JHSMINE Formulation version 1

In this document, the basic JHSMINE formulation will be shown. This whole formulation is a result of model development across multiple articles/reports. This is thus a combination of works of researchers from Hobbs group. The original formulation with two-stage stochastic programming was from [1]. Then in [2], this formulation was at the first time applied to WECC with DC OPF model enhancement. Details of a subsequent application that includes linearized unit commitment constraints [3] are in [4].

[1], A. H. van der Weijde, B. F. Hobbs, "The economics of planning electricity transmission to accommodate renewables: Using two-stage optimisation to evaluate flexibility and the cost of disregarding uncertainty," Energy Economics, vol. 34, no. 6, pp. 2089-2101, 2012.

[2] F. D. Munoz, B. F. Hobbs, J. L. Ho, S. Kasina, "An Engineering-economic approach to transmission planning under market and regulatory uncertainties: WECC case study," IEEE Transactions on Power Systems, col. 29, no. 1, pp. 307-317, 2014.

[3] S. Kasina, S. Wogrin and B. Hobbs, "Approximations to unit comment in planning models," INFORMS Annual Meeting, Minneapolis, 2013.

[4] J. Ho, B. F. Hobbs, P. Donohoo-Vallett, Q. Xu et al., "Planning transmission for uncertainty: Applications and lessons for the western interconnection," WECC, Salt Lake City, UT, 2016.

Set	**Description**
B	Bus set. Index: b
G	Generation type set. Index: g
G_I	Generation types to be invested on
G_R	Renewable generation types
G_{UC}	Generation types subject to unit commitment
H	Operation year set. Index: $h = 1\ or\ 2$
J	State/Province set. Index: j
L	Line set. Index: l
L_C	Candidate lines
L_{EA}	Existing AC lines
L_{ED}	Existing DC lines
L_E	Existing Lines
L_B	Backbone reinforcement candidate lines
L_R	Renewable access candidate lines
L_B	Backbone Candidates, subject to KVL
L_H	Non-limited unidirectional fictitious hub lines, connecting conventional generation to the grid
P	Period set. Index: t
PA	Path/Flowgate set. Index: u
RP	REC trading path set. Index: p
RG	Reserve group set. Index: m
S	Scenario set. Index: s

Parameters	**Description**
ACP_j^h	Alternate compliance payment rate for RPS, 100 \$/MWh
B_l	Susceptance of line l, GW
CX_l	Capital cost of line, Million \$
EM_g	Emission rate, Metric ton/MMBTU
$\overline{F}_l$	Line thermal capacity, GW
FOM_g	Fixed O&M cost, Million \$/GW-year
FOR_g	Forced outage rate
HW_t	Duration of t
$HR_{g,b}$	Heat rate of generation g on bus b, MMBTU/GWh
$IRPS_j$	Instate RPS requirement, as a fraction of total RPS
M_l	Large positive number, GW
MUT_g	Minimum up time, Hour
MDT_g	Minimum down time, Hour
OR_g	Operating Reserve limits as a fraction of capacity
$PW_{i,j}$	Population allocation weight per bus i to each state j
$QMIN_g$	Minimum run as a fraction of total capacity
RR_g	Ramp rate as a fraction of total capacity
$\overline{R_u}$	Existing Path rating/Flowgate limit, GW
$\overline{RX_l}$	Expansion of Path rating with candidate line l, GW
SUC_g	Startup cost, Million\$/GW
SDC_g	Shutdown cost, Million\$/GW
V^h	Number of years for operating stage h, $V^1 = 10, V^2 = 30$
$VOLL$	Value of load loss (100 Million \$/GWh)
$VOM_{g,b}$	Variable O&M cost, Million \$/GWh
$W_{b,g,t}$	Hourly capacity factors for wind solar and hydro
$Y_{b,g}^0$	Existing capacity at year 2014, GW
$\overline{YMAX_{b,g}}$	Maximum resource potential at bus b, technology g, GW
δ	Discount rate=0.05
$\Gamma_{b,m}$	Element of bus-operating reserve group incident matrix
$\Phi_{l,b}$	Element of line-node incidence matrix
$\Psi_{u,l}$	Element of Path/Flowgate-line incidence matrix
$\Omega_{p,j}$	Element of REC trading path-state incidence matrix

