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Near-Optimal Scheduling in Day-Ahead Markets: Pricing Models and Payment Redistribution Bounds

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***Abstract*—Near-optimal unit commitment (UC) scheduling is a practical reality in wholesale electricity markets. This paper revisits previous work that has found that minor differences in cost among near-optimal schedules can result in a large redistribution of market payments (i.e., changes in generator profits and consumer surplus). It has been believed that this instability is unavoidable, but previous studies have only calculated prices using what we call the Restricted pricing model. This paper compares previous results to three additional models that are based on integer relaxation, and which we call the Partial, Tight, and Loose Dispatchable pricing models. Results are presented for a suite of test cases, including four ISO-scale cases. Similar to previous findings, the Restricted and Partial Dispatchable models both result in large payment redistributions among alternative solutions. In contrast, theoretical and experimental results for the Tight and Loose Dispatchable models show that pricing models with unconditional integer relaxation will have bounded payment redistributions, and, further, this bound can become quite small by tightening the UC problem's convex relaxation. In the presence of market power, stable financial outcomes may improve market efficiency by reducing incentives to bid strategically.**

***Index Terms*—Unit commitment, nonconvex pricing, mixed integer programming, market design**

I. Introduction

CHANGES to traditional pricing methodologies in electricity markets continue to stir controversy. Wholesale electricity markets are often conceptualized as a uniform price auction, such that setting the price equal to the marginal system cost provides the correct incentives for all participants to produce and consume electricity at their socially efficient levels. However, the optimization problem used to schedule market participants, called unit commitment (UC), includes important nonconvexities in the production capabilities of many generating facilities. Uniform prices are not guaranteed to clear the market in this circumstance [1]. The market settlement often includes side-payments to ensure that generators do not suffer financial losses by following the socially efficient schedule [2] as well as rules to discourage production from generators who are not part of the least-cost schedule.

Thus, the crux of the pricing controversy is whether to adhere to the usual marginal pricing policy, or if an alternative pricing scheme with somehow better incentives can be formulated and adopted. These pricing schemes are implemented by first obtaining a physical schedule (i.e., production quantities) and then executing a separate pricing model. Most ISOs now have implemented some version of this two-step procedure. This paper illustrates how different pricing models affect the market settlements of sub- and near-optimal UC schedules.

Price formation issues attracted interest from FERC following severe weather events in the winter of 2014-2015. Those events highlighted the role of prices in aligning dispatch incentives, maintaining reliability, signaling efficient investments, and maximizing the market surplus [3]. A subsequent Notice of Proposed Rulemaking (NOPR) highlighted the inclusion or exclusion of nonconvexities in pricing methodologies, i.e., start-up and no-load operating costs, minimum output levels, and minimum run times.

This NOPR proposed to create uniform "fast-start" pricing rules for resources with quick response times [4]. Such resources are typically "block-loaded," operated at full capacity or not at all, and thus unable to set prices when the normal marginal cost criterion is used. All ISOs currently implement some form of fast-start pricing, but to varying degrees based on their resource mix and compatibility with existing ancillary service markets [4]–[10]. Rather than pursuing uniform rules in all six ISOs and RTOs under FERC's jurisdiction, FERC concluded the NOPR by opening new dockets to examine specific pricing rules for New York Independent System Operator (NYISO), PJM Interconnection (PJM), and Southwest Power Pool (SPP) [4]. While these issues are relevant in both the US and Europe, US markets solve nonconvexities in a centralized fashion whereas European markets require participants to internalize nonconvexities in their offer (with some exceptions, e.g., minimum income constraints in the Spanish electricity market [11]). This paper focuses on the treatment of nonconvexities as now undertaken in US markets.

The main contribution of this paper is to relate Gribik, Hogan, and Pope's initial paper on convex hull pricing [12] to an issue first discussed by Johnson *et al.* [13], and later by Sioshansi *et al.* [14], concerning the stability of financial outcomes in markets based on centralized UC. This paper measures stability, or rather instability, by the sum of absolute deviations of generator profits and consumer surplus compared to what would have occurred in an optimal UC schedule. This sum is referred to as the *payment redistribution quantity* throughout the paper. We address a previously unappreciated property, that convex hull pricing minimizes a bound on this redistribution quantity and thus stabilizes financial outcomes. Our results have significant implications in the ongoing elec-

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tricity pricing debate and, to our knowledge, have not been recognized previously in the convex hull pricing literature.

The paper is organized as follows. Section II provides additional background about the UC problem and then formulates a standard UC model and four pricing models. Section III derives upper bounds on the payment redistribution quantity that applies to two of the pricing models. Section IV illustrates these bounds in a simple example, and Section V demonstrates that the theoretical results are meaningful for a suite of larger test cases. Section VI concludes the paper.

II. Unit Commitment and Pricing

Fast-start pricing [4] is a specific instance of the nonconvex pricing problem, for which there is generally no completely accepted method for pricing and settlements. Difficulties in resolving nonconvex pricing issues stem from the presence of *lumpiness* or *indivisibilities* in the production sets of electric generators [1]. Examples of common instances include:

- has a minimum output constraint such that it cannot feasibly produce power at a level less than some threshold value, unless it produces exactly zero,
- incurs fixed costs that are required to begin producing power but are otherwise independent of the amount of power produced, or
- must remain on-line or off-line for a specified amount of time before shutting off or coming back on-line.

Rather than being rare or pathological examples, the above features are common to most thermal generating units. These nonconvexities can cause the need for side-payments, called "make-whole" payments, that are paid to participants who are part of the optimal UC schedule but do not recover their offered cost through the uniform energy price.

The standard market settlement, formally presented in [2], includes a locational marginal price-based energy payment and a make-whole payment to ensure recovery of as-bid costs. This has been called the "Restricted" model [12] since the marginal price is calculated by the dual problem of a linear program that fixes all binary variables to their value in the optimal UC schedule. One of the objections to this approach is that it may result in large make-whole payments, which is believed to distort market entry incentives [15]. Side-payments may also present incentives to distort supply offers, such as the well-known exercise of market power by JP Morgan in California that resulted in a $410 million settlement [16].

Reduction or elimination of these side-payments (which may be make-whole payments or may fall into a broader category of "uplift" payments) has been proposed through various optimization models [17]–[21] and equilibrium models [22]–[24]. Some of these methods ensure that the optimal primal solution is supported, but others can result in a changed and possibly suboptimal schedule [23], [24]. Market efficiency is a primary goal of federal policy, so US regulators are unlikely to approve schemes of the latter sort.

