increasing *RSI* and, ultimately decreasing mark-ups and prices (since *RSI* has a negative relationship with mark-up).

Thus, this approach to simulating an oligopolistic power market differs from the equilibrium models of Sections II and III, in that mark-ups in the latter models are calculated endogenously, rather than provided as an input to the market model. This four step empirical approach has been used to evaluate a hypothetical upgrade of Path 26 in California [3], [8] as well as the proposed Palo-Verde Devers 2 line between Arizona and California. Elsewhere, this general approach has also been used to calculate the benefits of an upgrade of the Sicily-mainland interconnector in Italy [42]. In all of these cases, a significant portion of the benefits arise from market power mitigation, and net benefits would be lower if instead competitive (price-taking) behavior was assumed.

The market model used in the TEAM evaluation of Path 26 [3], [8] was PLEXOS [16]. The model, formulated as a linear program, minimizes as-bid generation costs subject to transmission constraints represented as a linearized dc load flow. Demand is assumed to be fixed. Thus, PLEXOS is not a true oligopoly equilibrium model, unlike the models in Sections II and III, but rather simulates the effect of exogenous bid adders. The model has been calibrated for the WECC. Some features of that implementation include:

- calculation of flows on 17 450 lines;
- constraints upon flows on three DC lines, 284 high-voltage (500 kV) ac lines, and 129 interfaces;
- calculation of prices at 13 383 buses;
- representation of 57 phase shifters (seven optimized, 50 fixed);
- hourly dispatch of 760 generators over a 24-h day;
- bids of California plants based on empirical *RSI*-based mark-ups, with other plants bid competitively;
- optimal operation of eight pumped storage plants; and
- predetermined output schedules from 117 hydro plants.

As used in the Path 26 study, PLEXOS first calculated a market equilibrium using a lossless DC approximation based on PTDFs, and then estimated losses in a post-processor based on the estimated flows. As a result, the calculated prices omitted the effect of losses which, as noted earlier, can have as much effect on average prices through the year as congestion [36].

B. Comparison of Results With and Without Losses

Here we report on a comparison of the prices that result from that lossless representation with a set of prices resulting from a version of PLEXOS in which losses are calculated endogenously using the successive linearization approach of Section IV-B, above. This is done for all hours for the first week of August for the year 2008.

Fig. 2 shows the average prices by region in the west for this week for the without- and with-losses cases. On average through the west, the inclusion of losses raises cost to load by 2.3% in this hour. The variation of prices across the region also increases. This is indicated by the standard deviation of prices shown in the figure, which is 6.9% higher for the with-losses case. As the figure shows, this is because prices increase in high cost importing regions, especially California and the west coast, while actually decreasing in some low cost areas, mainly exporting regions in the Rocky Mountain west. The largest percentage price increase is 3.8% for Alberta, while the largest decrease is 6.1% in the Aquila region in the Southwest.

There is a significant computational cost associated with this improved representation of costs. Successive linearization means that a market model is solved more than once. The first

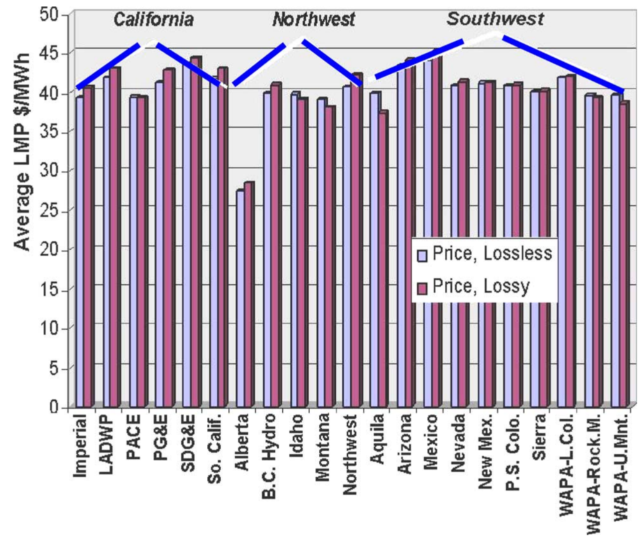

Fig. 2. Comparison of average wholesale prices by region without and with losses, WECC PLEXOS oligopoly run.

run uses some logical starting point (such as half of a line's capacity) for the linearization's initial guess for the flows f^o_{ji}, and subsequent iterations use the previous solution's flows. Since PLEXOS can take several hours to run the entire WECC market for an entire year, the resulting increase in execution time can be significant. Fortunately, however, the algorithm converges quickly, with no appreciable difference between the results of the second and subsequent iterations.[3] Thus, the computational effort required to perform the with-losses market simulation is about double that of the without-losses case (up to two hours on a modern work station for the former case, compared to 40 min for the latter). Depending on the number of runs made and the computational time per run, the more realistic estimates of prices, total generation, total emissions, and other statistics might be judged to be worth the extra effort.

