Appendix To
Negative Bidding by Wind: A Unit Commitment Analysis of Cost and Emissions Impacts

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This Appendix documents the unit commitment model we used in the paper.

1. NOMENCLATURE

Indices

t, τ: Index of time period (dispatch interval)

g: Index of generator

n: Index of starting step, n=1,2, 3, …, T_g

Decision Variables

$curt_t$: Amount of wind power curtailment at time t [MW]

$emis_{C_{g,t}}$: CO_2 [ton] emissions of generator g at t

$emis_{S_{g,t}}$: SO_2 [pound] emissions of generator g at t

$emis_{N_{g,t}}$: NO_x [pound] emissions of generator g at t

$k_{g,t}$: 0/1 variable indicating if the generator g has been offline for more than a certain amount of time Ω_g at t

pns_t: Amount of demand not supplied at time t [MW]

$o_{g,t}$: 0/1 variable indicating if g is off (1) or not (0) at t

$q_{g,t}$: Output of generator g at time t [MW]

$q'_{g,t}$: Output of generator g in excess of $QMIN_g$ at t [MW]

$q''_{g,t}$: Output of g below the min run level $QMIN_g$ at t [MW]

$sc_{g,t}$: Start-up cost incurred for generator g at t [$]

$w_{g,t}$: 0/1 variable indicating if g is starting up or not at t

$z_{g,t}$: 0/1 variable indicating if g is committed or not at t

Parameters

A_g: Marginal fuel consumption, generator g [MMBtu/MWh]

B_g: Fixed fuel consumption by g [MMBtu/h]

CC: Curtailment cost of wind (= -1*energy bid; so if bid is negative, this cost is positive) [$/MWh]

C_{pns}: Cost of unserved energy (1000 $/MWh)

DEM_t: Load at time t [MW]

EMC_g: CO_2 [ton/MMBtu] emission rate of generator g

EMS_g: SO_2 [pound/MMBtu] emission rate of generator g

EMN_g: NO_x [pound/MMBtu] emission rate of generator g

F_g: Primary fuel cost of generator g [$/MMBtu]

FS_g: Start-up fuel cost of generator g [$/MMBtu]

MD_g: Minimum down time of generator g [h]

P_C: Price of CO_2 [$/ton] emissions

P_S: Price of SO_2 [$/pound] emissions

P_N: Price of NO_x [$/pound] emissions

$QMAX_g$: Maximum output of generator g [MW]

$QMIN_g$: Minimum output of generator g [MW]

$Q^{su}_{g,n}$: Output of g during the n^{th} interval of the start-up [MW]

R_g: Multiplier for startup cost for generator g [-].

RD_g: Bound on downward ramp rate of generator g [MW/h]

RU_g: Bound on upward ramp rate of generator g [MW/h]

S_g: Start-up fuel use of generator g [MMBtu]

$SEMC_g$: Start-up CO_2 [ton] emissions of generator g [ton]

T_g: Start-up time of generator g [h]

$WIND_t$: Wind power production at time t if no curtailment takes place [MW]

Ω_g : A certain threshold value for offline time where the start-up costs increase by a certain factor [h]

2. UNIT COMMITMENT MODEL

A. Basic Model

The model is a mixed integer linear program with the following objective (1) and constraints (2)-(18).

$$Min\sum_t \left[CC\times curt_t + C_{pns}\times pns_t\right] + \sum_{g,t}\left[F_g\times\left(B_g\times z_{g,t} + A_g\times\left(z_{g,t}\times QMIN_g + q'_{g,t}\right)\right)\right] + \sum_{g,t}\left(emis_{C_{g,t}}\times P_C + emis_{N_{g,t}}\times P_N + emis_{S_{g,t}}\times P_S\right) + \sum_{g,t} sc_{g,t} \quad (1)$$