Of the alternatives to the Restricted model, convex hull pricing has attracted the most attention. This approach aims to minimize uplift payments payments that are based on lost opportunity costs and defined by individual profit maximization subproblems. These payments can are minimized by solving a computationally intensive Lagrangian dual problem [12], but it is not clear whether the minimization of wake-whole payments or lost-opportunity cost payments necessarily improves market efficiency [25].

Schiro *et al.* [25] describe hurdles to implementation of convex hull pricing. For example, its properties cannot be guaranteed because the convex hull prices are difficult to calculate accurately in realistic market scheduling problems [25]. Instead of solving the Lagrangian dual directly, we use a computationally efficient primal approach by implementing tight UC constraints from [26]–[29]. Tightening the UC problem to approach a primal convex hull formulation can become increasingly complex as additional time periods and resource details are considered. Examples of such details include combined cycle gas turbine (CCGT) transitions, time-dependent start up costs, hot- and warm-start-up types, and other operational details that won't be the focus of this paper. Ref. [30] discusses UC formulations in more detail.

The computational complexity of the UC problem often prevents ISOs from providing a provably optimal UC schedule. Pricing issues stemming from sub-optimal UC solutions were first identified by Johnson *et al.* [13], which shows that solving the UC schedule by Lagragian relaxation could result in multiple near-optimal UC schedules, each with significantly different prices. Sioshansi *et al.* [14] showed that the same problem occurs in more efficient mixed integer programming software, similar to what markets use today. Both papers question if this undermines incentives for participation in centralized UC since the financial outcomes can be heavily influenced by an arbirtrary choice made by the ISO [13], [14].

Good market design is multifaceted and requires careful analysis and balancing of a wider range of issues than are discussed here. The primary market design objective is usually to maximize social welfare (i.e., market surplus) [3]; any other objective is difficult to justify since the potential benefits may entail reducing social welfare. Efficient market design maximizes social welfare and depends largely on the incentive properties of the pricing mechanism, including short-term incentives for participation in day-ahead and real time markets as well as long-term incentives to retire old resources or invest in new ones. Additional design objectives might be important but are difficult to quantify, such as transparency, simplicity, fairness, or other stakeholder concerns. To date, there is no consensus that any centralized UC pricing mechanism performs best in all relevant design criteria [19]–[21], [25].

This paper does not propose a new market design, but rather contributes to the understanding of the properties of convex hull pricing and the effect of recently proposed pricing methodologies on the problems described by Johnson *et al.* [13] and Sioshansi *et al.* [14]. In particular, both papers state that the problems in electricity pricing stem from the use of centralized UC to achieve an efficient schedule. Our analysis shows that the magnitude of this problem depends crucially on the pricing methodology employed and is not an immutable property of centralized UC itself.

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A. Models

The scheduling software used by ISOs solves a MIP for a near-optimal UC schedule. Each day, ISOs collect bids and offers that define consumer valuations and producer costs, respectively, and are used to calculate price and quantity schedules. Demand is commonly assumed to be fixed, in which case minimizing production costs is equivalent to maximizing the market surplus. The UC model below follows this convention, but easily generalizes to include the case where demand plays an active role in the market.

$$\min \quad z = c^\top p + f^\top u \tag{1a}$$

$$\text{s.t.} \quad Ap \geq d \tag{1b}$$

$$Bp + Du \geq b \tag{1c}$$

$$u \in \{0, 1\} \tag{1d}$$

The model (1) minimizes total production costs z, given system constraints (1b) and generator constraints (1c) and (1d). Constraint (1b) is formulated so that all theoretical results in this paper can accommodate any linear equality (e.g., energy balance) or inequality (e.g., transmission and ancillary service) system constraints. Decision variables are generation quantities, p; and binary commitments, u. Parameters include marginal costs, c; fixed costs, f; system constraint matrix, A; energy demand and transmission capacity, d; generator constraint matrices, B and D; and generator limits, b. Price-responsive demand can be included by allowing some entries of p to be negative and defining the respective entries of c and f to be the marginal and fixed value of energy consumption, respectively. In that case, the model is equivalent to maximizing the market surplus. The index $n \in \mathcal{G}$ may be used to reference the vector and matrix entries that refer to individual generators. Feasible integer solutions to (1) will be denoted by (p^s, u^s), $s \in \mathcal{S}$, and $s = *$ will denote an optimal solution.

Given the time constraints of setting up and running a day-ahead market, the ISO's scheduling software either terminates as soon as a solution is proven to be within a tolerance of the optimal solution, and/or after it reaches a maximum time limit. The optimality tolerance is determined by a lower bound on the optimal solution's cost, $z^{LB} \leq z^*$. A solution to (1) is optimal if $z^s = z^{LB}$ or near-optimal if $z^s \leq (1 + \alpha)z^{LB}$, where $\alpha > 0$ can be any pre-determined optimality tolerance. It is often impractical for the software to verify optimality, so it is possible that a near-optimal solution could be optimal. We define the optimality gap δ^s_{opt} and MIP gap δ^s_{mip} as follows.

$$\delta^s_{\text{opt}} := z^s - z^* \leq z^s - z^{LB} =: \delta^s_{\text{mip}} \tag{2}$$

Since problem (1) is a MIP, there is no standard dual problem definition to calculate shadow prices [31]. Instead, ISOs calculate prices using convexified versions of (1). This paper compares prices from four convex pricing models, which we call Restricted (r), Partial Dispatchable (pd), Tight Dispatchable (td), and Loose Dispatchable (ld).

Each pricing model differs in constraints (1c) and (1d), as shown in Table II-A. Tight implementations for (1c) include minimum up-time and down-time [26], two-period ramp inequalities [27], variable upper bounds [28], and a convex envelope of the cost function [29]. The formulations in Ref. [32] do not affect the feasible region of (1) if u is binary, but could result in a larger feasible region if a continuous relaxation is applied to u. The r model results in the same prices for either of these two formulations.

TABLE I
PRICING MODEL DEFINITIONS

Model	Description	(1c) Formulation	(1d) Substitution
r	Restricted	[26]–[29]	$u = u^s$
pd	Partial Dispatchable	[26]–[29]	$0 \leq u \leq u^s$
td	Tight Dispatchable	[26]–[29]	$0 \leq u \leq 1$
ld	Loose Dispatchable	[32]	$0 \leq u \leq 1$

Prices in the four convex primal models are given by the dual variables to (1b), λ. In addition, we will denote ch for the true convex hull price derived by the Lagrangian dual formulation proposed in [12], and we denote each price by $\lambda^{ms}, m \in \{\texttt{r}, \texttt{pd}, \texttt{td}, \texttt{ld}, \texttt{ch}\}$, respectively. The dual problem of each pricing model constrains $\lambda \geq 0$ [33], but note that generator n's energy payment, $\lambda^\top A_n p_n$, and consumer i's energy charge, $\lambda_i d_i$, could be either positive or negative.