VII. Controllable DC Lines: The EU Interconnections

In contrast to the statistical approach to estimating oligopolistic bid mark-ups used in TEAM, the model used below is a true oligopoly equilibrium model. This analysis illustrates how inclusion of controllable DC lines can affect the results of such a model, using the European Union markets as an example

A. COMPETES Model Summary

The analysis of this section uses a simplified version of COMPETES EU-20 [27], which is based upon an earlier model of the northwest European market [23]. The full COMPETES

[3]This rapid convergence has been our general experience. For instance, another case study that involves the detailed California network and aggregated non-California network has 3959 buses, 5292 lines, and 672 generators, with losses simulated for 2018 lines with voltages of more than 115 kV. Seventy two N-1 transmission contingencies are enforced, and 105 interfaces are monitored. Twenty-four hours are simulated in each simulation day. Our experience is that loss modeling converges in three iterations for the first simulation day, and that the average number of iterations is two for the other simulation days (for which we use a "hot start" for the linear program, based on a previous day's solution). This experience reflects a wide variety of load and supply conditions in the western U.S., and so we are confident that similarly fast convergence will be achieved for other systems.

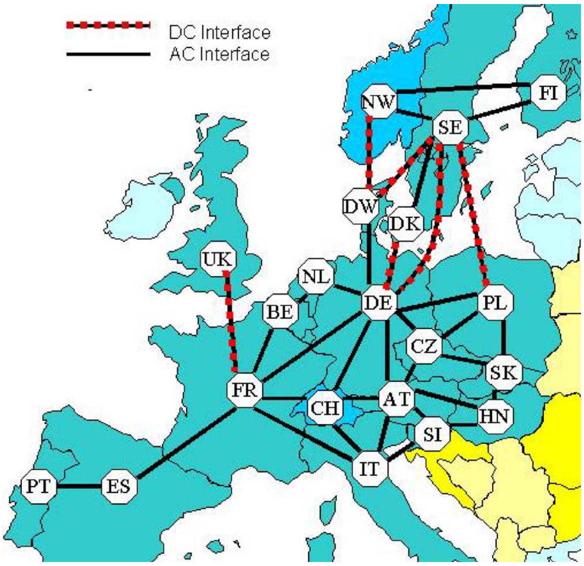

Fig. 3. COMPETES EU-20 network.

EU20 model is formulated as a complementarity model similar to the transmission-constrained Cournot model of Sections II and III, with additional capabilities to simulate conjectured supply function competition, manipulation of transmission congestion by generators, and imperfect transmission congestion management, such as path-based pricing. The EU20 version of COMPETES covers three synchronized systems: U.K., UCTE, and NORDPOOL. The model has a single node (zone or collection of buses) for each of 19 European countries, as shown in Fig. 3. The exception is Denmark, which has two nodes representing the parts that belong to two different synchronized systems (the west is in UCTE and the east is in NORDPOOL).

In this application of COMPETES, only the linearized dc load flow representation is considered, disregarding the path-based constraints which are often the most constraining limits in Europe. Losses are also not considered. Fig. 3 shows that we model intercountry links as either ac or controllable DC interfaces. The NORDPOOL ac network is simplified to a triangular three node system (NW-SE-FI) with a radial link to a fourth node (DK), while the UK market is a single node connected radially to the UCTE network. The UCTE system consists of 15 nodes on a meshed ac network. Flows within the NORDPOOL and UCTE ac networks are represented using a PTDF formulation.[4] For the UCTE system, the values of the PTDFs were obtained from Zhou and Bialek [45], while for the NORDPOOL system, we estimated them from NORDPOOL transmission system maps. Given a set of net injections (generation minus load), flows on the six DC interfaces between the three synchronized systems (Fig. 3) are decision variables that can be controlled by the system operator, unlike flows on the ac lines. (Since the UK-UCTE line joins those two markets radially, treating it as a controllable DC line would not affect the solution. Therefore, our analysis only addresses the effect of modeling the NORDPOOL-UCTE lines as being controllable.)

Our illustration uses a Cournot representation of competition among generators. Rather than solving for the Cournot market equilibrium as a complementarity problem, we use an equivalent quadratic program here, analogous to that at the end of Section III. To analyze the effects of the controllable lines, we compare the solution of the above described model with a version that substitutes uncontrollable ac lines for the six controllable DC interfaces in Fig. 3. (In effect, the latter model synchronizes Nordpool with UCTE, with the flows over the Nordpool-UCTE interfaces determined by PTDFs rather than being decision variables controlled by the operators.)