Subject to:

$$q_{g,t} = q''_{g,t} + q'_{g,t} \quad \forall g, \forall t \quad (2)$$

$$q''_{g,t} = z_{g,t}\times QMIN_g + \sum_n Q^{su}_{g,n}\times w_{g,t,n} \quad \forall g, \forall t, \forall n \in \left[1,2,3,\cdots T_g\right] \quad (3)$$

$$q'_{g,t} \le z_{g,t}\times[QMAX_g - QMIN_g] \quad \forall g, \forall t \quad (4)$$

$$z_{g,t} + o_{g,t} + \sum_n w_{g,t,n} = 1 \quad \forall g, \forall t \quad (5)$$

$$w_{g,t,n} = w_{g,t,n-1} \quad \forall g, \forall t, \forall n \in \left[2,3,\cdots T_g\right] \quad (6)$$

$$z_{g,t} \le z_{g,t-1} + w_{g,t-1,T_g} \quad \forall g, \forall t \quad (7)$$

$$o_{g,t} \le o_{g,t-1} + z_{g,t-1} \quad \forall g, \forall t \tag{8}$$

$$sc_{g,t} \ge \left(FS_g \times S_g + P_C \times SEMC_g + P_N \times SEMN_g + P_S \times SEMS_g \right) \times w_{g,t,1} \quad \forall g, \forall t \tag{9}$$

$$emis_{C_{g,t}} = EMC_g \times \left(B_g \times z_{g,t} + A_g \times (z_{g,t} \times QMIN_g + q'_{g,t}) \right) \tag{10}$$

$$emis_{S_{g,t}} = EMS_g \times \left(B_g \times z_{g,t} + A_g \times (z_{g,t} \times QMIN_g + q'_{g,t}) \right) \tag{11}$$

$$emis_{N_{g,t}} = EMN_g \times \left(B_g \times z_{g,t} + A_g \times (z_{g,t} \times QMIN_g + q'_{g,t}) \right) \tag{12}$$

$$q'_{g,t} - q'_{g,t-1} \le RU_g \quad \forall g, \forall t \tag{13}$$

$$q'_{g,t} - q'_{g,t-1} \ge -RD_g \quad \forall g, \forall t \tag{14}$$

$$o_{g,t+\tau} + o_{g,t-1} - o_{g,t} \ge 0 \quad \forall g, \forall t, \forall \tau \in \left[1, \cdots, \min\left(168 - t, MD_g - 1 \right) \right] \tag{15}$$

$$\sum_g q_{g,t} + WIND_t - curt_t = DEM_t - pns_t \quad \forall g, \forall t \tag{16}$$

$$WIND_t \ge curt_t \quad \forall t \tag{17}$$

$$DEM_t \ge pns_t \quad \forall t \tag{18}$$

The objective function, as shown in (1), is a cost function that includes fuel costs, costs of emissions, the penalty cost for curtailing wind energy and the cost of the non-served power, in case demand is not fully met by actual generation.

The motivation for using several variables rather than just one variable to express the generation output, as shown in (2), (3) and (4), is that it allows to represent specific start-up profiles of a generating unit in the model. We observed from the USEPA CEMS data that the heat input and generation have linear relationship when the generator is operated steadily (See Fig. 1 in the paper). However, this linear feature does not hold in the start-up period. Each generator has different start-up times and start-up ramp rates, and normally (especially for the coal generators) the heat input in the first several hours is for warming up without any generation output (see Fig. 2 in the paper). By using $w_{g,t,n}$ one can pin the non-zero, effective generation output values to a specific output $Q^{su}_{g,n}$ at each moment n of the start-up period, which allows the model to reflect the characteristics of different generators. The logic of this is further explained in Section B below.

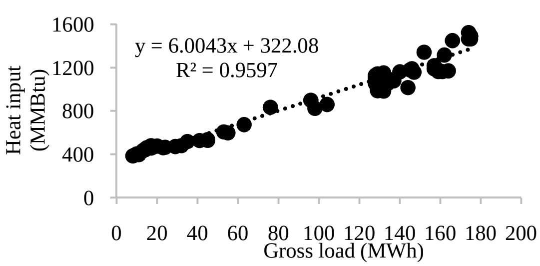


Fig. 1. The relationship between power generation and fuel consumption for an example thermal generation unit after start-up is completed