Parameters	**Description (scenario-based parameters)**
p_s	Probability of scenario s
CP_s	Carbon price per scenario, Million \$/metric ton
$CY_{b,g,s}^h$	Capital cost of generation, Million \$/GW
$D_{b,t,s}^h$	Forecast demand, GW
$FP_{b,g,t,s}^h$	Fuel price of generation g on bus b, \$/MMBTU
$MC_{b,g,t,s}^h$	Generation marginal cost. $MC_{b,g,s,t}^h = FP_{b,g,s,t}^h HR_{g,b} + CP_s HR_{g,b} EM_g + VOM_{g,b}$, Million \$/GWh
$RPS_{j,s}^h$	Renewable obligation of state j
$WRPS_s$	Region-wise RPS renewable obligation
$YR_{b,g,s}^h$	Cumulative forced retirement/increase of generation in operating stage h, GW

Decision Variables	**Description**
$cap_{b,g,s}^h$	Generation capacity, GW
$f_{l,t,s}^h$	Power flow, GW, unrestricted
$k_{b,g,t,s}^h$	Generation production, GW, positive

$lav_{l,s}^h$	Line availability, defined on L_C
$n_{j,s}^h$	Noncompliance of renewable target, GWh, positive
$pmin_{b,g,t,s}^h$	Minimum run at hour t, GW
$pup_{b,g,t,s}^h$	Minimum run started up at hour t, GW
$pdn_{b,g,t,s}^h$	Minimum run shut down at hour t, GW
$q_{s,p}^h$	Renewable energy credit trading, GWh, positive
$r_{b,t,s}^h$	Load curtailment, GWh, positive
$\theta_{b,t,s}^h$	Voltage angle, Unrestricted
x_l^1	Here and now line decision, online at $h=1$, Binary for L_B, Continuous 0-1 for L_R
$x_{l,s}^2$	Wait and see line decision, online at $h=2$, Binary for L_B, Continuous 0-1 for L_R
$y_{b,g}^1$	Here and now generation expansion anticipation, online at $h=1$, GW, positive, only on G_I
$y_{b,g,s}^2$	Wait and see generation expansion anticipation, online at $h=2$, GW, positive, only on G_I
$z_{b,g,t,s}^h$	Operating Reserve, GW, positive

Objective functions

$$Min\ \sum_s p_s \left[\left(\frac{1}{1+\delta}\right)^{10}(I_s^1+O_s^1)+\left(\frac{1}{1+\delta}\right)^{20}(I_s^2+O_s^2)\right] \tag{2}$$

$$I_s^h=\sum_l CX_l x_{l,(s)}^h+\sum_g CY_{b,g,s}^h y_{b,g,(s)}^h \quad \forall s \tag{3}$$

$$OC_s^h=\sum_{v=1}^{V_h}\left(\frac{1}{1+\delta}\right)^{v-1}\sum_{b,g,t} MC_{b,g,t,s}^h HW_t k_{b,g,t,s}^h \quad \forall s,h \tag{4}$$

$$OC_s^h=\sum_{v=1}^{V_h}\left(\frac{1}{1+\delta}\right)^{v-1}\sum_{b,g,t} HW_t\left(MC_{b,g,t,s}^h k_{b,g,t,s}^h+\frac{\left(SUC_g pup_{b,g,t,s}^h+SDC_g pdown_{b,g,t,s}^h\right)}{QMIN_g}\right) \tag{5}$$