For the purposes of this analysis of the effect of controllable DC lines, we represent the market by a very simplified set of 20 linear supply and demand curves, one pair per node. The supply (marginal production cost) curve for each node is defined by passing a line through the origin ($0 price, 0 MW) and the price-quantity combination resulting from the competitive version of the full EU20 COMPETES model [27] for the winter peak period. Since the full model is based upon an inventory of all power plants in those countries and their actual costs, the relative position of the price-quantity combinations in different countries does reflect differing supply conditions. The demand curve for each country also passes through the full model's competitive price-quantity combination, and has a slope consistent with a price elasticity of -0.4 at that price-quantity point. Thus, the supply and demand curves for each node intersect at the competitive price-quantity solution.

These supply and demand curves are not meant to be realistic depictions of the EU20 market. Instead they are intended to generally reflect differences in generation mixes, costs, and demand conditions across the continent, which is sufficient to illustrate the effects of considering controllable DC lines.

Similarly, our representation of market structure is highly simplified. We consider a situation where energy companies are either competitive (bid at marginal cost) or are "national champions" that can exercise market power *a la* Cournot. Fig. 4 shows the five such "champions" we simulate—a Portuguese-Spanish company that encompasses all generation within those two countries, France, a Dutch-German company that includes all those countries' generation, Italy, and Sweden. Sweden is shown as a dashed oval because we compare two scenarios: one in which Swedish producers behave oligopolistically like the other "champions" and another in which Swedish producers instead behave competitively by bidding at marginal cost. Considering these two scenarios allows us to compare the effect of market structure (the behavior of Sweden) with the effect of modeling the UCTE-NORDPOOL interconnections as controllable DC lines.

B. Impact of Controllable DC Lines

To gauge the relative importance of modeling DC lines as controllable compared to market structure assumptions (monopoly versus competition in Sweden), we do four runs representing each possible combination of {Sweden competitive, Monopoly} and {Controllable UCTE-NORDPOOL interties, Uncontrollable interties}. The aggregate capacity of

[4]Thus, the formulation differs from the model of Section III which instead uses analogies to Kirchhoff's laws. The PTDF formulation can be used here because losses are disregarded. As is well-known [38], for a lossless model, the PTDF and KCL/KVL formulations of the linearized DC model are mathematically equivalent, and yield the same flows, given the same injections.

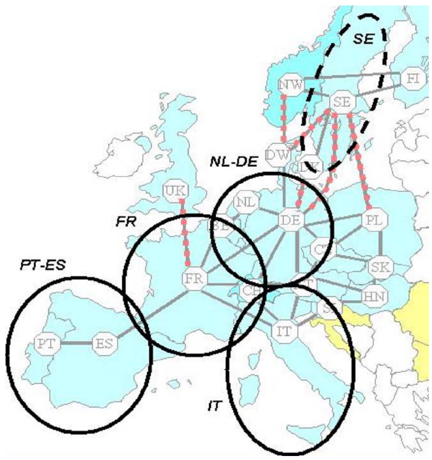

Fig. 4. National companies assumed to possess market power.

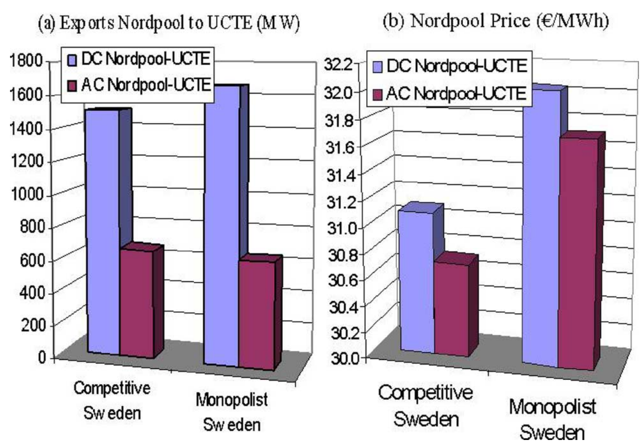

Fig. 5. COMPETES solutions under different levels of competition and controllability of NORDPOOL-UCTE interfaces.