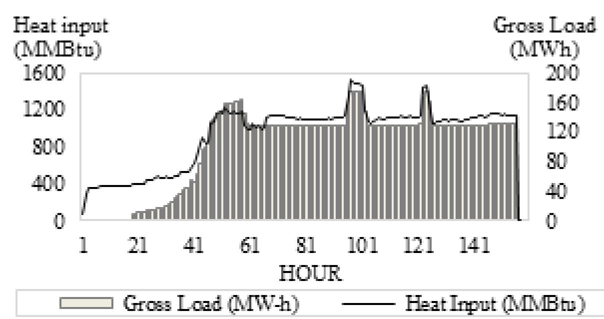


Fig. 2. Power generation and fuel consumption of a coal generator over time

A start-up time T_g of 3 hours is assumed in this model and thus the binary variables $w_{g,t,n}$ are limited to $w_{g,t,1}$, $w_{g,t,2}$ and $w_{g,t,3}$, although the model formulation accommodates more general start-up times. Any start-up time prior to those three hours is modeled by including it as part of minimum down time for the generator.

Equation (5), (6), (7), and (8) are used to ensure that a unit does not jump to normal operating mode (i.e., operation at *QMIN* or above before starting up) nor that it shuts down in the middle of a start-up. Several constraints are needed to enforce the logic and predetermined order of the binary variables.

Equation (9) is the start-up constraint. Every time a unit is turned on, a cost is added to the total cost function, represented by $sc_{g,t}$. This cost factor in combination with constraint (9) will be positive when $w_{g,t,1}$ becomes 1 at each start-up. The start-up cost consists of the amount of fuel S_g (MMBtu) that is burned until the unit has ramped up above minimum run level, multiplied with the cost of the start-up fuel FS_g (\$/MMBtu) (which might be different from the fuel used for operating above minimum run), and the cost of the emissions that are exhausted during the start-up, with P the price of the emissions and SEM the amount of emissions in tons (CO_2) or pounds (NO_x and SO_2). These start-up costs are added in the model only in the first hour of the start-up, but include the entire cost of the start-up period. Since there is no discounting, the time of occurrence does not matter. When analyzing the USEPA CEMS data, we noticed that units normally shut down immediately after producing at minimum run level or even at a higher output. Therefore, fuel costs and emissions during this short shutdown period (often less than 1 hour) can be neglected or considered included in the start-up cost.

Start-up fuel expenditure and emissions can vary greatly between cold starts and warm starts. However, the difference is not that obvious among the generators in the USEPA CEMS data base whose data we based the case studies upon. So we neglected the increase in fuel expenditures and emissions caused by longer downtimes of generators. But a more general model that includes the impact of downtime on start-up costs is presented in Section C, below.

CO_2 emissions ($emis_{C_{g,t}}$) when generators operate steadily show a linear relation with generation MW output and thus can be modeled as shown in (10), by multiplying the output by an emission rate EMC_g (ton/MMBtu). The same assumption and simplification method are applied also for NO_x and SO_2 emissions, and their emission equations are (11) and (12).

The hourly ramping rates of thermal units are limited by constraints (13) and (14), where RU_g is the maximum observed hourly difference in generation output when ramping up and RD_g is the maximum (negative) change when ramping down (MW/h).

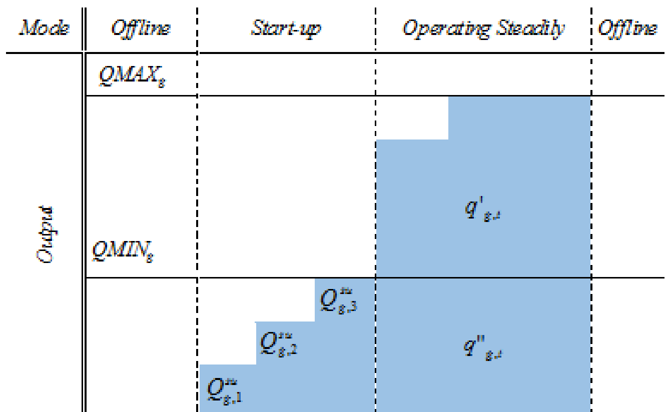
In this model there is also a downtime constraint, constraint (15) is included that requires units to remain off-line during a certain period of time MD_g (h) once it has been shut down in order to prevent boiler wear and damage. A minimum on-time constraint can also be included but has been disregarded in our paper.