Prices are set by by the marginal cost of online resources in the r model. By relaxing u's binary constraints, the pd model extends price-setting to reflect fixed costs of online resources, while td and ld models extend price-setting to include the fixed costs of all resources, regardless of commitment status in the selected schedule. Each of the pricing models are convex, but only the td and ld models are convex relaxations of (1).

The r model, formally described in [2], is considered standard practice in day-ahead markets. The td and ld models are two implementations of the 'Dispatchable' model introduced in [12], but they differ in how closely the generator constraints approximate the convex hull of the UC problem. The most notable adoption of this type of pricing is the Extended Locational Marginal Price (ELMP) in MISO [7], [25], which only permits price-setting by uncommitted resources in a narrow set of circumstances [7], and therefore resembles the pd model. The effect on market outcomes of conditional or unconditional integer relaxation has not previously been discussed in the literature and is part of our motivation for comparing these four pricing models.

A comprehensive review of various convex hull pricing implementations is beyond the scope of this paper, but references [25] and [29] provide overviews of convex hull pricing formulations and properties. The rules of each ISO market include many idiosyncrasies and are reviewed in [34]. Additional pricing proposals from the academic literature are reviewed in [21].

B. Side-Payment Policies

In the absence of side-payments, generators receive *quasi*-linear[1] profits π^{ms}_n, also referred to as "linear" profits.

$$\pi^s_n(\lambda) = (A^\top_n \lambda - c_n)^\top p^s_n - f^\top_n u^s_n \tag{3}$$

[1] Quasi-linearity denotes that revenues, $(\lambda^{ms})^\top p_n$, are linear and and costs, $c_n p_n + f_n z_n$, are nonlinear, both with respect to production level p_n.

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Due to the non-convex nature of UC, it often occurs that a generator's socially optimal schedule does not maximize its linear profit [1]. That is, given a UC solution s and a price vector λ, generator n may incur a lost opportunity cost, $U_n^s(\lambda)$, defined by the following profit maximization problem.

$$U_n^s(\lambda) = \sup_{(p_n,u_n)\in\chi_n} \{(A_n^\top\lambda - c_n)^\top p_n - f_n^\top u_n\} - \pi_n^s(\lambda) \tag{4}$$

where $\chi_n := \{B_n p_n + D_n u_n \geq b_n, u_n \in \{0,1\}\}$ is the set of generator n's internal constraints.

The derivation of convex hull pricing is based on minimization of the sum of lost opportunity cost payments [12]. An important rationale for paying lost opportunity costs to market participants is to ensure that the optimal commitment and dispatch are supported, i.e., that generators cannot profitably deviate from the ISO's schedule. However, consumers may have reasonable objections to being charged for lost opportunity costs. Lost opportunity costs payments may become very large if the market contains large nonconvexities [35] or if the convex hull price is poorly approximated [25], and such payments could go to unscheduled generators [25]. Unfortunately, a perfect resolution of all market participant desires may be unattainable in nonconvex markets [1].

The standard practice in ISOs is does not pay full lost opportunity costs, but only the portion of any scheduled generator's as-bid costs that is in excess of its energy market revenues. The make-whole payment, $\mu_n^s(\lambda)$, is defined as follows.

$$\mu_n^s(\lambda) = \max\{0, -\pi_n^s(\lambda)\} \tag{5}$$

Since producing nothing is a feasible solution to (4), $0 \leq \mu_n^s(\lambda) \leq U_n^s(\lambda)$. Actual profits will be distinguished from linear profit with a tilde.

$$\tilde{\pi}_n^s(\lambda) = \pi_n^s(\lambda) + \mu_n^s(\lambda) \tag{6}$$

An additional component of uplift, called Product Revenue Shortfall (PRS), can be required if the sets of binding inequality (e.g., transmission and/or ancillary services) constraints are different in the dispatch and pricing models [25], [36]. PRS charges perform a similar function to make-whole payments, preventing the underfunding of ancillary service providers and Financial Transmission Rights (FTR) holders [36]. This component is equal to $\lambda^\top(Ap - d)$ and is nonnegative since $\lambda \geq 0$ (from the pricing model dual) and $Ap \geq d$ (from (1)).

The following results will assume that charges for all generator side-payments and PRS will collected *pro rata* from consumers. Section III assumes only that $0 \leq \mu_n^s(\lambda) \leq U_n^s(\lambda)$, so the theoretical results will apply to a very generic side-payment policy. Numerical results in Section V will reflect side-payments only from the make-whole definition (5), consistent with current practice in most ISOs.

III. Theoretical Results

This section introduces bounds on the stability of individual profits when computational constraints prevent the calculation of an optimal UC schedule. We first prove a theorem that applies to any class of pricing mechanisms that uses the same price vector λ to settle both the optimal and near-optimal UC schedule. A corollary applies to any pricing model that is a convex relaxation of (1), such as the `td` and `ld` pricing models. Like [13], [14], we are unable to determine stability bounds for pricing models that depend upon the UC solution. Thus, the following theoretical results do not generally apply to the `r` and `pd` pricing models unless both UC solutions are known to result in the same λ, as shown in Section IV.

Changes to prices, commitment decisions, and make-whole payments all affect the change financial outcomes due to the suboptimality of the UC solution. We will assume that (1) accurately portrays the optimal operation of the power market. In actual operations, however, market operators will often manually adjust market schedules due to renewable energy forecast uncertainty or in order to satisfy reliability concerns, and such adjustments may complicate the market settlements even further [37].

The change in generator profits δ_n^{ms}, consumer surplus δ_{cs}^{ms}, and the solution cost δ_{obj}^{ms} are related by the balance equation.

$$\sum_n \delta_n^{ms} + \delta_{\text{cs}}^{ms} + \delta_{\text{obj}}^{s} = 0 \tag{7}$$

where,

$$\delta_n^{ms} := \tilde{\pi}_n^s(\lambda^{ms}) - \tilde{\pi}_n^*(\lambda^{m*}) \tag{8a}$$

$$\delta_{\text{cs}}^{ms} := (Ap^*)^\top\lambda^{m*} - (Ap^s)^\top\lambda^{ms} + \sum_n (\mu_n^*(\lambda^{ms}) - \mu_n^s(\lambda^{m*})) \tag{8b}$$

$$\delta_{\text{obj}}^{s} := z^s - z^* \tag{8c}$$

The quantity δ_{cs}^{ms} reflects not only changes to the consumer's direct energy payment, $\lambda^\top d$, but also changes to the PRS and any make-whole payments since these items are typically assumed to be allocated *pro rata* to consumers. The payment redistribution quantity, Δ^{ms}, is defined below.