$\forall s,h, only\ in\ Unit\ Commitment$

$$OP_s^h=\sum_{v=1}^{V_h}\left(\frac{1}{1+\delta}\right)^{v-1}\left[\sum_{b,g,t} VOLLHW_t r_{b,t,s}^h+\sum_j ACP_j^h n_{j,s}^h\right]\forall s,h \tag{6}$$

$$OM_s^h=\sum_{v=1}^{V^h}\left(\frac{1}{1+\delta}\right)^{v-1}\sum_{b,g} FOM_g cap_{b,g,s}^h \quad \forall s \tag{7}$$

$$O_s^h=OC_s^h+OP_s^h+OM_s^h \quad \forall s,h \tag{8}$$

Constraints

$$cap_{b,g,s}^1=y_{b,g}^1, cap_{b,g,s}^2=y_{b,g}^1+y_{b,g,s}^2 \quad \forall b,g\in G_I,s \tag{9}$$

$$lav_{l,s}^1=x_l^1, lav_{l,s}^2=x_l^1+x_{l,s}^2 \quad \forall l\in L_C,t,s,h \tag{10}$$

$$cap_{b,g,s}^h=\left(Y_{b,g}^0-YR_{b,g,s}^h\right) \quad \forall b,g\in G_E,s,h \tag{11}$$

$$y_{b,g}^1+y_{b,g,s}^2\le \overline{YMAX_{b,g}} \quad \forall b,g,s \tag{12}$$

$$k_{b,g,t,s}^h+z_{b,g,t,s}^h\le\left(1-POR_g\right)\left(1-FOR_g\right)W_{b,g,t}cap_{b,g,s}^h \quad \forall b,g,t,s,h \tag{13}$$

$$z_{b,g,t,s}^h\le\left(1-POR_g\right)\left(1-FOR_g\right)R_g W_{b,g,t}cap_{b,g,s}^h\ \forall b,g,s,h,t \tag{14}$$

$$\sum_g k_{b,g,t,s}^h+r_{b,t,s}^h+\sum_l \Phi_{l,b} f_{l,t,s}^h-D_{b,t,s}^h=0 \quad \forall b,t,s,h \tag{15}$$

$$\sum_b \Gamma_{b,m} z_{b,g,t,s}^h\ge 0.05*\sum_b \Gamma_{b,m} D_{b,t,s}^h \quad \forall b,t,s,h \tag{16}$$

$$\left|\sum_l \Psi_{u,l} f_{l,t,s}^h\right|\le\left[\overline{R_u}+\sum_l \overline{RX_l} lav_{l,s}^h\right] \quad \forall u,t,s,h \tag{17}$$

$$r_{b,t,s}^h\le D_{b,t,s}^h \quad \forall b,t,s,h \tag{18}$$

$$\left|f_{l,t,s}^h\right|\le\overline{F_l} \quad \forall l\in L_E,t,s,h \tag{19}$$

$$\left|f_{l,t,s}^h\right|\le\overline{F_l}lav_{l,s}^h \quad \forall l\in L_C,t,s,h \tag{20}$$

$$f_{l,t,s}^h\ge 0 \quad \forall l\in L_H,s,t,h \tag{21}$$

$$\theta_{1,t,s}^h=0 \quad \forall t,s,h \tag{22}$$

$$B_l\sum_b -1\Phi_{l,b}\theta_{b,t,s}^h-f_{l,t,s}^h=0 \quad \forall l\in L_{EA},t,s,h \tag{23}$$

$$\left|B_l\sum_b -1\Phi_{l,b}\theta_{b,t,s}^h-f_{l,t,s}^h\right|\le M_l\left(1-lav_{l,s}^h\right) \quad \forall l\in L_{FB},t,s,h \tag{24}$$

$$\sum_j\left(\sum_{g\in G_R,t,b} HW_t PW_{b,j} k_{b,g,t,s}^h+n_{j,s}^h\right)\ge WRPS_s\sum_{j,b,t} HW_t PW_{b,j}\left[D_{b,t,s}^h-r_{b,t,s}^h\right] \quad \forall s,h \tag{25}$$

$$\sum_{g\in G_R,t,b} HW_t PW_{b,j} k_{b,g,t,s}^h+\sum_p \Omega_{p,j} q_{s,p}^h+n_{j,s}^h\ge RPS_{j,s}^h\sum_{b,t} HW_t PW_{b,j}\left[D_{b,t,s}^h-r_{b,t,s}^h\right]\forall s,h,j \tag{26}$$