Demand balance constraint (16) couples individual generator output together with the demand and available wind power. The power output of each generator together with the wind power generation $WIND_t$ in each period equals the demand DEM_t. In case of excess wind power, one might opt for curtailing wind ($curt_t$) at a certain cost, as shown in the objective function. Furthermore curtailment of electricity demand is also possible if capacity is inadequate, which results in non-served power (pns_t). Constraints (17) and (18) ensure that one cannot curtail more wind than available nor curtail more demand than possible.

B. *Generation during Start-up Periods*

The output $q_{g,t}$ of generator g in time period t is modeled as the sum of the output $q''_{g,t}$ below the minimum stable production level ($QMIN_g$) and the output $q'_{g,t}$ in excess of that load (see Fig. A-1). If the generator unit is offline, $z_{g,t}$ and $w_{g,t,n}$ are all zero, so that output $q_{g,t}$ is zero as well, based on (3) and (4). When starting up, $q''_{g,t}$ becomes positive and follows the predetermined output $Q^{su}_{g,n}$, as illustrated by Fig. A-1 for a start-up time of 3 hours. Once the unit reaches the minimum run level, $z_{g,t}$ becomes 1 and all $w_{g,t,n}$ are zero so that $q''_{g,t}$ constantly equals the minimum run level during operating mode.

In operating mode ($z_{g,t} = 1$), the generator's output cannot exceed its maximum capacity or be below its minimum level, as forced by (4). When the unit shuts down, $z_{g,t}$ becomes zero and $o_{g,t}$ jumps to 1.



$o_{g,t}$	1	1	0	0	0	0	0	0	1
$w_{g,t,1}$	0	0	1	0	0	0	0	0	0
$w_{g,t,2}$	0	0	0	1	0	0	0	0	0
$w_{g,t,3}$	0	0	0	0	1	0	0	0	0
$z_{g,t}$	0	0	0	0	0	1	1	1	0

Fig. A-1. Illustration of Logic of Binary Variables and Generation Output

C. *Start-up Constraints with Hot- and Cold-Starts*

Start-up times, fuel expenditure and emissions can vary greatly depending on the time that a unit is off-line. Even though off-line times are not included in the model in our paper, we would like to introduce the way of adapting the model to the need of introducing the effect of off-line time. Certainly for steam units that need to reach a suitable boiler pressure and temperature in order to operate, this factor is non-negligible. In our analysis of USEPA CEMS data, we observed that for some units, the start-up fuel and emissions can increase by a factor of 2 to 3 depending on the off time. Therefore, the factor $R_g \times k_{g,t}$ is added to (9) and the startup cost equation changes to (19) in order to take this effect into account. $k_{g,t}$ is a binary variable that indicates if generator g has been offline for more than Ω_g hours, where Ω_g is a certain threshold value where the start-up costs increase by a certain factor, based on the value of R_g. For example, if start-up costs increase by a factor of 2 when offline exceeds 100 hours, the parameter R_g equals 1 and Ω_g equals 100. The mechanism that enables the binary variable $k_{g,t}$ to be equal to 1 when offline reaches or exceeds Ω_g is modeled by (20). When $w_{g,t,1}$ jumps to 1 at a start-up, the off-line hours before period t are counted by summing up all the offline hours $o_{g,t}$ of the last Ω_g hours and compared to $(\Omega_g - 1)$. This difference is divided by a "big M" and when it is positive, $k_{g,t}$ is forced to be larger than a very small number, making it equal to 1 because it is defined as a binary variable. $k_{g,t}$ will always equal zero if $w_{g,t,1}$ is zero since adding a very small number to -1 will never give an outcome bigger than zero.

$$sc_{g,t} \geq \left(FS_g \times S_g + P_C \times SEMC_g + P_N \times SEMN_g + P_S \times SEMS_g\right) \times \left(w_{g,t,1} + R_g \times k_{g,t}\right) \quad \forall g, \forall t \tag{19}$$

$$k_{g,t} \geq -1 + w_{g,t,1} + \frac{\sum_{\tau=t-\Omega_g}^{t-1} o_{g,\tau} - \left(\Omega_g - 1\right)}{M} \quad \forall g, \forall t \tag{20}$$