$$\Delta^{ms} := \left|\delta_{\text{cs}}^{ms}\right| + \sum_n \left|\delta_n^{ms}\right| \tag{9}$$

The Lagrange function plays a key role in the theoretical results, and is defined below.

$$L(\lambda) = \inf_{(p,u)\in\chi} \{c^\top p + f^\top u + \lambda^\top(d - Ap)\} \tag{10}$$

As shown in pages 28-29 of [12], the Lagrange function is directly related to the total lost opportunity cost and PRS of any arbitrary integer UC solution, derived as follows.

$$\sum_n U_n^s(\lambda) + \lambda^\top(Ap^s - d) \tag{11a}$$

$$= \sup_{(p,u)\in\chi} \{(A^\top\lambda - c)^\top p - f^\top u\} - (A^\top\lambda - c)^\top p^s + f^\top u^s + \lambda^\top(Ap^s - d) \tag{11b}$$

$$= \sup_{(p,u)\in\chi} \{(A^\top\lambda - c)^\top p - f^\top u\} + c^\top p^s + f^\top u^s - \lambda^\top d \tag{11c}$$

$$= -\inf_{(p,u)\in\chi} \{c^\top p + f^\top u + \lambda^\top(d - Ap)\} + z^s \tag{11d}$$

$$= z^s - L(\lambda) \tag{11e}$$

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Convex hull prices are defined by $\lambda^{\text{ch}} := \arg\max_{\lambda} L(\lambda)$ to minimize the sum of generator uplifts, $U_n^s(\lambda)$, and the PRS [12]. The resulting prices are inherently independent of the UC solution. The `td` and `ld` pricing models are convex relaxations of (1) and therefore share this second property, even if they do not calculate the true convex hull price. The following lemma and theorem apply to any two UC schedules with market settlements determined by the same λ.

Lemma 1. *Let $(p^{s'}, u^{s'})$, $s' \in \{s, *\}$, be a near-optimal and an optimal solution to* (1)*, and let λ be a single price vector of appropriate dimension. Suppose generator profits are $\tilde{\pi}_n^{s'}(\lambda) = \pi_n^{s'}(\lambda) + U_n^{s'}(\lambda)$ and the total consumer payment is $\lambda^\top A p^{s'} + \sum_n U_n^{s'}(\lambda)$. Then, the redistribution quantity Δ^{ms},* (9)*, is exactly the cost gap with the optimal solution, δ_{obj}^s.*

Proof. From the definition of lost opportunity cost (4),

$$\pi_n^s(\lambda) + U_n^s(\lambda) = \sup_{(p_n, u_n) \in \chi_n} \{(A_n^\top \lambda - c_n)^\top p_n - f_n^\top u_n\} \tag{12}$$

The RHS is independent of the UC solution, so $\sum_n |\delta_n^{ms}| = 0$. Since $\delta_{\text{obj}}^s \geq 0$, then (7) implies the following.

$$\left|\delta_{\text{cs}}^{ms}\right| = \delta_{\text{obj}}^s \tag{13}$$

Consequently, Δ^{ms} is simply δ_{obj}^s. □

The next lemma calculates bounds for Δ^{ms} if market settlements include payment of any portion of the lost opportunity cost, such as the make-whole payments defined by (5) or any policy such that $0 \leq \mu_n^s(\lambda) \leq U_n^s(\lambda)$.

Theorem 1. *Let $(p^{s'}, u^{s'})$, $s' \in \{s, *\}$, be a near-optimal and an optimal solution to* (1)*, both feasible, and let λ be a single price vector of appropriate dimension. Suppose generator profits are $\tilde{\pi}_n^{s'}(\lambda) = \pi_n^{s'}(\lambda) + \mu_n^{s'}(\lambda)$, such that $0 \leq \mu_n^{s'}(\lambda) \leq U_n^{s'}(\lambda)$, and the total consumer payment is $\lambda^\top A p^{s'} + \sum_n \mu_n^{s'}(\lambda)$. Then, the redistribution quantity Δ^{ms} is upper bounded by $2\delta_{obj}^s + 4\left(z^* - L(\lambda)\right)$.*

Proof. The net change in generator profits, $\sum_n \delta_n^{ms}$, is first decomposed, resulting in the following equality.

$$\begin{aligned}
\sum_n \delta_n^{ms} \\
&= \sum_n \lambda^\top A_n (p_n^s - p_n^*) - c^\top (p^s - p^*) \\
&\quad - f^\top (u^s - u^*) + \sum_n \left(\mu_n^s(\lambda) - \mu_n^*(\lambda)\right) && \text{(14a)} \\
&= \lambda^\top A (p^s - p^*) - \delta_{\text{obj}}^s + \sum_n \left(\mu_n^s(\lambda) - \mu_n^*(\lambda)\right) && \text{(14b)}
\end{aligned}$$

Then, substituting from (7) and (14b) to rewrite $|\delta_{\text{cs}}^{ms}|$,

$$\begin{aligned}
\left|\delta_{\text{cs}}^{ms}\right| &= \left|\delta_{\text{obj}}^s + \sum_n \delta_n^{ms}\right| && \text{(15a)} \\
&= \left|\sum_n \left(\mu_n^s(\lambda) - \mu_n^*(\lambda)\right) + \lambda^\top A (p^s - p^*)\right| && \text{(15b)}
\end{aligned}$$

Bounds for $|\delta_{\text{cs}}^{ms}|$ come from applying the triangle inequality and adding $\lambda^\top d - \lambda^\top d = 0$ to the last quantity,

$$\begin{aligned}
\left|\delta_{\text{cs}}^{ms}\right| \leq \sum_n &\left|\mu_n^s(\lambda) - \mu_n^*(\lambda)\right| \\
&+ \left|\lambda^\top (A p^s - d) - \lambda^\top (A p^* - d)\right|
\end{aligned} \tag{15c}$$

Since make-whole payments and PRS are both nonnegative, the triangle inequality implies that,

$$\begin{aligned}
\left|\delta_{\text{cs}}^{ms}\right| \leq \sum_n &\left(\mu_n^s(\lambda) + \mu_n^*(\lambda)\right) \\
&+ \lambda^\top (A p^s - d) + \lambda^\top (A p^* - d)
\end{aligned} \tag{15d}$$

Finally, $\mu_n^s(\lambda) \leq U_n^s(\lambda)$, which leads to the following simplification from (11).