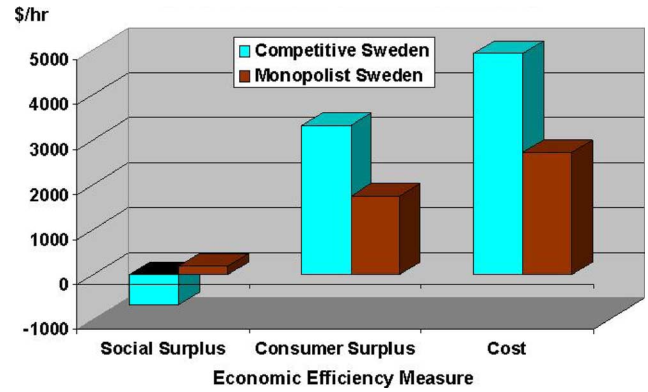

Fig. 6. Incremental benefits of controlling NORDPOOL-UCTE interfaces.

the UCTE-NORDPOOL interties is approximately 3000 MW in every run, so that differences arise only because of the assumption concerning whether flows on those interties are controllable, not because of capacity. The four runs are then compared in terms of MW exports from NORDPOOL to UCTE; average (quantity-weighted) NORDPOOL bulk power prices; and various economic efficiency measures (consumer surplus, production cost, and total social surplus, the latter equaling the sum of consumer surplus, producer surplus, and congestion surplus).

In Fig. 5, we compare NORDPOOL exports and exports across the four scenarios. Fig. 5(a) shows that NORDPOOL exports are doubled or more when the NORDPOOL-UCTE interfaces are represented as controllable DC lines. In contrast, the effect of monopoly in Sweden on those flows is much less (slightly increasing exports if DC lines are controllable, slightly decreasing otherwise).

On the other hand, whether Swedish generators are a monopoly has a bigger impact on NORDPOOL prices. Fig. 5(b) shows that making the UCTE-NORDPOOL interfaces controllable raises NORDPOOL prices by about 0.3 €/MWh because exports to UCTE are increased. The effect of changing Swedish company behavior from competitive to oligopolistic is three times as large (1 €/MWh).[5] This is even more true for economic efficiency measures (social surplus, consumer surplus, and production costs, results not shown): the impact of changing Swedish company behavior is five to ten times as large as controlling the interfaces.

In Fig. 6, we ask a different question: how do the benefits of a transmission improvement (here, a fictitious upgrade from noncontrollable ac interfaces to controllable DC lines) depend on market structure? A striking result here is that the changes in economic efficiency resulting from such an upgrade are much smaller in magnitude if Swedish power companies exercise market power. For instance, consumer prices throughout the EU20 market decrease more as a result of the upgrade if Sweden is competitive than if it is monopolistic (reflected in a larger increase in consumer surplus). If Sweden is competitive, the upgrade actually lowers net social welfare, the reason being that competitive (and relatively expensive, on the margin) Swedish exports displace power produced by oligopolistic producers in UCTE (whose marginal costs are relatively low, even though the prices they charge are high because they exercise market power). The increased exports lower producer profits more than they enhance consumer surplus. This is reflected in the result in Fig. 6 that making the lines controllable increases production costs, although that increase is also in part due to the increased loads stimulated by lower prices.

These results illustrate the fact that the interactions of networks and oligopolistic behavior in power markets can be quite complex and sometimes unexpected. They also confirm conclusions from elsewhere [3], [23], [37], [42] that the extent and nature of market power can make a large difference in benefit-cost analyses of transmission upgrades.

VIII. Conclusion

Although large scale models of oligopolistic competition among generators on networks have usually been based on linearized dc load flow approximations, more realistic representations of transmission costs and constraints are possible. A formulation that includes quadratic resistance losses, phase

[5]It might be surprising that the impact of a Swedish monopoly on NORDPOOL prices is only about 3%. One reason is the use of a high price elasticity (-0.4). Another is competition from UCTE producers who are assumed to be able to sell in the NORDPOOL market and also earn counterflow revenues by relieving congestion in the NORDPOOL to UCTE direction.

shifters, and controllable DC lines is presented, as well as linearized approximations. A simple application to a six node network illustrates the effect of including losses and phase shifters on prices and generation for both competitive and Cournot market models. In the competitive solution, congestion is more important than losses because higher loads under competition cause higher flows, which worsens congestion. In contrast, price differences due to losses are more important in the Cournot solution because of the high expense to the ISO of making up losses. This shows that it can be more important to consider losses in oligopoly models than in competitive market models.

These more realistic transmission models have also been used in simulations of large scale oligopolistic markets. The results of those simulations illustrate the practical impacts that these improved, more accurate transmission formulations can have on oligopolistic market outcomes. For a peak (summer) week, consideration of losses in the western North American market yields 2% higher prices and 7% greater price dispersion between submarkets. Meanwhile, inclusion of controllable DC lines between the Nordic countries and mainland Europe results in a doubling of NORDPOOL exports, decreases in Nordic prices, and, surprisingly, increases in production costs and ambiguous effects on social welfare. The complex results in the latter case are the result of interaction of the network with the spatial distribution of market power. These simulations show that improved transmission representations can change the results of oligopoly market models, and so have the potential to enhance the credibility and usefulness of those models.

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