$$\sum_{g\in G_R,t,b} HW_t PW_{b,j} k_{b,g,t,s}^h+\sum_p \Omega_{p,j}^{<0} q_{s,p}^h+n_{j,s}^h\ge IRPS_j RPS_{j,s}^h\sum_{b,t} HW_t PW_{b,j}\left[D_{b,t,s}^h-r_{b,t,s}^h\right]\forall s,h,j \tag{27}$$

$$pmin_{b,g,t,s}^h\le QMIN_g cap_{b,g,s}^h \quad \forall b,g\in G_{UC},s,h,t \tag{28}$$

$$pmin_{b,g,t+1,s}^h-pmin_{b,g,t,s}^h=pup_{b,g,t+1,s}^h-pdn_{b,g,t+1,s}^h \quad \forall b,g\in G_{UC},s,h,t \tag{29}$$

$$k_{b,g,t,s}^h+z_{b,g,t,s}^h\le\left(1-POR_g\right)\left(1-FOR_g\right)W_{b,g,t}\left(\frac{pmin_{b,g,t,s}^h}{QMIN_g}\right) \quad \forall b,g\in G_{UC},s,h,t \tag{30}$$

$$z_{b,g,t,s}^h\le\left(1-POR_g\right)\left(1-FOR_g\right)W_{b,g,t}R_g\left(\frac{pmin_{b,g,t,s}^h}{QMIN_g}\right) \quad \forall b,g\in G_{UC},s,h,t \tag{31}$$

$$\left(k_{b,g,t+1,s}^h+z_{b,g,t+1,s}^h-pmin_{b,g,t+1,s}^h\right)-\left(k_{b,g,t,s}^h-pmin_{b,g,t,s}^h\right)\le RR_g\left(\frac{pmin_{b,g,t,s}^h}{QMIN_g}\right)\forall b,g\in G_{UC},s,h,t \tag{32}$$

$$\left(k_{b,g,t+1,s}^h-pmin_{b,g,t+1,s}^h\right)-\left(k_{b,g,t,s}^h-pmin_{b,g,t,s}^h\right)\ge -RR_g\left(\frac{pmin_{b,g,t+1,s}^h}{QMIN_g}\right) \quad \forall b,g\in G_{UC},s,h,t \tag{33}$$

$$k_{b,g,t,s}^h\ge pmin_{b,g,t,s}^h \quad \forall b,g\in G_{UC},s,h,t \tag{34}$$

$$pmin_{b,g,t,s}^h\le QMIN_g cap_{b,g,s}^h-\sum_{\{t-MDT_g+1\le t'\le t\}} pdn_{b,g,t',s}^h \ \forall b,g\in G_{UC},s,h,t \tag{35}$$

$$\sum_{\{t-MUT_g+1\le t'\le t\}} pup_{b,g,t',s}^h\le pmin_{b,g,t,s}^h \ \forall b,g\in G_{UC},s,h,t \tag{36}$$

$$k_{b,g,t,s}^h-pdn_{b,g,t+1,s}^h\le\left(1-POR_g\right)\left(1-FOR_g\right)\frac{pmin_{b,g,t+1,s}^h}{QMIN_g} \quad \forall b,g\in G_{UC},s,h,t \tag{37}$$

$$k_{b,g,t+1,s}^h+z_{b,g,t+1,s}^h-pup_{b,g,t+1,s}^h\le\left(1-POR_g\right)\left(1-FOR_g\right)\frac{pmin_{b,g,t,s}^h}{QMIN_g} \quad \forall b,g\in G_{UC},s,h,t \tag{38}$$

Objective Function: Equation (2) to (8) show the objective function of JHSMINE. (2) documents that JHSMINE is minimizing the probability weighted system cost at present value, and that the system cost is composed of investment cost and operation cost. (3) is the overnight (transmission and generation) investment cost. (4) and (5) are the electricity production cost, with and without unit commitment enhancement respectively. (6) is the loss-of-load cost plus RPS non-compliance penalty. (7) is the fixed O&M cost for the generation capacity. The discounted summations of (3-7) imply that each operating year is assumed to be repeated 10 and 30 times for $h=1,2$ respectively. (8) is the summation of cost for each operating year.