$$\begin{aligned}
\left|\delta_{\text{cs}}^{ms}\right| \leq \sum_n &\left(U_n^s(\lambda) + U_n^*(\lambda)\right) \\
&+ \lambda^\top (A p^s - d) + \lambda^\top (A p^* - d) && \text{(15e)} \\
&= z^s + z^* - 2L(\lambda) && \text{(15f)}
\end{aligned}$$

The absolute change in profits can be calculated with respect to the conditions in Lemma 1, using similar steps as before.

$$\begin{aligned}
\sum_n \left|\delta_n^{ms}\right| \\
&= \sum_n \left|0 + \left(\mu_n^s(\lambda) - U_n^s(\lambda)\right) - \left(\mu_n^*(\lambda) - U_n^*(\lambda)\right)\right| && \text{(16a)} \\
&\leq \sum_n \left(\left|\mu_n^s(\lambda) - U_n^s(\lambda)\right| + \left|\mu_n^*(\lambda) - U_n^*(\lambda)\right|\right) && \text{(16b)} \\
&\leq \sum_n \left(U_n^s(\lambda) + U_n^*(\lambda)\right) && \text{(16c)} \\
&= z^s + z^* - 2L(\lambda) && \text{(16d)}
\end{aligned}$$

Combining (15f) and (16d),

$$\Delta^{ms} \leq 2\delta_{\text{obj}}^s + 4(z^* - L(\lambda)) \tag{17}$$

□

Given any solution to (1), δ_{obj}^s is fixed, so convex hull pricing will minimize bound in (17). However, computing convex hull prices by minimizing $L(\lambda)$ is computationally difficult and may be impractical [12], [25]. Instead, a corollary of Theorem 1 provides bounds that can be computed without calculating either an optimal UC solution or the Lagrangian dual.

Corollary 1. *Let (p^s, u^s) be a primal solution to* (1) *with a MIP gap $\delta_{mip}^s = z^s - z^{LB} \geq z^s - z^*$, and m be a convex relaxation of* (1) *with a dual solution λ and integrality gap $\delta_{int}^m = z^s - z^m$. Then, the payment redistribution Δ^{ms} is upper bounded by $2\delta_{mip}^s + 4\delta_{int}^m$.*

Proof. The new bound can be found by substitution from (17). Strong duality and the convex hull price definition imply that $z^{\text{ch}} = L(\lambda^{\text{ch}})$, where z^{ch} is the maximal objective function of the all convex relaxations of (1). Since $z^{LB} \leq z^* \leq z^s$ and $z^{\text{m}} \leq z^{\text{ch}}$, all of which are true by definition, then Δ^{ms} is also bounded by the following.

$$\Delta^{ms} \leq 2(z^s - z^{LB}) + 4(z^s - z^m) \tag{18}$$

□

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The theoretical results are summarized as follows. Lemma 1 shows that the redistribution quantity will be exactly equal to the cost gap if prices are independent of the UC solution and the market pays all lost opportunity cost payments. Meanwhile, Theorem 1 is a more general result that derives bounds on the redistribution quantity given any side payment definition up to the full lost opportunity cost, e.g., the typical make-whole payment policy. Finally Corollary 1 provides a relaxed bound that can be easily computed.

Following the convention of previous works [13], [14], the redistribution quantity is defined in terms of comparing a suboptimal UC solution with an optimal one. However, the same arguments in Lemma 1, Theorem 1, and Corollary 1 can also be immediately applied to the redistribution between any arbitrary pair of UC solutions by making the appropriate substitutions. That is, none of the steps rely on the optimality of z^*, but only on the fact that $z^* \leq z^s$.

Theorem 1 applies to any general price vector λ so long as it determines market settlements in both alternative schedules. In addition, it implies that a redistribution of payments may only arise if there is a nonzero optimality gap and/or duality gap. If there is only an optimality gap, then changes in payments are easy to identify from the difference in generator costs. When there is a duality gap, then the redistribution of payments comes from a reshuffling of commitment and dispatch decisions which will benefit some generators while disadvantaging others. The latter case is more difficult to detect because there may be no change in the schedule's total cost.

When convex relaxations of (1) (such as `td` and `ld`) are used as pricing models, Corollary 1 states that tightening the convex relaxation will also tighten the payment redistribution bound. In contrast to Theorem 1, the only information about the optimal UC solution required by Corollary 1 is a lower bound on the cost of the optimal UC schedule, z^{LB}, and it also does not require an evaluation of the Lagrange function, $L(\lambda)$, which is a large but decomposable MIP problem.

The bounds in Theorem 1 and Corollary 1 do not apply to pricing mechanisms that are in any way dependent on the UC schedule, such as for the `r` and `pd` pricing models. For such models, we were unable to define bounds that could be computed without first solving an optimal UC schedule. However, results in Sections IV and V demonstrate that the actual redistribution quantities of the `r` and `pd` pricing models are often larger than upper bounds that can be calculated for the `td` and `ld` pricing models.

IV. Example

This section presents a simple example to illustrate how scheduling changes with little or no effect on total costs can disproportionately affect financial outcomes of market participants, that is, because there is a nonzero payment redistribution quantity (9). The example consists of three types of generators that have each been replicated five times, shown in Table II. The demand quantity is 225 MWh, plus a small perturbation $\epsilon > 0$ to prevent degeneracy. Let $\mathcal{G}$ be the set of generators of each type $n \in \{1, 2, 3\}$ and replication $k \in \{1, \ldots, 5\}$. The single-period UC problem that implements (1) is written below.

TABLE II
Generator Attributes, Simple Example

Gen. $k \in \{1, \ldots, 5\}$	Min, $\underline{B}_{nk}$	Max, $\overline{B}_{nk}$	Cost, C_{nk}
Type 1 Output, p_{1k}	25 MW	25 MW	\$15/MWh
Type 2 Output, p_{2k}	0	25	10
Type 3 Output, p_{3k}	0	25	25

$$\min \quad z = \sum_{(n,k)\in\mathcal{G}} C_{nk} p_{nk} \tag{19a}$$

$$\text{s.t.} \quad \sum_{(n,k)\in\mathcal{G}} p_{nk} = 225 + \epsilon \tag{19b}$$

$$\underline{B}_{nk} u_{nk} \leq p_{nk} \leq \overline{B}_{nk} u_{nk}, \qquad \forall (n,k) \in \mathcal{G} \tag{19c}$$

$$u_{nk} \in \{0, 1\}, \qquad \forall (n,k) \in \mathcal{G} \tag{19d}$$

The optimal UC is simple enough to solve by hand. There are five optimal solutions; each entails scheduling four of the five Type 1 generators to their maximum output and scheduling the remaining Type 1 generator to zero. Prices are also easy to obtain by fixing or relaxing the appropriate binary constraints. The `r` and `pd` models set the price based on an ϵ dispatch from a Type 3 generator, so $\lambda^{\text{r},s} = \lambda^{\text{pd},s} =$ \$25/MWh. The `td` and `ld` models set the price based on an ϵ dispatch from a Type 1 generator. In fact, both the `td` and `ld` models calculate the exact convex hull price since the problem has strictly linear costs and there are no intertemporal constraints [29], so $\lambda^{\text{td}} = \lambda^{\text{ld}} = \lambda^{\text{ch}} =$ \$15/MWh. In each pricing model, the prices are constant across all five optimal solutions.

When the price is \$25/MWh, Type 1 generators make a profit of \$250 if committed or \$0 if left uncommitted. Type 2 generators each make a profit of \$375, and Type 3 generators break even or are not dispatched. As a result, the `r` and `pd` pricing models both result in profit redistribution quantities of \$500 because each alternative solution entails the the redistribution of profits from one Type 1 generator to another. Thus, the redistribution quantities are significant, being of the same order of magnitude as the profits themselves, in this case.

On the other hand, a \$15/MWh price causes Type 1 generators to make \$0 whether committed or not, Type 2 generators each make \$125 profit, and Type 3 generators receive, in total, a small make-whole payment of \$25ϵ to cover the cost of their physical dispatch. The cost of each solution is the same, and all market participants receive the same profit regardless of which schedule is selected by the ISO.

Since the `r` and `pd` models compute the same prices for each schedule, Theorem 1 implies that the resulting redistribution quantities are only possible because there is a nonzero duality gap when the price is $\lambda =$ \$25/MWh. At this price, the redistribution upper bound in (17) is \$1000. If the price is instead set to \$15/MWh (as in the `td` or `ld` pricing models), then the upper bound is \$40ϵ, i.e., arbitrarily small. The example therefore demonstrates that a redistribution of payments is possible even between two solutions with the same

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cost and same price vector, but only if that price vector results in a nonzero duality gap.

The redistribution of payments could plausibly influence bidder behavior. All ISO markets today allow self-commitments, i.e., allowing generators to pre-specify that they will be committed in the solution to (1). Solving a mixed strategy Nash equilibrium for the market described in Table II shows that `td` and `ld` models result in no strategic behavior, but the `r` and `pd` pricing models incentivizes strategic behavior that increases the schedule's cost by 1.4%, in expectation. Details are provided in an online expansion of this paper [38].

Small example problems like (19) can be helpful to illustrate concepts but can also be misleading or deliver contrived results. The small example in this section contains many identical generators and a negligible minimum duality gap, so it is admittedly susceptible to this objection. Accordingly, the following section presents similar results for a suite of more realistic test cases.

TABLE III
TEST CASE SUMMARY

Test Case	Num. Solns.	Min. $\frac{\delta^s_{\text{mip}}}{z^{LB}}$	Mean # Binding Flow Limits `r`	`pd`	`td`	`ld`
rts/spd/no	21	0.040%	0	0	0	0
rts/spe/no	21	0.050%	0	0	0	0
rts/sud/no	9	0.029%	0	0	0	0
rts/sue/no	2	0.014%	0	0	0	0
rts/wid/no	11	0.030%	0	0	0	0
rts/wie/no	19	0.046%	0	0	0	0
rts/spd/tx	20	0.046%	4.0	4.0	0	0
rts/spe/tx	12	0.057%	7.8	7.8	8	0
rts/sud/tx	10	0.029%	3.3	3.8	1	0
rts/sue/tx	6	0.021%	16.0	16.3	16	0
rts/wid/tx	12	0.033%	0.3	0.3	0	0
rts/wie/tx	21	0.061%	6.0	6.0	5	0
pjm/su/no	23	0%	0	0	0	0
pjm/wi/no	16	0%	0	0	0	0
pjm/su/tx	17	0%	399.6	392.6	393	326
pjm/wi/tx	15	0%	146.0	144.0	142	143

V. TEST CASES

Unit commitment, pricing, and market settlements were solved for a suite of test cases, listed in Table III. The first set (`rts`), from the IEEE 1996 reliability test sysem [39], [40], consists of 96 generators, and 24-hour load shapes for spring, summer, and winter (`sp-, su-, wi-`), and weekdays and weekends (`-d, -e`). It was solved with and without transmission limits (`tx, no`), for a total of 12 `rts` test cases. The second set (`pjm`), made available by FERC [32], consists of two 24-hour snapshots of the PJM day-ahead from summer and winter of 2009 (`su, wi`), each including about 1,000 generators. It was also solved with and without transmission limits (`tx, no`), for a total of four `pjm` test cases.

Each test case was either solved to a 0% optimality tolerance or terminated after a 1,000 second time limit. All feasible integer solutions found during the MIP solver's algorithm were saved if they met a 0.1% optimality tolerance at the end of the algorithm's execution, resulting in 164 `rts` solutions and 71 `pjm` solutions. In the following results, it will be assumed that z^* denotes the cost of the best known solution for test cases in which the MIP solver terminated before an optimal solution could be verified.

For computational efficiency, test cases with transmission limits were formulated using transfer distribution factor (PTDF) transmission constraints [41]. Transmission limits in the `rts` cases were reduced to 90% of their nominal values in order to induce transmission congestion. The last four columns of Table III show the average number of binding transmission constraints in each test case and pricing model.

A. Results Overview

Fig. 1 shows load-weighted hourly prices in each of the four `pjm` cases. The mean of those prices across all solutions is shown for all four pricing models, and bars for coefficient of variation (c.v.) are shown for the `r` and `pd` pricing models (c.v. is zero for the `td` and `ld` models). The summer and winter price curves are both typical for each respective season. Price variation tends to be highest near peak periods in both the `r` and `pd` pricing models. However, price variations can also persist throughout the day, as in Fig. 1a.

The `ld` model tends to result in lower prices than the other three models despite allowing prices to be set by fixed costs. On the other hand, `r`, `pd`, and `td` pricing models result in very similar prices on average, especially in the summer cases. Morning and evening peak prices diverge more significantly among the four pricing models, but without an obvious pattern. In the `pjm` test cases, the average energy payments by load were 1.76, 1.79, 1.76, and 1.71 times system cost for the `r`, `pd`, `td`, and `ld` models, respectively, leading to differences short-run generator profits.

Fig. 2 shows potential side-payment quantities for the `rts` and `pjm` cases, with make-whole payments (MWP, (5)) displayed as a fraction of the total lost opportunity cost (LOC, (4)). The various pricing models based on integer relaxation (`pd`, `td`, and `ld`) are often motivated by the desire to lower make-whole payments, and indeed, the `pd` and `td` models are mostly successful on this front. On average, although the `ld` model lowers the total make-whole payments in the `rts` cases, it increases total make-whole payments in the `pjm` cases due to the fact that it has lower prices in each of those cases, as Fig. 1 shows.