Generation and transmission operation constraints: Constraints (9-24) are constraints simulating generation and transmission operation per hour per scenario. Constraints (9-10) and (11) track the generation capacity of candidate generation and existing generation respectively. (12) is limit upon generation expansion. (13) and (14), respectively, are the operating limits on generation and spinning reserve. (15) is Kirchhoff's Current Law. (16) is the spinning reserve constraint of each area. (17) is the flowgate or path limits. (18) is the load curtailment limit. (19-20) are thermal limits for existing lines and candidate lines respectively. (21) is the unidirectional limit for hub lines, which connects the conventional generation hub to every bus in the same path region. (See additional discussion in the body of the paper). Constraints (22-24) are Kirchhoff's Voltage Law in the DC "B-theta" form, with (24) being the disjunctive constraints for transmission candidates.

RPS policy: Constraints (25-27) are RPS constraints with renewable electricity credit (REC) trading. (25) is the general RPS constraint for the whole WECC. (26) is the RPS requirement for each state, while (27) is the in-state RPS requirement. These three constraints are imposed for each region and each year, specifying that all renewable generation is above certain amount of total load. For (26), each state has the ability to sell or buy RECs from other states (if allowed to do so). In (27), the RECs sold to other state are subtracted from the total renewable generation in the states, and bought RECs are not allowed to be used to comply with in-state RPS requirements.

Linearized unit commitment: (28-38) are linearized unit commitment constraints. We define $pmin_{b,g,t,s}^{h}$ as the minimum capacity that is in operation for each generator, so that the maximum capacity that is in operation for each generator is $\frac{pmin_{b,g,t,s}^{h}}{QMIN_g}$. Thus, the upper limit for $pmin_{b,g,t,s}^{h}$ is $QMIN_g cap_{b,g,s}^{h}$, as shown in (28). (29) is the start-up and shut down constraints. (30) and (31) are generation capacity and reserve limit for generators that are limited by unit commit problem. (32) is the ramp up constraint: we assume that newly started up capacity at $(t+1)$ cannot be ramped up, thus the amount of generation that can be ramped up is limited by the previous capacity. (33) is the ramped down constraint: we assume that newly shut down capacity at $(t+1)$ must be at minimum run at the beginning of $(t+1)$, and cannot provide ramp down capacity. Thus, the free capacity (able to be ramped down), excluding the shutdown part, is just the current capacity. (34) is minimum run limit. (35) and (36) are minimum shut down time and minimum start up time for generators. (35) is showing the maximum capacity that can be started up is limited by some shut-down decision in the previous hours. For example, if minimum shutdown time is 2 hours, for $t = 3$, this constraint becomes

$$pmin_{b,g,3,s}^{h} \leq QMIN_g cap_{b,g,s}^{h} - pdn_{b,g,2,s}^{h} - pdn_{b,g,3,s}^{h}$$

which is exactly showing, only the shutdown decision at hour 2 and 3 will affect the minimum run at 3, shut down decision at $t = 1$ has no effect on this constraints. Similar deduction can be done at constraints (36). (37) guarantees that if shut down $pdn_{b,g,t+1,s}^{h}$ happens at $t+1$, then this part of capacity must be operated at minimum run (equals to $pdn_{b,g,t+1,s}^{h}$) in previous hour. Thus, upper limit of capacity that can be operated is at t is just

$$(1 - POR_g)(1 - FOR_g)\frac{pmin_{b,g,t+1,s}^{h}}{QMIN_g}.$$

Similarly, in (38), if $pup_{b,g,t+1,s}^{h}$ happens at $t+1$, then the maximum electricity provided (excluding the newly started up capacity) by this generator at $t+1$ is actually the maximum capacity at t, the previous one. All constraints that is with chronological feature, i.e., t and $t+1$, are implemented in a snake-biting-tail way, which is to say, the next hour past $t = 24$ for one day will be $t = 1$.