In both the `pjm` and `rts` test case sets, the `td` model lowers the mean total lost opportunity costs (and by extension, make-whole payments) to less than the make-whole payments resulting from the `r` pricing model. Lowering lost opportunity costs is an expected outcome for pricing models based on convex hull pricing [12]. An unexpected outcome is that the lowering of lost opportunity costs also has the effect of limiting the redistribution of payments that occur among near-optimal solutions, as shown in the next section.

B. Payment Redistributions

Because sub- and near-optimal solutions are a practical reality in ISO markets, market designers may prefer to adopt pricing models that redistribute less of the market surplus due to negligible cost differences in the chosen schedule.

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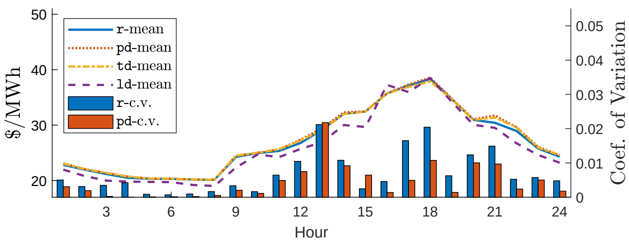
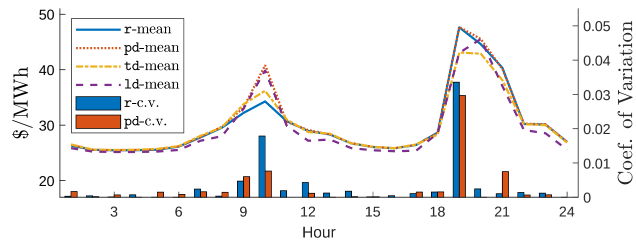
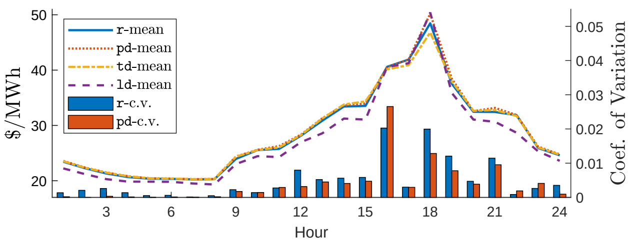


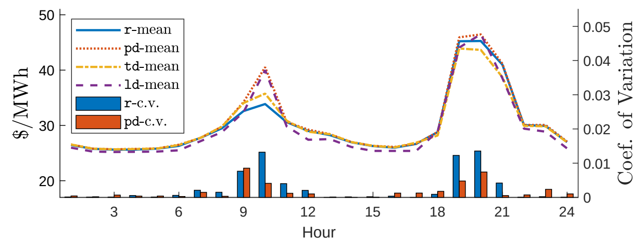
(a) Case `pjm/su/no` price mean and c.v.



(b) Case `pjm/wi/no` price mean and c.v.

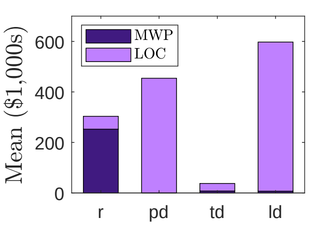


(c) Case `pjm/su/tx` price mean and c.v.

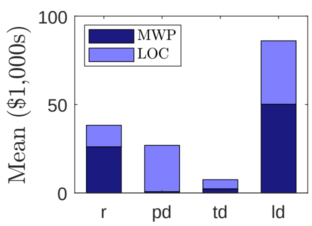


(d) Case `pjm/wi/tx` price mean and c.v.

Fig. 1. ISO-scale test cases from the `pjm` data set. Mean prices are similar in many hours, but `r` and `pd` include significant inter-solution variablity.



(a) `rts` cases



(b) `pjm` cases

Fig. 2. Make-whole payments, MWP=$\sum_n \mu_n^s(\lambda)$, and lost opportunity costs, LOC=$\sum_n U_n^s(\lambda)$.

Or, put differently, there may be good reason to consider whether the pricing model may cause some market participants to be significantly worse off because of small scheduling inefficiencies, among other market design considerations.

The redistribution of payments between near-optimal solutions can become very complex; it is affected not only by changes to the price vector, but also changes to generator schedules, side-payments, and, as shown in Section III, the presence of a duality gap. According to (7), payments may be redistributed from consumers to generators, from generators to consumers, and possibly from generators to other generators.

Table IV shows the average value of Δ^{ms}, (9), for all `pjm` and `rts` test cases. The payment redistribution quantities in the `rts` cases were much larger than in the `pjm` cases, on average. However, in both sets of test cases, `td` pricing model's bounds from (18) are relatively tight and thus the payment redistribution quantities are quite small.

The `td` pricing model's theoretical bounds from (18), reproduced here, can be used as a common basis of comparison of the other pricing models.

$$\widehat{\Delta}^s := 2(z^s - z^{LB}) + 4(z^s - z^{\text{td}}) \tag{20}$$

TABLE IV
MEAN PAYMENT REDISTRIBUTION

	Mean Δ^{ms}/z^{LB}, (%)			
Test Case Subset	`r`	`pd`	`td`	`ld`
`rts` cases	10.1%	25.1%	0.12%	2.28%
`pjm` cases	0.80%	0.48%	0.01%	0.02%

While only the `td` pricing model will guarantee $\Delta^{ms} \leq \widehat{\Delta}^s$, comparing all pricing models to the `td` model's bound provides for a comparison that controls for the possibility that the redistribution of payments may be larger in lower quality solutions. The proportion of solutions that satisfy some multiple of this bound, $\tau\widehat{\Delta}^s$, for some $\tau > 0$, is then used to compare each pricing model's relative effect on the redistribution of payments. This proportion will be called $\Delta^m(\tau)$ and is calculated as follows,

$$\Delta^m(\tau) = (1/S)\sum_s 1_{\{\Delta^{ms} \leq \tau\widehat{\Delta}^s\}} \tag{21}$$

where S is the number of sampled solutions and $1_{\{.\}}$ is a counting operator.

Fig. 3 shows the proportion of solutions that satisfy the bound $\tau\widehat{\Delta}^s$. As must be the case, `td` model satisfies the bound in all solutions (shown by the vertical line at $\tau = 1$), and in fact its redistribution quantities are about 10% of the bound, at most. On the other hand, the majority of redistribution quantities from `r` and `pd` pricing models, which are not bounded Theorem 1 or Corollary 1, do not even satisfy the `td` model's worst case bound (at $\tau = 1$) and some are up to 10-100 times higher than the `td` model's bound.

Although the `ld` and `fd` models are both convex relaxations of (1), they result in very different payment redistribution quantities in the `rts` case. Corollary (1) suggests that pricing models that are looser relaxations of (1) can result in larger

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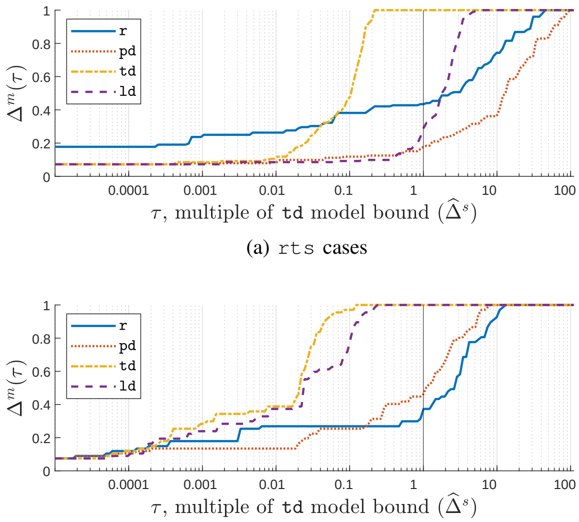
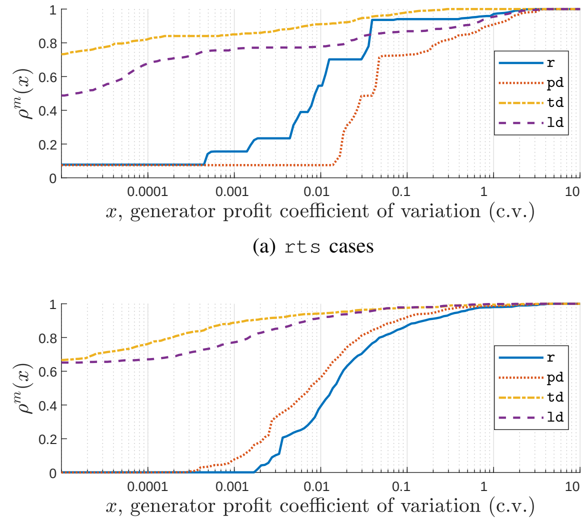


(a) rts cases

(b) pjm cases

Fig. 3. Payment redistribution incidence compared to multiples of the theoretical guarantee of the td model.



(a) rts cases

(b) pjm cases

Fig. 4. Fraction of generators with profit c.v. $\leq x$.

payment redistributions. Indeed, the td model's average integrality gap was quite small, 0.28% when averaged across all solutions in the rts cases, and the ld model's average integrality gap was significantly larger, at about 30%. According to Corollary 1, many of the redistribution quantities for the ld model would not be possible but for this difference in integrality gaps. No analysis was performed to determine which constraints (from [26]–[29]) were most responsible for the difference in integrality gaps, but an implication remains that achieving a tight relaxation of (1) is not a trivial task.

C. Effects on Individual Market Participants

Next, we show that the redistribution of payments does not affect all market participants evenly, but tends to have the largest effect on the profits of a small subset of participants.

To assess the variability of individual generator profits, we compute the coefficient of variation (c.v.) of each generator's profits. Sample mean and variance are computed from the pool of near-optimal solutions. Let $\bar{\pi}_n^m = \sum_s \tilde{\pi}_n^s(\lambda^{ms})/S$ and $(\sigma_n^m)^2 = \sum_s(\tilde{\pi}_n^s(\lambda^{ms}) - \bar{\pi}_n^m)^2/(S-1)$ be the mean and variance, respectively, of generator n's profit when prices are determined by pricing model m. The profit c.v. is defined as $\rho_n^m = \sigma_n^m/\bar{\pi}_n^m$, and we define the test case sample cumulative distribution as follows.

$$\rho^m(x) = \sum_n 1_{\{\rho_n^m \leq x\}} / \sum_n 1_{\{\bar{\pi}_n^m > 0\}} \quad (22)$$

Cumulative distributions of the c.v. of generator profits are shown in Fig. 4 for each pricing model. Profit variation is consistently low for settlements determined by the td pricing model. The ld model resulted in consistently low profit variation in the pjm cases but less so in the rts cases. The pd model produced high levels of profit variation in the rts cases, some exceeding 1 (i.e., standard deviation greater than mean profits).

Because ISO markets are nonconvex, a single generator's profitability can be highly dependent on the characteristics of other generators in the market. Some generators may be complements, in that one is likely to be profitable when the other is also profitable, or they may be substitutes if the opposite occurs. However, knowing which generator characteristics will be associated with higher or lower profit variability requires a detailed analysis of a large number of UC solutions, and is left for future work.

VI. Conclusion

It has long been recognized that sub-optimal solutions can have significant distributional implications in markets with nonconvexities, and unit-commitment based electricity markets in particular [13], [14]. What hasn't been explored is whether those implications are very different among alternative methods for determining prices and settlements in such markets.

Results in this paper demonstrate that, indeed, different pricing models can result in payment redistributions that are significantly different in magnitude. This was shown for a suite of test cases, showing that some pricing models can result in a sizable redistribution of payments, but, unlike previous analyses, other pricing models often result in almost no redistribution of payments compared to settlements in an optimal UC schedule. In particular, the td pricing model, a tight convex relaxation of the UC problem (1), resulted in almost no redistribution of payments in any of the test cases that were attempted.

The paper's theoretical contributions show that such results should be expected for any pricing model that is a tight convex relaxation of the UC problem. More specifically, if energy prices are determined by a convex relaxation of the UC problem, then a redistribution of payments can only occur if there is an optimality gap and/or duality gap, and the magnitude of the redistribution can be bounded by a weighted sum of these two quantities. These bounds remain valid for a broad range of side-payment policies, including make-whole

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payments or any portion of lost opportunity costs, that might accompany market settlements.

The paper's theoretical results do not apply to pricing models that are dependent on the UC schedule, such as the `r` and `pd` pricing models. In that case, we were unable to define bounds that could be calculated without first solving the optimal UC schedule, and test case results were consistent with previous analyses.

A larger question is whether the adoption of a new pricing policy is likely to improve market efficiency. To help answer this question, Theorem 1 and Corollary 1 in this paper may be useful tools for future analyses of equilibria in electricity markets, as the payment redistribution bound also places an upper bound on the incentives for certain types of strategic behavior. Further analysis is required to determine if this strategic behavior is actually possible or likely in a realistic test case.

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